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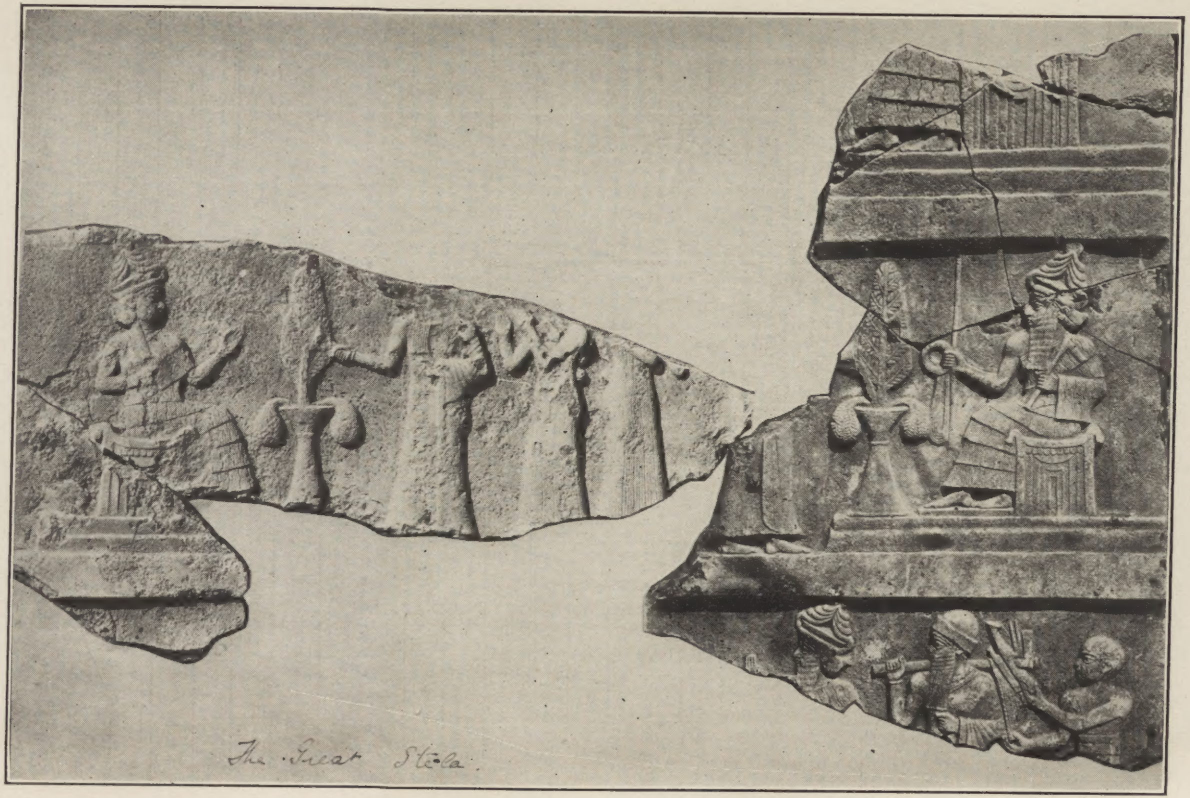
MASONRY STRUCTURES



The Great Stela of King Ut-Engur, Showing Workmen Carrying Tools to Build the Tower of Ziggurat in Mesopotamia.
One of the Most Recent Excavations of One of the Most Ancient Masonry Structures—2300 B. C.
(By Courtesy of the Joint Expedition of the British Museum—University Museum, Philadelphia.)

[*Frontispiece*]

MASONRY STRUCTURES

BY

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SECOND EDITION, REVISED AND ENLARGED

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PREFACE TO SECOND EDITION

In the preparation of the Second Edition of this work, the aim has been to preserve the author's individuality and to adhere as closely as possible to his original plan; to simplify, to clarify, and to extend the text to meet the requirements of the many users of the book.

It will ever be the fate of text-books to lag slightly behind current practice. The author is loath to leave unmentioned usages which may have been abandoned in one part of our large country while they still persist in another. Newer forms of construction, such as reinforced concrete, are certain to change rapidly in the early stages of their development, but gradually they take on standardized methods of handling and changes become less radical and less frequent. The latest report of the Joint Committee of Standard Specifications for Concrete and Reinforced Concrete (October, 1924) has evoked considerable discussion and has brought out an unusual number of valuable suggestions. When adopted in final form, it will serve for a much longer period than any of its predecessors.

Thanks are due to many users of the book for helpful suggestions, especially to Professor Perry M. Teeple of the University of South Carolina for many corrections of typographical errors, and to Professor S. D. Sarason of Syracuse University for constructive criticism.

A. Lincoln Hyde.

Columbia, Missouri,
January, 1926.

PREFACE TO FIRST EDITION

This book is designed to present, in a brief and systematic manner, the fundamental principles involved in the design and construction of masonry structures.

The term Masonry has been construed to include concrete, and the field covered by the title is a very wide one. It has therefore been necessary to select for discussion those types which seem most adequately to illustrate the principles, and no attempt has been made to cover fully the details of all classes of masonry structures. The purpose has been to provide an introduction to the subject, which may later be followed by intensive study in more detailed works upon the various branches. This gives a general view of the subject as a whole, and is the natural method of approach.

The Author has derived much assistance from a number of books which deal more fully with various portions of the subject. These are mentioned at the ends of articles or chapters to which they specially relate. They should be studied by students desiring a more complete presentation of the subject.

Special acknowledgment is also due to Professors A. Lincoln Hyde and Guy D. Newton of the University of Missouri for reading and criticising portions of the manuscript and for assistance in preparing the illustrations.

F. P. Spalding.

Columbia, Missouri,
September, 1920.

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MASONRY STRUCTURES

CHAPTER I

DEVELOPMENT OF MASONRY CONSTRUCTION

ART. 1. INTRODUCTION

1. Definition.—The term *masonry* in its original significance means "a construction of dressed or fitted stones and mortar." It is thus properly limited to stone masonry. Custom has, however, extended the use of the term to cover any construction composed of pieces of inorganic non-metallic material fitted together into a monolithic block. This includes all structural work in stone, brick, terra-cotta, and tile, as well as concrete construction.

The word brick was formerly used to designate a small block of burned clay. Similar blocks of other materials have recently come into use, and we now have several kinds of bricks—as clay brick, sand-lime brick, cement brick, etc. Glazed and other ornamental and surfacing tiles are commonly employed, while hollow tiles of various kinds are rapidly coming into use. All construction formed of bricks or tiles cemented together may be classed as *brick masonry*.

The term *stone masonry* is used to designate any work in which stones are fitted and cemented together so as to form a structure. Stone masonry is further subdivided into rubble masonry, squared-stone masonry, and ashlar or cut-stone masonry.

Concrete is ordinarily formed by mixing broken stone or gravel with cement mortar to a mobile condition and placing it in forms in the position in which it is to be used. It is then allowed to harden and forms a monolithic block.

Ordinary concrete cannot economically be employed where tensile stresses are developed in the structure on account of the low tensile resistance of the concrete. It is therefore common, when it is desired to use concrete in such situations, to embed steel rods in the concrete to take the tensile stresses, relying on the concrete to carry compression only. This construction is known as *reinforced concrete*.

2. Uses of Masonry.—Masonry in some form is now used in nearly every kind of engineering and architectural construction. The selection of the type of masonry to be used in any particular structure ordinarily is largely a matter of cost. This factor depends upon the suitability of the construction to the use which it is to serve, and the availability and costs of the necessary materials and labor. These are subject to local variation and need to be considered in each instance.

Brick masonry is largely used in the construction of buildings, being usually cheaper than stone, and when of good quality, showing both strength and durability. Very pleasing architectural effects are readily obtained by proper selection and arrangement of materials in brickwork. Brick masory is frequently used in the construction of large sewers and in the arch ring of small arched bridges, and is readily adapted to such uses, but is gradually giving way to concrete.

Hollow-tile construction is being quite commonly applied in building operations, and is replacing ordinary brickwork in many instances. It is sometimes faced with brick in exterior walls, and is used for partitions and in solid floor construction on account of its lightness and low cost.

Stone masonry is largely used in architectural construction, where the appearance and permanence of the structure are of special importance. It is almost universally employed in monumental construction, being at once the most durable material known to man and the one capable of producing the most imposing and most beautiful effects.

Many engineering structures such as retaining walls, bridge piers and abutments, and arch bridges are often constructed of stone masonry, or are faced with stone. Concrete is, however, gradually replacing stone masonry for such work on account of lower cost and facility of construction, except where facing of stone is used for appearance or durability.

Concrete is almost universally employed in foundations, having replaced stone masonry for this purpose. In the construction of tunnels, subways, and other underground work, it is usually the cheapest and most convenient material. In heavy masonry, such as retaining walls, dams, piers, and abutments, concrete is commonly used alone, or with a facing of stone masonry.

The use of reinforcement makes it possible to apply concrete in many types of construction to which masonry has heretofore been inapplicable. For short-span bridges reinforced concrete is rapidly replacing wood and steel, and on account of its durability, is a much

more economical material for such use. Reinforced concrete is extensively used in fireproof building construction for floors, beams, and columns, and is frequently used in connection with hollow tile for this purpose. It is sometimes used for the walls of buildings, but is apt to be more expensive than brick, on account of the forms necessary in such work.

ART. 2. EARLY HISTORY

3. Ancient Masonry.—The art of masonry construction dates from the earliest records of authentic history. The most fruitful source from which to obtain a knowledge of the history of the more ancient peoples is a study of the remains of their masonry structures.

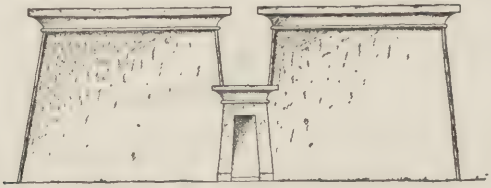


FIG. 1.—Egyptian Temple.

The earliest important constructions of which we have any remains are probably those of Egypt [1] (about 2700 B.C.), Chaldea [2] (about 2000 B.C.), and Assyria [2] (about 1600 B.C.). Alongside of these are the remains of the second Babylonian Empire, founded about 600 B.C. The dates of few of them are known with accuracy. Timber was scarce in Egypt. Granite, basalt, and limestone were to be had from the mountain ridges to the east and west of the Nile, but they were difficult to quarry, transport, and dress, precluding their use for structures other than mausoleums and temples. Nubian sandstone was easier to work, but this fact was not learned until later. Clay was abundant and naturally became the common material of construction. The earliest rude huts were no doubt made of mud or clay, with outer walls diminishing in thickness toward the top, a style which persisted in Egyptian architecture. It was discovered that wet clay could be molded into crude bricks which could be dried or baked by the heat of the sun, and that by their

[1] "The Manners and Customs of the Ancient Egyptians," by Sir J. Gardner Wilkinson, D.C.L., F.R.S., F.R.G.S.

[2] "The Five Great Monarchies of the Ancient Eastern World," by George Rawlinson, M.A.

use a better class of construction could be obtained. Bricks which were made of mud from the Nile required straw as a binder to prevent them from cracking, while those which were made of clay from the torrent bed on the edge of the desert did not require the straw. As the sun-dried bricks were rarely exposed to rains, they served for private dwellings, granaries, enclosing walls, and similar structures. The manufacture of bricks became a government

Fig. 2.—Egyptian Construction Showing Use of Column and Entablature. A Corner of the Second Court of the Temple of Ramses III at Medinet Habu (Thebes, West Bank)—about 1200 B.C.
(Courtesy of Prof. Walter Miller, A.M., LL.D.)

monopoly and the bricks were marked with the name of the reigning monarch. More bricks are found bearing the name of Thotmes III (about 1600 B.C.) than of any other ruler. The sun-dried bricks of Egypt were of various sizes, some of them being from 16 to 20 in. square and from 3 to 4 in. thick. Mud from the Nile was used for filling the joints. Of the seventy-five pyramids which are known, three were built of brick. They were evidently originally cased with stone, but as the structures of one dynasty became the quarries of later dynasties, the casing disappeared. The granite facing of some of the stone pyramids was " borrowed " for use in later struc-

tures. Kiln-burned bricks are found in Egypt, but they are believed to date back only to the Roman occupation, about 44 B.C.

The public structures, that is, the mausoleums and temples, were built of stone, the earlier ones of Egyptian limestone and the later of sandstone from Silsilis. The ancient Egyptians acquired extreme skill in working stone. Their structures were built of large blocks, well squared and dressed. The blocks were from 2 to 5 ft. thick, and some of them as much as 30 ft. long. The weight of the stones and the excellence of the workmanship obviated the necessity of mortar for the joints. The Egyptians quarried and transported large blocks of granite

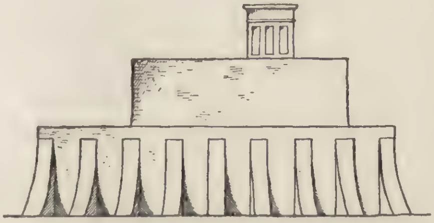


FIG. 3.—Mugheir Temple—Restored. (Rawlinson.)

FIG. 4.—Typical Chaldean Temple Construction of Sun-dried Bricks Faced with Kiln-burned Bricks—about 2500 B.C. (Restored by Ch. Chipiez.)

great distances, and they carved and polished this material. The great pyramid has a base 764 ft. square, and is approximately 482 ft. high. It is estimated that it required the labor of 35,000 men for thirty years to complete it.

The Egyptians developed a massive style of architecture for their temples in which the column and entablature were emphasized. Figure 1 shows the type of temple built by the Egyptians, while Fig. 2 illustrates the style in detail.

Chaldea was deficient in timber, minerals, and stone, and was entirely destitute of metals. Sandstone was to be had in adjacent Arabia and basalt in the more distant parts, but transportation across marshy land and the lack of tools for dressing the stone prevented the common use of either of these materials. Occasionally stone was carried by boat up the river, but to an extremely limited extent. As in Egypt, clay was plentiful, and was used for public as well as for private buildings. In the earlier and more crude structures sun-dried bricks of rough form were used; later kiln-burned bricks were employed. In some of the earlier buildings both kinds are found, the kiln-burned having been used as a facing to protect the sun-dried. The burned brick facing in some instances was as much as 10 ft. thick.

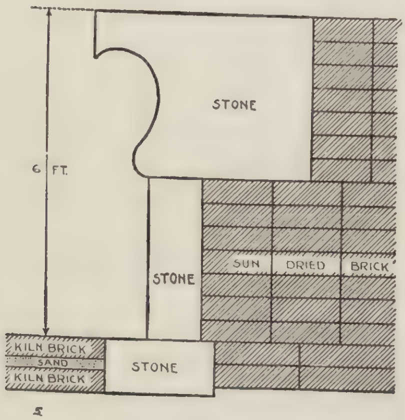


FIG. 5.—Typical Assyrian Temple Construction. Detail Showing Use of Sun-dried Bricks Faced with Limestone.

The kiln-burned bricks varied from 11 to 13 in. square and from $2\frac{1}{2}$ to 3 in. thick, the sun-dried being somewhat larger. The burned bricks varied in color, the earliest being pale red; others were blue-black, and some a light yellow similar to modern fire-brick. The kiln-burned bricks of the earliest times are still found to be sound and hard, and many of the sun-dried retain their shape and original character, in some cases offering stubborn resistance to archaeological investigators.

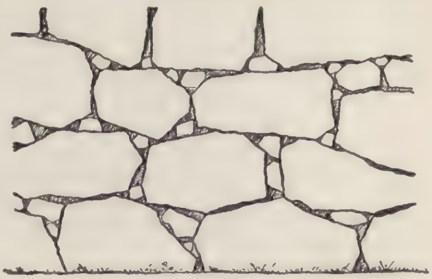


FIG. 6.—Cyclopean Masonry.

The cementing material commonly used in Chaldea was clay mixed with chopped straw for the sun-dried bricks, while bitumen,

which was found in inexhaustible pools, was used for the kiln-burned. Excavators have found bricks so firmly united with bitumen that they can with difficulty be separated. The type of temple built by the early Chaldeans is shown in Fig. 3. This is a restoration of the temple at Mugheir (Ur of the Scriptures). The third story is conjectural. The name Mugheir means "the bitumened." The style of temple constructed by the later Chaldeans shows considerable advancement, and is illustrated in Fig. 4.

Fig. 7.—Polygonal Masonry.

The Assyrians were plentifully supplied with both limestone and clay, but they followed the custom of the Chaldeans in the use of sun-dried and kiln-burned bricks, no doubt for economical reasons. In some cases limestone was used as facing, and stone headers extending into the crude brick body of the construction have been disclosed. Fig. 5 is a detail sketch showing the method of using limestone for facing. The style of architecture developed by the Assyrians was similiar to that of the Chaldeans, though there were deviations. The great halls of their temples were freely ornamented with sculptures; the entire walls, in some cases to a height of 10 or 12 ft., were covered with figures in relief, representing scenes from life, and usually commemorating the greatness of the monarch for whom they were erected.

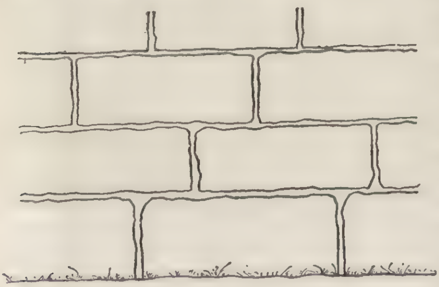


Fig. 8.—Rectangular Masonry.

In the later Babylonian construction the character of materials shows further advancement, and elaborate ornamentation was accomplished by carving and enameling bricks, and by the use of colors. One writer states that lime mortar was used.

In the ancient Grecian archipelago the Myceneans, a primitive people of Pelasgic origin, antedated the true Greeks. Three distinct styles of Pelasgic masonry have been observed, and all three were used in building the walls of the city of Mycenae. The earliest was of a crude type, consisting of huge stones of irregular shape, the

spaces between them being filled with smaller stones without mortar. This is known as Cyclopean and is shown in Fig. 6. The next in point of age was the "polygonal," in which the stones retained the irregular shape, but were carefully dressed and fitted as illustrated in Fig. 7. The third shows the Egyptian influence with blocks more nearly rectangular in form and laid in horizontal courses as shown in Fig. 8. A portion of the Mycenean wall in which the Cyclopean

FIG. 9.—Grecian Construction Showing Primitive Use of the So-called "Tirynthian Arch." Window in the Southeast Wall of the Cyclopean Fortifications of Mycenae. (About 1500 B.C.)
(Courtesy of Prof. Walter Miller, A.M., LL.D.)

style was used is illustrated in Fig. 9. The window opening is spanned by a false or Tirynthian arch.

True Grecian construction dates from the age of Pericles, about 429 B.C. The Greeks received their inspiration from the Egyptians. They used rectangular stones laid in horizontal courses, closely jointed, and equal to the best Egyptian workmanship. They developed the carving of artistic forms to the highest degree of excellence. The beauty of their moldings and sculptures has never been

surpassed. The Greeks introduced the pediment and improved the artistic design of buildings, bringing the proportions of such structures to a most wonderful perfection. Figure 10 shows the use of the pediment.

In Etruria, in central Italy, there grew up apparently from the same Pelasgic root, but independently, an art of building construction which is said to have been inferior in every way to that of the Greeks. Etruscan masonry closely paralleled the Mycenean through

FIG. 10.—Grecian Construction Showing Use of Pediment. The Treasury of Athens on the Sacred Way at Delphi. Erected by the Athenians in the Latter Part of the Sixth Century, B. C.
(Courtesy of Prof. Walter Miller, A.M., LL.D.)

the Cyclopean, polygonal, and rectangular, but the extreme refinement which was developed in Greece was not attained in Etruria. The Etruscans were of northern origin, but they were influenced by the Asiatics. The voussoir, or true arch, which had been used by the Egyptians, Chaldeans and Assyrians, for spanning minor openings, was adopted and developed by the Etruscans, and passed on by them to the Romans.

4. Roman and Medieval Construction.—The art of masonry construction was greatly advanced during the Roman period. The

Romans were influenced both by the Etruscans on the north and the Greeks on the south. They took the voussoir arch from the formei and carried it to a high state of development. The introduction of the arch changed the whole system of construction. In Romanesque architecture, the circular arch was the principal feature, the buildings consisting mainly of heavy masonry walls supporting semicircular arched roofs.

FIG. 11.—Roman Construction Showing the Use of the Semicircular Arch. Aqueduct at Troy. Built about the Second Century, A.D. (Courtesy of Prof. Walter Miller, A.M., LL.D.)

While lime mortar is said to have been used in Babylonian construction, it was not brought into general use in Rome until two or three centuries before the Christian era. Later the Romans discovered that Pozzuolan, a silicate of alumina of volcanic origin, found at Pozzuoli near the foot of Mount Vesuvius, when added to lime mortar imparted to it the property of hardening under water. This discovery no doubt brought about the first use of concrete, which later became one of their most extensively used building materials both at home and abroad. Their concrete was composed of sand, lime, Pozzuolan, and water, together with broken stones, bricks, or tiles. Their walls were built of rough cemented rubble or coarse

concrete, and were usually faced with brick or marble. Sometimes, in less important construction, small blocks of tufa, set irregularly, formed the surfaces of the walls, which were stuccoed on their interior surfaces. Roman arches, which were used for buildings, bridges, viaducts and aqueducts, were constructed of brick, cut stone, or concrete. Figure 11 shows the use of the semicircular arch in aqueduct construction.

The pointed as well as the circular arch had been used in Egypt and Asia for spanning openings of 15 feet or less, but the pointed arch did not gain a foothold in Europe until the middle of the twelfth century A. D. At that time arched ribs for roofs, with piers and buttresses to transmit the loads to the foundations, came into use in the construction of numerous cathedrals. The new form was opposed by the adherents of the classic style of Greece and Rome. The Goths, who had invaded Italy some centuries before, being held in contempt by the inhabitants of that country, the architects of the Italian Renaissance gave vent to their opposition by applying the term Gothic to the new type of construction. The epithet was entirely inappropriate. The Goths had nothing to do with its introduction or development, and time has proved it to be one of the noblest and most complete styles of architecture ever produced. In Europe the term "Pointed" has been adopted as a substitute, and among English-speaking people, the term Gothic has come to signify that which is beautiful, delicate, and graceful in building construction. The new style made possible a better disposition of materials, and led to more economical construction.

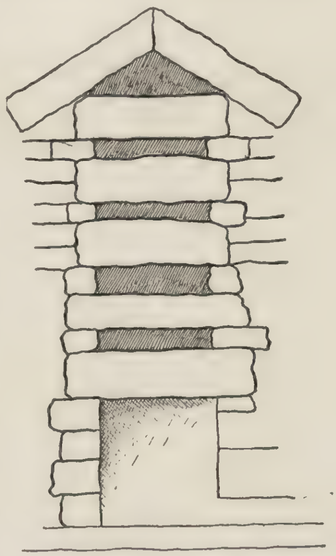


FIG. 12.—Cheops Pyramid. (Cross-section of Interior Chamber.)

During medieval times the use of stone masonry was brought to a high state of perfection. Random ashlar or rubble was commonly used in buildings in preference to coursed ashlar. Beautiful and imposing effects were obtained by the use of materials of comparatively small size, and great skill was developed in the cutting of artistic and ornamental forms.

5. Development of the Arch.—The origin of the arch is unknown. Evidence of its use dates back to the earliest times. It is known that among the vegetable products of the ancient eastern countries there were enormous reeds rising often to heights of 14 to 15 feet. The natives of the marsh regions are believed to have formed their habitations from these reeds, binding their stems together and bending them into arches to make the skeletons of their houses. This may have been the first suggestion of the arch.

Wilkinson maintains that the use of bricks led to the invention of the arch, the lack of timber having pointed out the necessity for some substitute for wood in spanning openings. Examples of the semicircular voussoir or true arch built of bricks are found in the small chapels at the foot of the pyramids and in the ruins of ancient granaries. But the simplest form of the true arch, that of two stones bearing obliquely against one another, was used in the Cheops pyramid to relieve the load on the ceiling of the interior of the chamber, as shown in Fig. 12. The voussoir arch in its flat form was also used at a very early date as illustrated in Fig. 13, which is a sketch showing the restoration of a stone arch roof over a tomb near the pyramids and believed to have been built during the fifth dynasty, about 2700 B. C. The pointed voussoir arch of kiln-burned bricks of rectangular shape has been found spanning an opening for a drain, as shown in Fig. 14. In other cases wedge-shaped bricks were molded especially for use in arch construction.

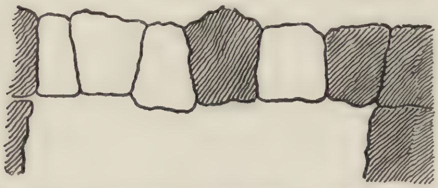


Fig. 13.—Primitive Flat Arched Roof of Tomb Near Pyramids—about 2500 B.C. (Wilkinson.)

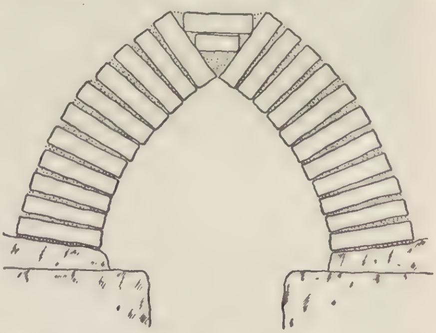


Fig. 14.—Ancient Assyrian Pointed Arch over Drain.

Examples of the false or corbeled arch are found in both brick and stone in the ancient eastern countries. In some cases the lower corners of the stones are cut away to simulate the true arch form. Famous examples in stone are found at Tiryns and Mycenae in Greece, where this type is given the name of Tirynthian arch. See Fig. 9, p. 8.

ART. 3. RECENT DEVELOPMENTS

6. The Cement Industry.—The discovery by the Romans of the hydraulic properties of volcanic lava, and the location of other materials possessing the same properties, made possible the construction of subaqueous masonry work. No considerable progress, however, was made in such work.

About the middle of the eighteenth century Smeaton, a noted English engineer, discovered that lime made from certain limestones containing clay possessed hydraulic properties. This discovery opened new possibilities in under-water work, and these hydraulic limes were used to a limited extent during the next half century.

In 1796 James Parker, an Englishman, burned limestone containing a larger proportion of clay and ground the product. He thus produced the first natural cement, which he called Roman cement. This process was patented, and the manufacture of natural cement resulted.

In 1818 Canvas White, an engineer of the Erie Canal, located rock suitable for making natural cement in Madison County, New York, and the first cement produced in the United States was made in the same year. Five years later the manufacture of natural cement was begun at Rosendale, New York. The production of cement in this region extended, and cement was thus provided for most of the hydraulic construction in this country for a considerable period. Later, as demands for cement increased, deposits of cement rock were found at many other places. Natural cement plants were established along the James River in Virginia; in the Lehigh Valley, in Pennsylvania; at Louisville, Kentucky; Utica, Illinois; Milwaukee, Wisconsin; and a number of other localities.

In 1824 Joseph Aspdin, of Leeds, England, discovered that by burning a mixture of slaked lime and clay at high temperature, hydraulic cement was produced. Aspdin named this material *Portland Cement*, on account of its resemblance to Portland stone, then largely used in England. In 1845 the manufacture of Portland cement was begun on a commercial scale by J. B. White & Sons, in Kent.

During the period between 1830 and 1850 Vicat, in France, made a number of studies which were of great value in extending knowledge of the new material. Plants were soon established in France and Germany for the manufacture of Portland cement, and the industry became an important one throughout Europe. During the next few years, 1865 to 1880, John Grant made a series of investigations of the properties of Portland cement and methods of using

it in mortars and concrete. His papers before the Institution of Civil Engineers had a marked influence in shaping the methods of use of cement.

From 1880 to 1900 the Portland cement industry developed rapidly in Europe, and numerous studies were made concerning the composition and properties of the material. LeChatelier, Alexandre, Candlot, and Feret, in France, Tetmajer in Switzerland, Michaelis and Bohme in Germany, Faija in England, and a number of others, investigated all phases of the subject, greatly improving the quality of the cement and showing methods of employing it in construction to secure the best results.

In 1875 Mr. D. O. Saylor began the manufacture of cement at Coplay, Pennsylvania. From this beginning, the American Portland cement industry has developed. Great improvements in methods of manufacture and in the control of the character of the product have been made in this country. The studies of Newberry, Richardson, and others have contributed to definite knowledge of the proper composition of the material, while committees of the National Engineering Societies and many independent investigators have perfected methods of testing cement and of using it in construction.

This industry has now reached immense proportions in the United States, and the use of Portland cement has extended in all directions, modifying greatly both the types and methods of construction.

7. Reinforced Concrete.—In its early use concrete was commonly employed as a filler in heavy construction, and was not possessed of great strength. Walls of concrete were usually protected by facings of stone or brick masonry. In recent years, however, the availability of cementing materials of high grade has made possible the use of concrete in many classes of construction for which stone or brick masonry was formerly employed. The facility with which concrete may be applied to many uses makes it highly desirable material, and since the introduction of Portland cement its use has rapidly increased. This use has been further extended in the past few years by the development of reinforced concrete construction. In 1850 Lambot, in France, constructed a boat of reinforced concrete, and in 1855 patented his invention in England. François Coignet, in 1861, applied reinforced concrete to the construction of beams, arches, pipes, etc.

In 1861 Joseph Monier, a gardener of Paris, constructed tubs and small water tanks of concrete in which a wire frame was

embedded. In 1867 Monier patented his reinforcement, which consisted of a mesh formed of wires or rods placed at right angles to each other. He also exhibited some work at the Paris Exposition in the same year. Nothing came of this invention for a number of years, but in 1887 Wayss and Bauschinger published in Germany the results of an investigation showing the value of the Monier system, and giving formulas for use in design.

The next few years saw considerable development of this type of construction in Austria, and Melan, an Austrian engineer, invented a system of reinforcement for arches in which I-beams were bent. to the form of the arch and enclosed in concrete. (See Fig. 113, p. 397). Hennebique, in France, began making reinforced concrete slabs about 1880, and patented his system of slab reinforcement in 1892.

The first use of reinforced concrete in the United States seems to have been by Ernest L. Ransome, in 1874. The next year W. E. Ward constructed a building in New York, in which reinforced concrete walls, roof, and floor beams were used. In 1877 H. P. Jackson used reinforced concrete in building construction in San Francisco. About 1884 Ransome began applying reinforced concrete to important work in California, and in that year took out a patent for the first deformed bar.

In 1894 the Melan system of arch-bridge construction was introduced into the United States by Mr. Fr. von Emperger, who built the first important arch bridges. At about the same time Mr. Edwin Thacher began the construction of arch bridges, using bar reinforcement.

During the period from 1890 to 1900 the use of reinforced concrete steadily increased, while the application of plain concrete had been extending rapidly, as the increasing supply of cement provided material for a better grade of construction.

Since 1900 the use of reinforced concrete has rapidly increased. The use of massive slab construction for railroad bridges was introduced by the C. B. & Q. Railroad at Chicago. Fireproof building construction of concrete has become common, and concrete has become the standard material for short-span highway bridges. Many investigations have been made concerning the properties of the materials and the strengths of various structural forms; the work of Considère, in France, and of Talbot at the University of Illinois, being specially notable. Principles for rational design have been established and recognized standards of practice are rapidly forming.

CHAPTER II

CEMENTING MATERIALS

ART. 4. LIME

8. Classification.—The cementing materials employed in the construction of masonry and concrete structures include *common lime, hydraulic lime, Portland cement, natural cement,* and *puzzuolan.* These materials are formed by the calcination of limestones, or of mixtures of limestones with siliceous or argillaceous materials, and their properties vary with the nature and porportions of the substances combined in them.

Common Lime.—When limestone composed of nearly pure carbonate of lime is burned, the resulting clinker, known as *quicklime,* possesses the property of breaking up, or *slaking,* upon being treated with a sufficient quantity of water. The slaking of lime is due to its rapid hydration when in contact with water, and the process is accompanied by a considerable increase in the volume of the mass of lime and by a rise in temperature. If the quantity of water be only sufficient to cause the hydration of the lime, the quicklime is reduced to a dry powder; while if the water be in excess it becomes a paste.

The slaked lime thus formed possesses the further property, when mixed to a paste with water and allowed to stand in the air, of hardening and adhering to any surface with which it may be in contact. This hardening of common limes will take place only when exposed to the air and allowed to become dry.

When lime is nearly pure and its activity very great it is known as *fat lime.*

If the lime have mixed or in combination with it considerable impurities of inert character, which act as an adulteration to lessen the activity of the lime, causing a partial loss of the property of slaking and diminishing its power to harden, it is known as *meager* or *poor lime.*

Hydraulic Lime.—When the limestone contains about 10 to 20 per cent of silica or clay mixed with the carbonate of lime, the

material resulting from the burning is known as hydraulic lime. This clinker will slake when treated with water like common lime, but with reduced activity. The slaked lime thus obtained possesses the further property, when mixed with water to a paste, of hardening under water and without contact with the air.

In hydraulic lime the silica and alumina are combined with a portion of the lime, forming compounds which harden under water, while part of the lime is left uncombined. This free lime expands when hydrated by addition of water, causing the material to slake.

Hydraulic Cement.—When the proportion of siliceous or argillaceous materials in limestone, or mixed with it, is sufficient to combine with all the lime, leaving no lime in a free state, the product of burning is known as hydraulic cement. This clinker will not slake, but must be reduced to powder by grinding. The cement powder, when mixed with water, has the property of setting and hardening under water, and of adhering firmly to any surface with which it may be in contact.

Portland Cement is the name given to hydraulic cement which is formed by burning and grinding an intimate mixture of powdered limestone and argillaceous matter in accurately determined proportions. In making Portland cement, the ingredients are carefully proportioned to secure the complete combination of the lime with the silica and alumina into active material, and it is necessary to reduce the materials to a very fine state and secure uniform incorporation of the ingredients before burning.

Natural Cements are made by burning limestones which contain proper proportions of argillaceous materials, and grinding the resulting clinker to powder. Natural cements are less rich in lime than Portland cements, complete combination of the argillaceous materials not being effected. They are burned, like lime, without the pulverization of the raw materials, and require a much lower temperature in burning than Portland cement.

The term *Pozzuolan* is commonly applied to a class of materials which, when made into a mortar with fat lime or feebly hydraulic lime, impart to the lime hydraulic properties and cause the mortar to harden under water. It derives its name from Pozzuoli, a city of Italy near the foot of Mount Vesuvius, where its properties were first discovered. It was extensively used by the Romans in their hydraulic constructions, being mixed with slaked lime for the formation of hydraulic mortar. Pozzuolan is essentially a silicate of alumina in which the silica exists in a condition to be attacked

readily by caustic alkalies, and hence easily combines with the lime in the mortar.

Pozzuolana Cement is formed by mixing slaked lime with Pozzuolan and grinding the mixture to a fine powder. Certain materials of volcanic origin are frequently used for this purpose in Europe, while considerable quantities of cement of this class have been made by the use of blast furnace slag, both in Europe and the United States.

9. Common Lime.—Common lime is such as does not possess hydraulic properties. It is divided into fat or rich lime and meager lime, according to the quantity of impurities of an inert character it may contain. When made into paste and left in air it slowly hardens. The process of hardening consists in the gradual formation of carbonate of lime through the absorption of carbonic acid from the air, accompanied by the crystallization of the mass of hydrated lime as it gradually dries out. In common lime the final hardening takes place very slowly, working inward from the surface, as it is dependent upon contact of the mortar with the air. When the lime is nearly pure the resulting carbonate is likely to be somewhat soluble, and consequently to be injured by exposure. Nearly all limes, however, contain small amounts of silica and alumina, and these ingredients, even when in quantities too small to render the lime hydraulic, impart a certain power to set, causing the hardening to take place with greater rapidity and without entire dependence upon contact with air. It also renders the material less soluble and more durable in exposed situations.

Nearly pure limes, consisting mainly of calcium oxide, are very caustic and become hydrated very rapidly when brought into contact with water. This hydration, or slaking, produces a rise in temperature and increase in volume, which vary in amount according to the purity of the lime, the volume being doubled or tripled for good fat lime. When the lime is derived from a magnesian limestone, it may contain a considerable proportion of magnesia mixed with the lime. Limes containing more than about 15 per cent of magnesia are usually called magnesian limes. The presence of magnesia has the effect of rendering the lime less active, causing it to expand less upon slaking. The magnesian limes harden more slowly, but usually gain a higher ultimate strength than the high-calcium limes.

The common method of slaking lime consists in covering the quicklime with water, using two or three times the volume of the lime. This method is known as drowning. The lime is usually

spread out in a layer perhaps 6 or 8 inches thick, in a mixing box, the water poured over it and allowed to stand. Sufficient time must be allowed for all of the lumps to be reduced. When the lime contains much foreign matter, the operation frequently requires several days. Too great quantity of water is to be avoided, the amount being such as will reduce the lime after slaking to a thick pasty condition. All the water should be added at once, as the addition of water after the hydration is in progress causes a lowering of temperature and checks the slaking. For the same reason, the lime should be covered after adding water, and not stirred or disturbed until the slaking is completed. The covering is often effected by spreading a layer of sand over the lime, the sand being afterward used to mix with it in making mortar.

A second method of slaking is sometimes employed having for its object the reduction of the slaked lime to powder, and known as slaking by immersion. This is accomplished in two ways. By the first method, the lime is suspended in water in baskets for a brief period to permit the absorption of the necessary water, after which it is removed and covered until slaking takes place and the lime falls to powder. By the second method, sprinkling is substituted for immersion, the lime being placed in heaps and sprinkled with the necessary quantity of water, then covered with sand and allowed to stand.

Lime is commonly sold as quicklime, and should be in lumps and not air slaked. When it is old and has been exposed to the air it is likely to have absorbed both moisture and carbonic acid, thus becoming less active, the portion combined with carbonic acid being inert. A simple test of the quality of quicklime is to immerse a lump for a minute, then place in a dish and observe whether it swells, cracks, and disintegrates, with a rise of temperature.

Slaking some days in advance of use is desirable in order to insure the complete reduction of the lime, and it is quite common to slake lime several weeks before it is to be used.

Common lime is ordinarily used in construction as a mortar, mixed with sand. The quantity of lime in the mortar should be just sufficient to fill the voids in the sand, without leaving any part formed entirely of lime. Mortar of rich lime shrinks in hardening, while masses composed entirely of lime on the interior are likely to remain soft, so that an excess of lime may be an element of weakness. If too little lime be used the mortar may be porous and weak. The proportions ordinarily required are between one part lime to two parts sand, and one part lime to three parts sand.

In mixing lime mortar, sand is spread over the lime paste and worked into it with a shovel or hoe. The proper proportions of sand and lime may be judged by observing how the mortar works. If too much sand be used it will be brittle, or " short "; while too much paste will cause it to stick and cake so that it will not flow from the trowel.

Mortar of common lime should not be employed in heavy masonry or in damp situations. Where the mass of masonry is large, the lime mortar will become hardened with great difficulty, and after a long time. The penetration of the final induration due to the absorption of carbonic acid is very slow. The observations of M. Vicat showed that carbonization extended only a few millimeters the first year and afterward more slowly. The induration of the lime along the surfaces of contact with a harder material is usually more rapid than in the interior of the mass of lime, and the strength of adhesion to stone or brick is often greater than that of cohesion between the particles of mortar.

10. Hydraulic Lime.—Hydraulic lime is obtained by burning limestone containing silica and alumina in sufficient quantities to impart the ability to harden under water. The hydraulic elements are present in such quantities that they combine with a portion of the lime, forming silicates and aluminates of lime, leaving the remainder as free lime in an uncombined state.

The hydraulic activity of a lime or cement, that is, its ability to harden under water, depends primarily upon the relative proportions of the hydraulic ingredients and of lime. Silica and alumina are considered to be the effective hydraulic ingredients, and it is common to designate the ratio of the sum of the weights of silica and alumina to that of lime in the material as its *hydraulic index*. The hydraulic index gives, therefore, within certain limits, a measure of the hydraulicity of the various classes of limes. It is to be remembered, however, that there are other factors to be considered in judging of the action of lime than this simple proportion. The other ingredients may by their combinations withdraw portions of the active elements so as to modify the effective ratio between them, while the activity of the lime depends largely upon the state of combination in which the active elements exist. This is not shown by analysis, and may be greatly modified by the manipulation given the material during manufacture.

Limes with hydraulic index less than 10/100 possess little if any hydraulic properties, and are known as common limes. When the hydraulic index is between 10/100 and 20/100 the lime is feebly

hydraulic, and may require from twelve to twenty days to set under water. Hydraulic lime proper includes that of index from about 20/100 to 40/100. These may harden in from two to eight or ten days.

The quantity of free lime in the material is dependent upon the degree of burning, as well as upon the amount of lime contained by the stone. If the stone be underburned, the combination of the hydraulic elements with the lime is not complete, and more of the lime remains in a free state. For this reason, a stone of high hydraulic index may, when underburned, yield a lime, but burned at a high temperature becomes unslakable. The best limes are usually those which can be burned at a high temperature to complete the chemical combinations. It is necessary that sufficient free lime be present to cause the lime to slake properly, but it is also desirable that the quantity of uncombined lime be as small as possible, as the setting properties are due to the silicates and aluminates, while the hydrated lime remains inert during the initial hardening of the mortar.

According to Professor LeChatelier, limestone for hydraulic lime should contain but little alumina, as the aluminates are hydrated during the slaking of the lime, while the silicates are not affected, the heat of the slaking preventing their hydration.

The following is given as an average analysis of the best French hydraulic lime:

Silica	22
Alumina	2
Oxide of iron	1
Lime	63
Magnesia	1.5
Sulphuric acid	0.5
Water	10
	100

It is important that the slaking be very thorough, as the presence of unhydrated free lime in the mortar while hardening is an element of danger to the work. Any lime becoming hydrated after the setting of the mortar may, by its swelling, cause distortion and perhaps disintegration of the mortar.

After the lime has been reduced to powder by slaking, it is forced through sieves which permit the passage of all pulverized particles but hold those of appreciable size, including the underburned rock

and the overburdened parts which refuse to slake. The residue left from the sifting of hydraulic lime is known as *grappiers*. This material is mainly composed of hard material more rich in silica and alumina that the other portions of the lime. The grappiers are frequently found and sold as cement, and when properly handled may form cement of fairly good quality.

11. Hydrated Lime.—When quicklime is slaked with the quantity of water necessary to completely hydrate it, and the resulting material is bolted to remove all unslaked particles, the result is a very fine white powder, commercially known as hydrated lime. This lime is sold on the market in barrels or bags, and it is in convenient form for use. Lime in this form may be kept for considerable periods without deterioration, provided it is protected from contact with moisture.

Hydrated lime ordinarily weighs about 40 pounds per cubic foot, and contains approximately 75 per cent of quicklime. By mixing with about an equal weight of water, it may be reduced to lime paste, or *lime putty*, as it is commonly called in building operations. Lime paste occupies a slightly greater volume than the hydrated lime from which it is prepared.

The use of hydrated lime for mixing with cement mortar in ordinary masonry construction is rapidly increasing. It is also frequently used in small proportions in Portland cement concrete to make the concrete flow more smoothly, and sometimes to decrease the permeability of the mortar. (See Art. 23.)

12. Specifications for Lime.—In ordinary building operations lime is commonly employed in the form of quicklime and slaked where used. Usually the quality of the lime has been judged by its activity in slaking and no particular tests are specified. Tests of composition by chemical analysis and of completeness of slaking by washing through sieves are, however, frequently employed.

Hydrated lime is now largely used for mixing with cement mortar and for plastering work, and this use is rapidly extending. The tests employed for hydrated lime include chemical analysis, fineness, permanence of volume or soundness, and consistency.

The American Society for Testing Materials has adopted standard specifications giving methods for making these tests. These specifications are given in the Book of Standards of the Society or may be obtained in pamphlet form from the Secretary of the Society. They have been revised and were adopted in amended form in 1924. (Serial Designation: C 6-24.)

ART. 5. HYDRAULIC CEMENT

13. Setting and Hardening of Cement.—When cement powder is mixed with water to a plastic condition and allowed to stand, it gradually combines into a solid mass, taking the water into combination, and soon becomes firm and hard. This process of combination among the particles of the cement is known as the setting of the cement.

Cements of different character differ very widely in their rate and manner of setting, some occupying but a few minutes in the operation, while others require several hours. Some begin setting immediately and take considerable time to complete the set, while others stand for considerable time with no apparent action and then set very quickly.

The points where the set is said to begin and end are necessarily arbitrarily fixed, and are determined by finding when the mortar will sustain a needle carrying a specified weight. The initial set is supposed to be when the stiffening of the mass has become perceptible; the final set, when the cohesion extends through the mass sufficiently to offer such resistance to any change of form as to cause rupture before deformation can take place.

After the completion of the setting of the cement, the mortar continues to increase in cohesive strength over a considerable period of time, and this subsequent development of strength is called the *hardening* of the cement.

The process of hardening appears to be quite distinct from, and independent of, that of setting. A slow-setting cement is apt, after the first day or two, to gain strength more rapidly than a quick-setting one; but it does not necessarily do so. The ultimate strength of the cement is also quite independent of the rate of setting. A cement imperfectly burned may set more quickly and gain less ultimate strength than the same cement properly burned, but of two cements of different composition the quicker-setting may be the stronger.

There is as wide variation in the rate of hardening of different cements as in the rate of setting; some gain strength rapidly and attain their ultimate strengths in a few weeks, while others harden much more slowly at first and continue to gain in strength for several years. The rate of early hardening gives but little indication of the ultimate action of the cement, as the final strength of the mortar may be the same however rapidly the strength is attained.

The rate at which cement sets seems to depend upon the pres-

ence of certain aluminates of lime, the rapidity of set increasing with the percentage of alumina in the material. The final hardening is attributed mainly to the silicates of lime, which are the important elements in giving strength and durability to the mortar. The formation of these active elements in the cement depends upon the manipulation of the material in manufacture, as well as upon the composition of the raw materials. In an underburned cement, the relative proportions of aluminates to silicates is large and the set is rapid.

Calcium Sulphate.—The addition of a small amount of sulphate of lime to cement has the effect of slackening the rate of set. Such addition is frequently made by manufacturers to reduce the activity of fresh cement, by grinding a small amount of gypsum with the cement.

Effect of Sand.—Cement is ordinarily employed in mortar formed by mixing it with sand, and the action of the mortar is necessarily largely affected by the nature and quantity of sand used.

When the cement is finely ground and the sand of good quality, a mortar composed of equal parts of each, as a general thing, finally attains a strength as high as, or higher than, that of neat cement. Cements of different characters, however, vary considerably in their power to "take sand" without loss of strength; some of the weaker ones may not be able to take more than half their weight of standard sand, while others can be mixed with considerably more than their own weight without loss of strength at end of six months or one year after mixing. All have a certain limit within which they may be made stronger by an admixture of good sand than they would be if mixed neat.

Clean and sharp sand usually gives higher strength in mortar than that containing admixtures of clay or earth, or that composed of rounded grains, coarse sand usually giving greater strength than that which is very fine. It is often difficult, however, to judge of the quality of sand without experimenting with it. In some cases a small amount of fine clay appears to increase the strength of mortar, while a judicious mixture in the sand of grains of various sizes may be of value in reducing the volume of interstices. Mortar composed of sand and cement usually possesses greater ability to adhere to other surfaces when coarse sand is used than when the sand is fine.

Effect of Water.—The quantity of water used in mixing mortar is one of the most important elements; the less the quantity, provided there be sufficient to thoroughly dampen the mass of cement,

the quicker the set. With some Portland cements, changing the quantity of water used in mixing from 20 to 25 per cent of the weight doubles or even triples the time required for the mortar to set.

When the quantity of water used in mixing is sufficient to reduce the mortar to a soft condition, the hardening as well as the setting becomes slow, and the strength during the early period is less than when a smaller quantity of water is used. This difference disappears to a considerable extent with time, and the mortar mixed wet may eventually gain as much strength as though mixed with less water.

Cement mortar kept under water hardens more rapidly in the early period than that exposed to the air. Nearly any cement mortar will harden more rapidly and gain greater strength if kept moist during the operation of setting and the first period of hardening than if it be exposed at that time to dry air. Sudden drying out about the time of completing setting causes a considerable loss of strength in cement mortar, and frequently the mortar so treated is filled with drying cracks. This result is usually more marked when the mortar has been mixed quite wet.

Effect of Temperature.—The temperature of the water used in mixing and that of the air in which the mortar is placed during setting has an important bearing upon the time required for setting; the higher the temperature, within certain limits, the more rapid the set. Some cements which require several hours to set when mixed with water at temperature of 40° F. will set in a few minutes if the temperature of the water be increased to 80° F. Below a certain inferior limit, ordinarily from 30° to 40° F., the mortar sets with extreme slowness or not at all, while at a certain upper limit, in some cements between 100° and 140° F., a change suddenly occurs from very rapid to very slow rate of set, which then decreases as the temperature increases until the cement ceases to set.

The temperature of the air or water in which the mortar is immersed while hardening has a very important effect upon the gain in strength. Heat accelerates the action, while at temperatures near the freezing-point of water the gain in strength is very slow.

14. Portland Cement.—The term Portland cement is used to designate material formed by burning to incipient fusion a finely ground mixture of definite proportions of limestone and argillaceous materials, and grinding the clinker so formed to fine powder. Several classes of materials are used for this purpose. Hard limestone or chalk, consisting of nearly pure carbonate of lime, is frequently employed, mixed with clay or shale to furnish the hydraulic ingredients. In the Lehigh District in Pennsylvania cement rock, con-

sisting of limestone containing silica and alumina in sufficient quantities to make natural cement when burned alone, is mixed with nearly pure limestone to obtain the proper Portland cement composition. In the Michigan district marl and clay excavated in soft and wet condition are used. In a few instances limestone is mixed with blast-furnace slag for the production of Portland cement. This is quite distinct from the manufacture of slag cement (so called) in which the materials are not burned together.

To make good Portland cement it is always necessary that the ingredients be very carefully proportioned and that the mixture be homogeneous. This requires the pulverization of the materials and their uniform incorporation into the mixture before burning.

The burning of Portland cement requires high heat to insure complete combination of the lime with the silica and alumina. In underburned cement, a part of the lime may be left as caustic lime, uncombined with the clay. This is apt to produce unsound cement, which may swell and crack after being used.

The action of Portland cement seems to depend upon the formation, during burning, of certain silicates and aluminates of lime which constitute the active elements of the cement, the other ingredients being considered impurities. The ideal cement would be that in which the proportion of lime is just sufficient to combine with all the silica and alumina in the formation of active material. If there be a surplus of clay beyond this point, it forms inert material. Any surplus of lime remains in the cement as free lime and constitutes one of the chief dangers in the use of cement, as, although it may not prevent the proper action of the cement when used, it may cause the mortar to swell afterward and become cracked and distorted as the lime slakes.

As perfect homogeneity is not attainable in practice, it is always necessary that the clay be somewhat in excess in order that free lime be not formed. The amount of excess of clay necessary depends upon the thoroughness of the burning and the evenness which may be reached in the mixture of the raw materials.

The normal composition of Portland cement is usually within the following limits:

Silica	20	to 25 per cent
Alumina	5	to 9 per cent
Iron oxide	2	to 5 per cent
Lime	59	to 65 per cent
Magnesia	0.5	to 3 per cent
Sulphuric acid	0.25	to 2 per cent

After the cement clinker resulting from the burning is sufficiently cooled, it is put through grinders and reduced to a fine powder. The degree of fineness to which the cement is ground is always very important in its effect upon the strength of mortar made from the cement. The valuable part of the cement is that which is ground extremely fine—to an impalpable powder. The coarse parts are not altogether inert, but are more or less active, depending upon the size of the grains of which they are composed.

Cement when used is commonly mixed with sand and the attainment of strength in sand mortar, rather than paste of neat cement, is of importance. The more finely ground the cement, the greater its resistance when mixed with sand, both in the earlier and later stages of hardening, and also the sooner will it reach its ultimate strength. The effect of fine grinding is much greater when the proportion of sand to cement is large, as the power of the cement to "take sand" without diminution of strength is thereby greatly increased. The coarser particles of the cement may be considered as practically inert material, which acts as sand rather than as cement in the mortar. The ability of the cement to harden and develop strength in sand mortar is thus dependent upon the amount of fine material contained in it.

Portland cement made from materials containing very small percentages of iron oxide are very light in color or white. These cements usually contain high percentages of alumina, and are consequently quick setting. They are lower in strength than normal Portlands.

15. Early Strength Cement.—About the middle of the year 1924 there was placed on the market in the United States a new cementing material known as Lumnite Cement. The principal ingredient is obtained from Bauxite, a high-grade aluminum ore. The chief feature of the new product is its ability to gain strength rapidly. For a 1 : 3 mortar in which standard sand was used, tests have shown in twenty-four hours a 95 per cent value of its twenty-eight day strength; and for 1 : 2 : 4 and 1 : 3 : 6 concretes in which the aggregates were sand and pebbles, about 77 per cent values were attained in the same period. The strength at twenty-four hours seems to be about the same as Portland cement concrete at from twenty-one to twenty-eight days. Though hardening rapidly, the setting properties and methods of mixing and handling are about the same as for Portland cement.

In situations where early strength is desired the new material is meeting a demand. Forms for foundations, walls, and abutments may be removed in twenty-four hours, while those for beams and

girders may be removed after three days. Floors and pavements may be used twenty-four hours after pouring.

It is also claimed for Lumnite Cement that it can be used in colder weather than ordinary Portland cement, and that it resists the chemical action of sea water and sulphate-bearing ground water.

A similar rapid hardening product, known as high-grade Portland cement, has been used successfully in Europe, principally in France, for more than ten years. Two plants were engaged in its manufacture in Germany in 1923, and so great is the confidence in the new material in that country that the number of plants has been increased to fifteen. Standard methods of testing the rapid hardening cements have not yet been developed.

16. Natural Cement.—The term *natural cement* is used to designate a large number of widely varying products formed by burning rock without pulverization or the admixture of other materials. These cements contain larger proportions of argillaceous materials, with less lime, than Portland cement, and are burned at a lower temperature.

The term *Roman Cement* is used in Europe to designate a class of quick-setting cements formed by burning, at a comparatively low temperature, limestone containing a high percentage of clay. The proportion of alumina in these materials is large and possibly accounts for the quick set. Materials of this character become inert when the temperature of burning is increased to the point where the chemical reactions would become complete.

A class of materials intermediate between the Roman cements and the Portland cements is called in Europe *Natural Portland Cement.* In composition they are similar to Portland cement, but contain less lime. They are burned at a higher temperature than Roman cements, and are usually slower setting. Natural cements made in the Lehigh region are of this character. These materials may be made into Portland cement by the addition of a limestone consisting of more nearly pure carbonate of lime.

Magnesian Natural Cements are formed by burning Magnesian limestones. The composition of these cements varies from that of the Roman cements to that in which the proportion of magnesia is as great as that of lime. The action of cements of this class is somewhat similar to that of the Roman cements. They may be either slow or quick setting, and gain strength rather slowly, reaching a much less ultimate strength than Portland cement. Magnesian cements are but little used in Europe, but in the United States they constitute the larger part of the natural cements in use.

and many of them have been found by experience to be very useful and reliable materials.

The rock from which natural cements are made differs greatly in character in the same locality, and in different strata in the same quarry. In some of the mills the nature of the product is regulated by mixing, in proper proportions, the clinker obtained by burning rock from different strata. Each portion of the rock must be burned in such degree as is suited to its composition, and hence, as the material is not pulverized before burning, it must be burned separately and mixed afterward. To produce uniformly good cement, therefore, requires close and careful attention; for this reason there is often considerable difference in the quality of cement made by works in the same locality and from very similar materials.

Mixed Cements.—In localities where both Portland and natural cements are made by the same works, mixtures of the lower grades of Portland with natural cements are sometimes made. These are usually sold as natural cements under the name *Improved Cements.* The effect of the mixture is to make the setting slower, and to somewhat increase the strength of the natural cement.

17. Pozzuolana Cement.—Pozzuolana cement is formed by mixing and grinding together definite proportions of slaked lime and pozzuolan. In Germany pozzuolana cement is made by the use of a natural product called trass, consisting of a volcanic earth. In the United States cement of this character is made by the use of specially prepared blast-furnace slag. This cement is sometimes called slag cement. Basic slag, containing lime in excess of the silica and with a high alumina content, is used for this purpose. It is made granular by quenching in cooling.

It is very important, in making slag cements, that the slag be ground very fine, and be very intimately mixed with the lime. The lime is slaked and bolted and then ground mechanically with the slag so as to insure thorough incorporation into the mixture. In some of the European plants the slag is finely ground and bolted through fine sieves before being mixed with the lime, but more common practice is to slake and bolt the lime and mix with the granular slag before grinding, or to do the pulverizing of the slag in two stages and make the mixture between the first and second grinding.

Pozzuolana cement is usually very finely ground, and is slow in setting. It is sometimes treated with soda to quicken the set. When allowed to harden in dry air, it is likely to shrink and crack. When used for under-water work, mortar of pozzuolana cement frequently gives nearly the same strength as good Portland cement.

It is essentially a hydraulic material, and it is specially important that it be kept damp during the early period of hardening, in order that the water necessary to proper hardening may not evaporate.

The composition of slag cement usually differs from that of Portland cement in having a smaller quantity of lime, more silica and alumina and more alumina in proportion to silica.

18. Sand Cement.—Sand cement is the name given to material formed by grinding together Portland cement and silica sand to extremely fine powder and a very intimate mixture. It is claimed that a considerable amount of sand may be thus mixed with the cement without materially reducing the strength of mortar made by mixing the resulting cement with the usual proportions of sand. The additional grinding reduces all of the cement to impalpable powder, thus increasing the amount of active material.

Sand cement as ordinarily made contains equal proportions of Portland cement and silica sand. Cement of this character has recently been made in California by grinding volcanic rock, or tufa, with Portland cement. The tufa used is a pozzuolan, and it is claimed that it reacts with the lime of the cement. The results of tests indicate that mortar made from this cement is equal in strength to that of the original Portland. Cement of this kind is now being made by the U. S. Reclamation Service in some of the Western States to reduce the cost of concrete work where Portland cement is expensive and difficult to get.

Similar methods are employed in Germany where a pozzuolan called trass is used, and in Italy where volcanic lava is ground with the cement. These cements are used for work in sea water to lessen the action of the sea salts upon the lime salts of the Portland cement.

Sand cement has frequently been used for the purpose of securing impermeable mortar where waterproof work is needed. It is useful for this purpose on account of its extreme fineness.

19. Soundness of Cement.—The permanence of any structure erected by the use of cement is dependent upon the ability of the cement, after the setting and hardening processes are complete, to retain its strength and form unimpaired for an indefinite period. Experiment has shown that mortars made from cement of good quality frequently continue to gain strength and hardness through a period of several years, or at least that there is no material diminution in strength with time; and that changes of temperature, or changes in the degree of moisture surrounding it, produce no

injurious effects upon the material. This durability in use is commonly known as the *permanence of volume or soundness* of the cement.

When mortar which has been immersed in water is transferred to dry air, a slight contraction may take place in volume, together with an increase in strength; while a transfer the other way may produce the opposite result; but no distortion of form or disintegration of the mortar will take place in either case if the cement be of good quality.

Sometimes cement when made into mortar sets and hardens properly, and later, when exposed to the action of the atmosphere or water, becomes distorted and cracked or even entirely disintegrated. If the composition deviates but slightly from the normal, this process of disintegration may not show itself for a considerable time and proceeds very slowly. It thus becomes an element of considerable danger, as it is liable to escape detection in testing the cement.

The presence of small quantities of free lime in cement is doubtless one of the most common causes of disintegration in cement mortar. The lime being distributed through the cement in small particles is hydrated very slowly after the cement has set, causing, through its swelling during slaking, strong expansive forces on the interior of the mortar, and producing an increase of volume, loss of strength, and perhaps final disintegration.

Free magnesia in cement is supposed to act very much like free lime. The action of magnesia, however, is much slower than that of lime, and for this reason is a more serious defect. Specifications for Portland cement frequently limit the amount of magnesia that may be present in the cement.

Most Portland cements probably contain small amounts of the expansive elements, which when in very small quantity act with extreme slowness and perhaps produce no visible effect for several months after the use of the cement; then occurs a decrease of strength, which disappears with time. Cements which gain strength rapidly are quite apt to act in this manner, a depression in the strength curve occurring at from six months to one year after the mortar is made.

Cements for use in sea water should contain very little alumina. Some of the salts in the sea water attack these alumina compounds, causing disintegration of the cement and giving rise to expansive action which cracks and breaks up the work.

The presence of expansive elements in Portland cement is prob-

ably due to incomplete burning or lack of uniformity in the incorporation of the ingredients rather than to defective composition.

The fineness of the cement modifies the action of the free lime, as finely divided material will slake more quickly than coarse grains, and the lime is more apt to become hydrated before setting; or, if the cement be exposed before use, the lime in a fine state will sooner become air slaked.

20. Chemistry of Cement.—Professor LeChatelier was the first to explain the composition of Portland cement. He studied sections of clinker under the microscope, and examined the properties of the various compounds formed by the principal ingredients. He concluded[1] that the tricalcium silicate, $3CaO, SiO_2$, is the only silicate that is really hydraulic, and that it is the essential active element in cement. In Portland cement he finds it to be the principal component, occurring in cubical crystals. It is formed by combination of silica and lime in presence of fusible compounds formed by alumina and iron.

" The dicalcium silicate, $2CaO, SiO_2$, possesses the singular property of spontaneously pulverizing in the furnace upon cooling. This silicate does not possess hydraulic properties and will not harden under water.

" There are various aluminates of lime, all of which set rapidly in contact with water. The most important is the tricalcium aluminate, $3CaO, Al_2O_3$."

Professor LeChatelier gives two limits within which the quantity of lime in Portland cement should always be found. These are, that the proportion of lime should always be greater than that represented by the formula

$$\frac{CaO+MgO}{SiO_2-Al_2O_3-Fe_2O_3}=3,$$

and that it should never exceed that given by the formula,

$$\frac{CaO+MgO}{SiO_2+Al_2O_3-Fe_2O_3}=3.$$

The symbols in these formulas represent the number of equivalents of the substances present, not the weights.

Messrs S. B. and W. B. Newberry from a study of the compounds

[1] Annales des Mines, September, 1893.

of silica and alumina with lime reached the following conclusions:[2]

(1) Lime may be combined with silica in proportion of three molecules to one and still give a product of practically constant volume and good hardening properties, though hardening very slowly. With $3\frac{1}{2}$ molecules of lime to one of silica the product is not sound and cracks in water.

(2) Lime may be combined with alumina in the proportion of two molecules to one, giving a product which sets quickly but shows good hardening properties. With $2\frac{1}{2}$ molecules of lime to one of alumina the product is unsound.

Assuming that the tricalcic silicate and the dicalcic aluminate are the most basic compounds which can exist in good cement we arrive at the following formula:

$$X(3CaO,\ SiO_2)+Y(2CaO,\ Al_2O_3),$$

in which X and Y are variable quantities depending upon relative proportions of silica and alumina in materials employed.

$3CaO$, SiO_2 corresponds to 2.8 parts of lime by weight to 1 of silica, while $2CaO$, Al_2O_3 corresponds to 1.1 parts of lime to one of alumina.

$$\text{Per cent lime} = \text{Per cent silica} \times 2.8 + \text{Per cent alumina} \times 1.1.$$

Mr. G. A. Rankin, in an extended study of the composition of Portland cement[3] finds the essential constituents to be the tricalcium silicate, $3CaO$, SiO_2; the dicalcium silicate, $2CaO$, SiO_2; and the tricalcium aluminate, $3CaO$, Al_2O_3. He finds that in burning Portland cement, after the carbon dioxide has been driven off, the lime combines with silica and alumina, forming first a fusible aluminate, $5CaO$, Al_2O_3, and the dicalcium silicate. At higher temperatures these compounds unite with additional lime, forming the tricalcium aluminate and silicate. When the material is not thoroughly burned, and complete equilibrium is not reached, the clinker will contain free lime, CaO, and the aluminate, $5CaO$, Al_2O_3. Magnesia and iron oxide have little influence on the final main constituents of the cement, but act as fluxes and lower the temperature at which the reactions take place.

Too high proportion of lime causes cement to be unsound through the presence of free lime. The same results are caused by underburning or by irregular incorporation of the raw materials into the mixture. As perfect uniformity in the mixture of the ingredients is not attainable in the manufacture of cement, it is necessary that the amount of lime be somewhat less than the theoretic maximum to avoid unsoundness in the cement. The desirable proportion of lime seems to be that which will change the dicalcium silicate

[2] Journal Society of Chemical Industry, Nov. 30, 1897.

[3] Journal Industrial and Engineering Chemistry, June, 1915.

to tricalcium silicate as completely as possible without producing unsoundness.

The ratio of silica to the sum of alumina and iron in cement materials is known as the *silica ratio.* It is desirable that the silica ratio be at least 2.5 or possibly 3 in Portland cement.

Very little is definitely known concerning the chemical reactions which take place in the setting and hardening of cement mortars. Studies are in progress which it is hoped may throw light upon the subject and tend to more accurate knowledge of the requirements for such materials.

On January 15, 1924, the Board of Direction of the American Society of Civil Engineers authorized the appointment of a Special Committee on Cement and on March 16, 1925, the personnel of the committee was completed with Mr. Thaddeus Merriman as Chairman. The scope of the work of the Committee is indicated by the following extract from the January 15, 1924 report of the Research Committee:

"It is recommended that a Research Committee on Cement be constituted. The field of the Committee may well include the constitution, nature, and properties of cement, both as a material and as an ingredient in engineering construction, and the means of determining its quality for both general and specific purposes.

"In making this recommendation, the Committee desires to emphasize the need of making a study of the fundamentals relating to cements and their action. Notwithstanding that time has been given to its study, our knowledge of the chemical action of cements is very inadequate, and no great progress is being made. The composition of cements is quite diverse, the chemical action of setting and hardening under different conditions and the nature of the resulting compounds are varied and complex, and the resulting products may differ markedly in strength and durability qualities. It is apparent that a thorough and systematic chemical, physical, and petrographic study of the action of cement should furnish information of the greatest value, knowledge that is essential to a proper consideration of the qualities that may reasonably be expected or required in this construction material and helpful in a consideration of the proper or allowable methods in its use. Such a study would be only one part of the work of a committee. Although Portland cement is the principal material to be considered, other cements would naturally be included in the work. It is expected that the Committee would deal with cement rather than with mortar and concrete, although, of course, the scope should include the action of the cement and its resulting properties for all the various conditions to be found in mortar and concrete and all matters relating to the durability and permanency of construction as affected by the nature and use of the cement. Such conditions as amount of water used, both mixing water and water retained, exposure, expansion and contraction, and changes in moisture content would seem to be involved in the investigation. The chemical reactions and the physical changes under a variety of conditions with cements of diverse composition would need to be

studied. Naturally, the Committee should try to learn whether tests of cement specified at present are conclusive and sufficient tests and whether other forms of tests may well supplant any of the present ones. This enumeration is not intended to fix or limit the scope of the Committee's work, but to suggest something of the field. It is not the thought, however, that the Committee would report on proper and improper methods of proportioning, making, and placing concrete, this subject being one that might well be assigned to an engineering practice committee."

For progress report of Special Committee on Cement see Proceedings of the American Society of Civil Engineers for March, 1926, p. 210.

ART. 6. SPECIFICATIONS AND TESTS FOR CEMENT

21. Standard Specifications.—The specifications of the American Society for Testing Materials are now commonly recognized as standard and used in the purchase of cement in the United States. These specifications were adopted in 1904 and revised in 1908, 1909, 1916, and 1921. In specifications for construction of masonry and concrete it is usual to require that the cement meet the requirements of the American Society for Testing Materials, although in ordinary work it is not common to actually apply all the tests. The tests of chemical analysis and specific gravity are used only when special reasons exist for their application in the character of the work to which the cement is to be applied or doubt as to the material offered.

It is frequently necessary, on important work, to modify the specifications to suit the peculiarities of the particular construction, This is particularly the case in work to be subjected to the action of sea water, or unusual conditions of service.

The general specifications adopted in 1909 were modified in 1916 and 1921 as to Portland cement only, those for natural cement being left unchanged. The methods of making the tests for Portland are to be also applied to natural cement.

The 1916 specifications make some important changes from those previously used. The No. 100 sieve is dropped from the test for fineness and the requirements somewhat increased for the No. 200 sieve. The Gillmore needles are introduced as an alternate method in the test for rate of setting. Tensile tests of cement paste are dropped and sole dependence placed on the 1 to 3 mortar test, requirements for which are somewhat increased. The normal test for soundness which had previously been the final test is dropped and the steam test is made the standard.

The specifications have been gradually developed through experi-

ence with a number of different methods of testing which have been changed from time to time as knowledge of the material has increased and manufacturers have improved the quality of the material they are able to produce. The reliability of the cement on the market has markedly improved within a few years past and the likelihood of finding poor cement and consequently the necessity for tests under ordinary circumstances has greatly diminished. The application of tests where feasible and upon all important work is, however, desirable.

The specifications for Portland cement adopted in 1916 and modified slightly in 1921, are the result of several years' work of a Joint Committee of the American Society of Civii Engineers, the U. S. Government Engineers, and the American Society for Testing Materials. They are published in the Book of Standards of the Society for Testing Materials, and are also reprinted for distribution to those interested in cement testing by the Portland Cement Association.

22. Purpose of Standard Tests.—The tests imposed by the standard specifications are chemical analysis, specific gravity, fineness, normal consistency, time of setting, tensile strength, and soundness. Specifications covering all of these are usually employed for cement to be used in important work. The making of the tests for chemical analysis and specific gravity are often omitted when the cement proved satisfactory upon the other tests.

The *chemical analyses* employed for Portland cement are intended to determine whether the cement has been adulterated with inert material, such as slag or ground limestone, and whether magnesia or sulphuric anhydride are present in too large amounts.

The test for *specific gravity* when used for Portland cement is intended mainly to detect adulteration with materials of lower specific gravity. It may also aid in determining the true character of the material and whether the cement is well burned. The specific gravity of Portland cement is usually between 3.10 and 3.20, that of a natural cement 2.75 to 3.10, and pozzuolana cement 2.7 to 2.9. Good Portland cement may be lowered in specific gravity by long exposure to the air without serious injury to the cement. For this reason, the specifications allow a second test upon an ignited sample of cement failing upon a first test.

The test for *normal consistency* is made to determine the proper quantity of water to be used in the paste or mortar for tests of time of setting or strength. In the preparation of paste or mortar for these tests, variations in the quantity of water used, or in the methods of mixing and molding the specimens, may produce considerable

differences in results. A standard method is therefore prescribed.

The *time of setting* is tested for the purpose of determining whether the cement is suitable for a given use, rather than as a measure of the quality of the cement. Testing for time of setting consists in arbitrarily fixing two points in the process of solidification called the initial set and the final set. This is accomplished by noting the penetration of a standard needle carrying a given weight into the mass of cement.

The test for *fineness* is to determine whether the cement is properly ground. Only the extremely fine powder is of value as cement. The coarse parts, while having some cementing value, are practically inert when used in sand mortar.

The test for *tensile strength* of cement pastes and mortars is made for the purpose of demonstrating that the cement contains the active elements necessary to cause it to set and harden properly. Cement is not usually subjected to tensile stresses in use, but the tensile test has commonly been employed because it offers the easiest way to determine strength, and seems to give a satisfactory means of judging the desired qualities.

The proper conduct of any test for strength is a matter requiring care and experience. There are a number of points connected with the conditions and manipulation of the tests which have important effects upon the results. These are: the form of the briquette, the method of mixing and molding, the amount of water used in tempering the mortar, the surroundings in which the mortar is kept during hardening, the rate and manner of applying the stress, the temperatures at which all the operations are performed. In order to secure uniform results, it is essential that the tests be standardized in all these particulars.

Soundness is the most important quality of a cement, as it means the power of the cement to resist the disintegrating influences of the atmosphere or water in which it may be placed. Unsoundness in cement may vary greatly in degree, and show itself quite differently in different material. Cement in which unsoundness is very pronounced is apt to become distorted and cracked after a few days, when small cakes are placed in water. Those in which the disintegrating action is slower may not show any change of form, but after weeks or months gradually lose coherence and soften until entirely disintegrated.

The object in the tests is to accelerate the actions which tend to destroy the strength and durability of the cement. As the tests

must be made in a short time, it is necessary to handle the cement in such manner as to cause these qualities to show quickly.

Normal Test.—The method which has been commonly employed is to make small cakes, or pats, of cement paste about 3 inches in diameter and ½ inch thick at the center, with thin edges, upon a plate of glass about 4 inches square. These pats are kept twenty-four hours in moist air and then allowed to stand for twenty-eight days in water, or in the air. The pat during this period should show no signs of cracking, checking, distortion, or disintegration. This is known as the normal test, and has been relied upon as the final test for soundness. This test is defective in requiring too much time and also, in some instances, fails to discover defective material in which the action is very slow.

Accelerated Tests.—Numerous tests have been proposed for the purpose of hastening the hardening of the cement and causing unsoundness to show more quickly. In most of these tests, heat is employed to accelerate the changes taking place in the cement, and they are known as accelerated tests.

These tests have usually been made by subjecting small pats of the cement to the action of hot water or steam and observing whether cracking or disintegration takes place. Sometimes small bars of cement are used and the increase in length of the bar measured after exposure to the hot water or steam. The expansion of unsound cement should be much greater than that of sound cement. The tensile strengths of briquettes of cement which have been exposed to hot water or steam are sometimes measured and compared with the strengths of similar briquettes kept at normal temperatures. The heat should cause a considerable increase in strength of sound cement.

The *standard steam test* consists in observing the effect of steam at about 100° C. upon small pats of the cement. This test was recommended by a committee of the American Society of Civil Engineers in 1904. It has since been included in the specifications of the American Society for Testing Materials in conjunction with the normal pat test, which was the deciding test. In the modified specifications for Portland cement adopted in 1916 and 1921, the normal test is discontinued and the steam test becomes the standard.

The methods for making the standard tests are described in detail, with the specifications, in the Book of Standards of the American Society for Testing Materials, and in the reprint published by the Portland Cement Association.

23. Tests of Compressive Strength.—Tests of compressive

strength are seldom used in specifications for cement, on account of the greater ease of making the tensile test and the lighter machines that may be employed for the purpose. These tests have frequently been made for purposes of comparison or to determine special qualities of the material. The standard test piece has usually been a 2-inch cube, prepared in the same manner as the tension specimens. This was recommended by a committee of the American Society of Civil Engineers in 1909.

As cement mortar is usually employed in compression, some engineers prefer to use the compression test in their specifications. A new tentative specification with methods of testing was recommended by a committee of the American Society for Testing Materials in 1916. This has not been adopted by the society as a standard, and may be further modified before such adoption. It is probable that such a standard will be adopted, to be used in conjunction with or to replace the tension test. This proposed specification with the method of making the test is given in Volume I of the Transactions of the Society for 1909. Reprints may be had from the Secretary of the Society.

24. Special Tests.—The tests ordinarily employed in determining the quality of cement are enumerated in the preceding sections. Other tests are frequently made to determine special qualities or for the purpose of investigating properties of cements and mortars.

Transverse Strength.—Tests of the strength of cement mortar under transverse loading are seldom employed as a measure of the quality of the material, but are frequently made with a view to determining the action of the material in service. Propositions have often been made to substitute the transverse for the tensile test in the reception of material. These suggestions have usually been based upon the simplicity of the test and of the apparatus with which it may be carried out. The specimen usually employed for this purpose is 1 inch by 1 inch and 6 inches long. It is tested by placing upon knife edges 5 inches apart and bringing the load upon the middle section. Professor Durand-Claye, from a large number of comparative tests, found the unit fiber stress under transverse load to average about 1.9 times the unit stress for tension.

Adhesive Strength.—The ability of cement mortar to adhere firmly to a surface with which it may be placed in contact is one of its most valuable properties and quite as important as the development of cohesive strength. Tests for adhesive strength are not employed as a measure of quality, because of the uncertain character of the test and the difficulty of so conducting it as to make

it a reliable indication of value. The adhesive properties of the cement are to a certain extent called into play in tests of sand mortar, and may be inferred from comparison of neat and sand tests.

Experiments upon the adhesion of mortars to various substances are sometimes made, both for the purpose of comparing the cements or methods of use, and to study the relative adhesions to various kinds of surfaces. Such experiments are quite desirable with a view to the extension of knowledge of this very important quality.

The common method of making this test is to prepare briquettes of which one half the briquette is of cement paste or mortar and the other half a block of stone, glass, or other material to be used. The cement half is made in the ordinary form for tensile specimens. The other half is made to fit the cement mold at the middle and arranged at the end to be held by a clip in the testing machine.

ART. 7. SAND FOR MORTAR

25. Quality of Sand.—As hydraulic cement is commonly mixed with certain proportions of sand, when used in construction, the nature and quality of sand used, and the method of manipulating the materials in forming the mortar have quite as important an effect upon the final strength of the work as the quality of the cement itself.

In testing cement a standard sand is employed. This sand may be obtained quite uniform in quality. In the execution of work, however, local sand must generally be used; this varies widely in character, and should always be carefully considered upon any work where the development of strength and lasting qualities are of importance.

Effect of Impurities.—Sand for use in mortar should be clean, and free from loam, mud, or organic matter. When the only sand obtainable contains any of these objectionable impurities, it should be carefully washed and tested. A very small amount of vegetable matter in sand has sometimes caused the failure of mortar to harden properly.

Size of Sand Grains.—It is usual to class as sand all material less than $\frac{1}{4}$ inch in diameter, pieces larger than this being classed as gravel. Coarse sand is superior to fine sand for use in cement mortar. Coarse sand presents less surface to be coated with cement and the interstices are more easily filled with cement paste. Fine sand requires more water in mixing to the same consistency, and usually gives weaker and more porous mortar than coarse sand.

The use of a mixture of grains of different sizes is usually desirable, resulting in fewer voids to be filled by the cement; and it is frequently found, when the cement is not in considerable excess, that the strength obtained by such a mixture is much greater than that given by either large or small grains alone. Sand of mixed sizes, giving a minimum of voids, requires less cement to make a mortar of maximum density and strength than that of more uniform sizes.

Shape of Grains.—Sand with angular grains usually give better results in mortar than that with rounded grains. However, the specification requiring *sharp* sand has been discontinued.

Stone Screenings.—The screenings from crushed stone are frequently used in place of natural sand. Ordinarily screenings from hard stone of good quality give mortar of rather better strength than natural sand. The sharpness of the grain is favorable to the screenings. When the screenings are derived from soft rock, dust, that is, material that will pass the 100 or even the 200 sieve will be present and will need to be screened or washed out before the screenings can successfully be used.

Chemical Composition.—Sands as commonly used for mortar are composed mainly of silica. In most cases, sand which has a proper granulometric composition is satisfactory for use. The failure of concrete work has, however, in a number of instances been found to be due to the use of sand low in silica. Sand containing less than 95 per cent silica needs to be carefully tested before being used, although some sands as low as 75 per cent silica have given good results. The compositions of sands have not been sufficiently studied to determine the differences which cause failure in one case and success in another.

The presence of mica in sand or screenings is supposed to affect injuriously the strength of mortar in which the material is used. The results of experiments upon the effect of mica are not conclusive, although they seem to indicate that mica may sometimes be injurious. Sand containing mica should be carefully tested before being used.

Selection of Sand.—Sands differ so greatly in their qualities that it is difficult by mere superficial inspection of the materials to judge of their relative values for use in mortar. In choosing sand for use in important work, it is desirable not only to determine fully the physical characteristics of the available materials, but also to make actual tests of mortar by their use.

26. Tests for Sand.—Tests intended to determine the mortar-making qualities of sand may be made in several ways:

1. *Cleanness Test,* to determine the presence and extent of deleterious matter.

2. *Specific Gravity Test.*

3. *Voids Test,* to determine the probable quantity of cement required to fill completely the voids among the particles of sand.

4. *Granulometric Test,* to determine the percentage of the particles of various sizes.

5. *Mortar Density Test,* to determine the percentage of the volume of solid materials contained in the mortar.

6. *Mortar Strength Test,* to determine the value of the sand in accepting the adhesion of the cement.

The value of sand depends mainly upon its granulometric composition. The sand which, mixed with a given proportion of cement, gives the most dense mortar yields the strongest mortar. The sand which requires the least cement to make a mortar of maximum density is the most economical, when the mortar is properly proportioned.

The purpose in testing the sand should be to determine the proportions of cement to sand necessary as well as to choose the best sand.

27. Cleanness Test.—Two methods of procedure may be followed for determining the presence and quantity of undesirable impurities in sand, the washing test and the colorimetric test.

Washing Test.—When it is necessary to examine sand for impurities, the silt may be removed from the sand by washing. This is done by shaking a sample of the sand in a bottle with water, letting it settle for a few seconds, and then decanting off the turbid water. This is done repeatedly until the suspended matter is all removed. The wash water is then evaporated, and the amount of silt determined.

The silt is ignited in a platinum crucible and the loss on ignition is the percentage of organic matter present.

A very small amount, not more than 1 per cent, of organic matter may be a serious detriment. Sand containing such impurities should be carefully tested and may need to be washed in order to give satisfactory results in use.

Colorimetric Test.—A simple method of detecting the presence organic impurities in sand was devised by Professor Duff A. Abrams, and Oscar E. Harder, chemist, of the Structural Materials Research Laboratory, Lewis Institute, Chicago. The method was printed in full in the Concrete Highway Magazine for February, 1918.

Briefly, the test consists in shaking the sand thoroughly in a dilute solution of sodium hydroxide (NaOH) and observing the resultant color after the solution has been allowed to stand for a few

hours. It is suggested that a 12-ounce graduated prescription bottle be used and filled to the $4\frac{1}{2}$-ounce mark with sand to be tested. A 3 per cent solution of sodium hydroxide is added until it stands at the 7-ounce mark after thorough shaking. It is recommended that the solution be then allowed to stand for twenty-four hours, though a good idea of the quality of the sand may be gained in a shorter period of time. A color chart against which the color may be checked is desirable.

28. Specific Gravity.—The specific gravity of siliceous sand is quite uniformly 2.65, or the weight per cubic foot of the solid rock is 165 pounds. To assume these values in determining the voids in such sand involves slight error in any case. Sands not strictly siliceous may vary in specific gravity from about 2.6 to 2.7.

The determination of specific gravity is made by immersing a sample of the material in water at 68° F. and dividing the weight of the sand by the weight of water displaced. This is most conveniently done by sifting the sand into the water in a graduated glass cylinder, and reading the increase of volume of the liquid in the cylinder. Care must be used to introduce the sand slowly so as to eliminate all air bubbles.

29. Voids Test.—The method most commonly used for void determination is known as the *wet method*, which consists in filling a measure with the sand to be tested and pouring in water until the voids are completely filled. The volume of water required to fill the voids divided by the volume of sand and multiplied by 100 is the percentage of voids; or the weight of water poured into the sand divided by the weight of water required to fill the measure and multiplied by 100 is the percentage of voids. It is very difficult to eliminate completely the air from the sand in making this test. The test is therefore liable to considerable error unless great care be used in manipulating it.

Dry Method.—A more accurate method of determining voids is to compare the weight of a measured volume of the sand with the weight of an equal volume of solid material of which the sand is composed. In measuring the volume of sand, it is necessary to use care to secure the proper degree of compactness. For ordinary comparisons the sand should be well compacted by shaking and jarring the measure. The weight of the solid rock is obtained by multiplying the weight of an equal volume of water by the specific gravity of the sand. The difference between the weight of the rock and that of the sand divided by the weight of the rock and multiplied by 100 is the percentage of voids.

If R is the weight of the solid rock and S, the weight of the sand, percentage of voids is

$$\frac{R-S}{R}\,100.$$

This test supposes the sand to be dry. When it is desired to obtain the voids in moist sand, a weighed sample of the sand should be dried at 212° F. and the loss of weight determined. The weight of moisture in the measure of sand to be used in the test many then be computed. This weight is then to be subtracted from the total weight of the moist sand to find the weight of solid material in the sand.

If m is the weight of moisture in the volume of sand under test, percentage of voids is

$$\frac{R-(S-m)}{R}\,100.$$

30. Granulometric Test.—To determine the relative sizes of grains composing sand, the material is screened through a series of sieves of varying degrees of fineness. The sieves are made of standard size, 8 inches in diameter by $2\frac{1}{4}$ inches high, those with openings smaller than $\frac{1}{10}$ inch being made of woven brass wire, while the larger sizes are preferably drilled circular openings in sheet brass. These sieves are designated by numbers corresponding to the number of meshes to the linear inch, the size of the opening depending upon the diameter of the wire used. The size openings usually employed are those given by the Tyler series, and are approximately as follows:

Number of sieve..................	100	50	30	16	8	4
Size opening, inch................	.0058	.0116	.0232	.0464	.093	.185

The sieves are made to fit together in nests with a cover and tight bottom to catch the residue from the finest sieve. The sifting may be done by hand, by shaking and jarring the sieves, or mechanical shakers may be used. These may be obtained to work by hand or with small electric motors attached.

In making the tests, a sample weighing 50 grams is dried to a constant weight at a temperature of not more than 110° C. (230° F.) and is then sifted through the sieves, so as to separate the grains into various sizes and determine the percentage of each by weight. The material properly classed as sand is that which passes through the No. 4 and is retained on the No. 100 sieve. Material passing the No. 100 sieve is called dust.

Analysis Curves.—Comparisons of granulometric composition of sands were formerly made by platting the results of the sieve analyses as curves, the sizes of the openings as abscissae and the percentages passing each size as ordinates.

Fineness Modulus.—Professor Duff A. Abrams of Lewis Institute has introduced a method of granulometric comparison which he calls the " fineness modulus " method. By his method the percentages coarser than a given sieve are added and the sum is divided by 100. This procedure gives index numbers which are readily compared. Professor Abrams has found that sands varying considerably in sizes of grains may have the same index number, and he also found that under the same conditions of making the tests that different sands with the same index number gave practically the same results.

A comparison of sands by use of the fineness modulus method is shown in Table I, which is taken from the table on p. 5 of Bulletin 1 of the Structural Materials Research Laboratory.

TABLE I.—FINENESS MODULUS OF SAND

Sieve Size.	Per Cent of Sample Coarser than a Given Sieve.		
	Fine. (A)	Medium. (B)	Coarse. (C)
100-mesh	82	91	97
50-mesh	52	70	81
30-mesh	20	46	63
16-mesh	0	24	44
8-mesh	0	10	25
4-mesh	0	0	0
Fineness modulus	1.54	2.41	3.10

31. Mortar Density Test.—Comparative tests of sands may be made by determining the volume of mortar produced by definite weights of cement and dry sand. The sand that for a given weight of materials, when mixed with the same proportion of cement to the required consistency, produces the smallest volume of mortar gives the most dense mortar. In making this test, molds in which the height is large in comparison with the section are convenient, the relative heights to which the mold is filled giving the proportionate volumes. The volume of mortar after setting is what is required,

but the measurement before setting, unless the mortar is quite wet, will give practically the same result.

Determination of Density.—The term density, as commonly applied to mortar, means the ratio of the volume of solid materials contained in the mortar to the whole volume of mortar. The density is obtained by weighing the ingredients before mixing and calculating their solid volumes from these weights and their specific gravities. The weight and volume of the resulting mortar are then measured. The weight of mortar should equal the sum of the weights of the several ingredients. The density equals the sum of the solid volumes of sand and cement divided by the measured volume of the mortar.

The densities of mortars made from three different sands, in the proportion by volume of one part cement to three parts of sand, are shown in Table II.

TABLE II.—DENSITIES OF MORTAR

Sand No.	Weights Used, Grams.			Mortar Volume, c.c.	Density.
	Cement.	Sand.	Water.		
1	358	1026	178	670	0.75
2	358	1128	163	735	0.73
3	358	972	180	730	0.66

Sand No. 1 was a well-graded sample; No. 2 was slightly finer; and in No. 3 the finer grains decidedly predominated.

The method of computation is as follows:

Taking specific gravity of cement as 3.1 and specific gravity of sand as 2.65,

$$\text{Density of No. 1 is } \frac{\frac{358}{3.1}+\frac{1026}{2.65}}{670}=.750.$$

32. Mortar Strength Tests.—Tests of the strength of mortars made from sands are the most conclusive evidence of the mortar-making properties of the sands. These tests to be of real value should extend over a period of at least twenty-eight days. They are made in the same manner as the mortar tests for judging cement, and comparisons are sometimes made with the results of tests with standard sand. Table III gives comparative results of tests of the sands shown in Table II.

TABLE III.—RESULTS OF SAND TESTS

Sand No.	Packed Sand.		Density, 1 : 3 Mortar.	Tensile Strength.			
				1 : 2 Mortar.		1 : 3 Mortar.	
	Per Cent Voids.	Weight, Cu. Ft.		28 Days.	6 Months.	28 Days.	6 Months.
1	36.1	106.4	0.75	523	603	443	495
2	28.6	117.5	0.73	379	493	253	339
3	38.0	100.8	0.66	223	343	153	265
Standard..	....	.....	....	326	477	268	318

33. Specifications for Sand.—Tests have seldom been used as means of judging sand for use in masonry construction. The requirements have usually been that the sand be coarse, clean, and sharp; the requirement of sharpness is now commonly omitted.

Granulometric and void tests are frequently made for the purpose of judging the qualities of available sands on important work, and to aid in properly proportioning mortar, but such tests are not usual in specifications.

The Joint Committee of the Engineering Societies on Concrete and Reinforced Concrete in its report dated Aug. 14, 1924, has suggested the following as requirements for sand to be used as fine aggregate in concrete work:

Fine aggregate shall consist of sand, or other inert materials with similar characeristics, or a combination thereof, having clean, hard, strong, durable, uncoated grains and free from injurious amounts of dust, lumps, soft or flaky particles, shale, alkali, organic matter, loam, or other deleterious substances.

Fine aggregate shall range in size from fine to coarse, preferably as follows: Passing through No. 4 sieve, not less than 85 per cent; passing through No. 50 sieve, not more than 30 per cent nor less than 10 per cent; weight removed by decantation, not more than 3 per cent.

The sieves and method of making sieve analysis shall conform to the "Standard Method of Test for Sieve Analysis of Aggregates for Concrete" (Serial Designation: C41–24) of the American Society for Testing Materials.

The decantation test shall be made in accordance with the "Tentative Method of Decantation Test for Sand and Other Fine Aggregates" (Serial Designation D136–22T) of the American Society for Testing Materials.

Fine aggregate shall be of such quality that mortar briquettes, cylinders, or prisms, consisting of one part by weight of Portland cement and three parts by weight of fine aggregate, mixed and tested in accordance with the methods described in the "Standard Specifications and Tests for Portland Cement" (Serial Designation: C9–21) of the American Society for Testing Materials, will show a tensile or compressive strength at the ages of 7 and 28 days not

less than per cent* of the 1 : 3 standard Ottawa sand mortar of the same plasticity made with the same cement.

Sand when tested in accordance with the "Standard Method of Test for Organic Impurities in Sands for Concrete" (Serial Designation: C40–22) of the American Society for Testing Materials, shall show a color not darker than the standard color unless it complies with the strength test.

* As prescribed by the Engineer.

Fine aggregate which does not conform to the above requirements for grading, mortar strength, or color, may be used only when approved by the Engineer and then in such proportions as he may require.

ART. 8. CEMENT MORTAR

34. Proportioning Mortar.—An ideal cement mortar may be said to be composed of a certain volume of suitable sand, oven dried and thoroughly compacted, with sufficient cement to coat completely each grain of sand and fill the interstices among the grains, together with just enough water to bring about the hydration of the cement; the mass to be mixed vigorously for the length of time required to produce proper plasticity and then thoroughly compacted.

The coating of each grain of sand with cement will increase the sizes of the particles and the volume of the resulting compacted mortar will be greater than that of the original compacted sand. To insure the complete filling of the voids a quantity of cement somewhat in excess of the minimum may be necessary. For the sake of economy in placing, a quantity of water above that required for hydrating the cement may be necessary, but it must be kept in mind that arithmetical additions in volume of water produce geometrical reductions in strength of mortar.

In specifying the proportions of ingredients for cement mortar to be used in construction, it is ususal to give the ratio of the parts of cement to those of sand by volume. The relative proportions of sand and cement to be used in any instance depend upon the nature of the work and the necessity for developing strength or watertightness in the mortar. The proportions commonly used in ordinary work are: for natural cement, one part cement to one part or two parts sand; for Portland cement, one part cement to two parts or three parts sand. In common practice these ratios are chosen without reference to the particular materials used and the resulting mortars vary widely in character.

Good sand in 1 to 3 mortar frequently shows greater strength than a poorer one mixed 1 to 2, and gives equally good results in use.

The methods of measuring materials also vary, and the relative quantities of cement and sand in the mortar differ correspondingly.

Measuring Cement.—Cement should always be measured by weight, on account of the variation in volume of the same quantity of cement with different degrees of compactness. In specifying proportions by volume, therefore, it is always desirable to state the weight of cement to be taken as unit volume.

Portland cement is usually packed in wooden barrels or in canvas bags. A barrel of cement contains 376 pounds of cement, while a bag contains 94 pounds, or one-quarter barrel. Natural cement is ordinarily packed in barrels of 282 pounds, or bags of 94 pounds (one-third barrel) each.

Portland cement as packed in barrels weighs a little more than 100 pounds per cubic foot. A cubic foot of cement paste requires from 95 to 110 pounds of cement. It is common to consider a cubic foot of Portland cement to weigh 94 pounds in porportioning mortar. A bag of cement is then mixed with 2 cubic feet of sand to form 1 to 2 mortar, or with 3 cubic feet of sand to form 1 to 3 mortar. This assumes the volume of a barrel of cement to be 4 cubic feet. This is the recommendation of the Joint Committee of the Engineering Societies. Some engineers use 3.8 cubic feet as the volume of a barrel, or 100 pounds as the weight of a cubic foot.

In the same way, 70 pounds is frequently used as the weight of a cubic foot of natural cement. This makes the volume of a sack of natural cement $1\frac{1}{3}$ cubic feet. A barrel of natural cement would then have the same nominal volume as a barrel of Portland, 4 cubic feet. The actual volume-weight of natural cement varies considerably for different brands.

Measuring Sand.—It is usual to measure sand by volume. The method of measuring to be used in any particular instance depends upon the method of mixing and handling the mortar. Very commonly the measuring is done in the barrow or bucket in which the sand is carried to the mixer or platform. Measuring boxes without bottoms are often employed to set on the mixing platform, and after filling are removed, leaving the measure of sand. Whatever method of handling the sand is employed, it is important that careful attention be given to securing the correct proportion of sand for the mortar.

Effect of Moisture.—In proportioning mortar by volume, the

moisture content of the sand may be a matter of importance. Damp sand weighs less per unit volume than dry sand. When sand is moistened with a small quantity of water, the grains of sand are coated with a thin film of water, which separates the grains, causing the sand to occupy more space than when dry. When the amount of water becomes sufficient to coat all the grains of sand (about 4 to 7 per cent with ordinary sands), a maximum effect is reached, and an increase in amount of water beyond that point causes a reduction of volume. At saturation (10 to 20 per cent of water), it becomes slightly less in volume than when dry.

The solid content in a given volume of moist sand is less than that of the same volume of dry sand, and a mortar mixed with the moist sand will be richer in cement than that mixed with the same sand when dry. This effect is greater with fine than with coarse sand. A given volume of sand measured dry may contain 10 per cent to 15 per cent more solid material than the same volume of the same sand measured in a moist condition.

The extent to which differences in moisture condition may affect the volume of the sand depends upon the position in which the sand is placed and the way it is handled in measuring. If dry sand in a bin, or a pile, be moistened with a small quantity of water, the sand will not appreciably swell in the pile, as the particles are held by the weight of the mass above—they are not free to move and the water fails to separate them. If the sand be loosened in moving to a new position, it will be found to have increased in volume and will not return to its former dimensions until it has become dry, or wet to saturation.

Proportioning by Weight.—In Germany it has been quite common to measure the material for mortar by weight. This has been applied in some instances in the United States, and reduces largely the variations in the proportions due to moisture. On important work it may frequently be possible to arrange for weight measurement without materially increasing the cost of handling the material.

The ratio of cement to sand is commonly arbitrarily fixed with reference to the particular use to which the mortar is to be put, without considering the character of the sand to be used. For ordinary masonry, or massive concrete, Portland cement is usually employed in 1 to 3 mixtures. When high strength is needed, as in reinforced concrete work, the mixture is 1 to 2. Under specially trying conditions, or sometimes when cement grout is being used, a 1 to 1 mixture may be employed. With natural cement, the mixtures are 1 to 2 for ordinary work and 1 to 1 where greater strength

is needed. Natural cement is not used for reinforced concrete work. The choice of ratios has usually been well on the side of safety, and good results have been obtained in practice by this method, although equally good work at less cost might in many instances have been obtained by more careful study of the materials in proportioning the ingredients of the mortar.

In comparing the mortar-making qualities of various sands, it is found that the amount of cement necessary to make mortar of the same strength from different sands depends mainly upon the fineness and density of the sands. The office of the cement paste in mortar is to coat the grains of sand and fill the voids between them. In fine sand the surface to be coated with cement is greater than in coarse sand. Dense sand, with grains of varying sizes, presents fewer voids to be filled than more uniform sand.

It is desirable that careful study be given to the sands to be used in any important work before finally deciding upon the proportions of the materials, and that final judgment be based upon actual tests of the mortar itself.

Frequently a mixture of a fine with a coarse sand, or of crusher dust with sand may be so proportioned as to give economical results in the saving of cement, while at the same time improving the mortar.

35. Mixing Mortar.—In mixing mortar by hand a water-tight box or platform is used. The required quantity of sand is spread over the floor of the box and the cement distributed evenly over the sand. The cement and sand are then mixed together with a hoe or shovel until the cement is uniformly distributed through the sand, as shown by the even color of the mixture dry. It is important that the dry materials be very thoroughly mixed before water is added. A uniform mixture will not otherwise be obtained. When the mixing of dry materials is complete, water is added and the mass worked into a stiff paste. The quality of the mortar is materially affected by the vigor with which it is worked in bringing it to the proper consistency. After the water has been absorbed by the cement, vigorous working will make the mass more plastic, and working should continue until a permanent condition is reached.

Quantity of Water.—The quantity of water to be used in mixing mortar can be determined only by experiment in each instance—it depending upon the nature of the cement and sand, and the proportion of cement to sand. The quantity of water used should be the least consistent with reducing the mortar to the required condition of plasticity by vigorous working. Additional water should not be used to save labor in working.

Mixing should be quickly and energetically done, only such quantity being mixed at once as can be used before initial set takes place. A considerable quantity is sometimes mixed dry and left to stand until needed before adding water. If this is done with damp sand, the cement may be acted upon by the moisture in the sand to the injury of the mortar. Quick-setting cements are particularly liable to injury from this cause.

Retempering.—Masons frequently mix mortar in considerable quantities, and if the mass becomes stiffened before being used, add more water and work again to plastic condition. After the second tempering the cement is much less active than at first, and remains a longer time in a workable condition. This practice is not approved by engineers and is not permitted in good engineering construction, although there is some dispute as to the extent of the injurious effects.

Cement when retempered becomes very slow in action, both in setting and hardening. The quicker-setting cements are usually more affected than the slow setting. The strength during the earlier periods of hardening is lessened, although the final strength may not be impaired. Portland cement may ordinarily be used for two, or sometimes three hours after mixing without appreciably affecting its action. When retempered after a longer period it will usually become slower in action, but may in some cases gain as much strength in thirty to sixty days.

Continuous working materially improves the strength of mortar, and when allowed to stand after mixing it should be frequently worked.

Grout.—Mortar when made thin, so that it can be poured into cracks or small openings, is known as grout. Mixtures of cement and sand used in this manner are difficult to handle without separation of the materials. They should be used only under exceptional circumstances and when stiffer mortar cannot be applied.

36. Yield of Mortar.—The volume of mortar formed by mixing given quantities of cement and sand depends mainly upon the densities of the materials. It is affected by the method of preparing the mortar, the uniformity of the mixture, and the degree of compactness. The net volume of materials entering into the composition of mortar is readily found from their weights and densities, but it represents only approximately the resulting volume. An accurate knowledge of the yield of any particular mixture is to be obtained only by experimenting upon the materials to be employed.

The amount of cement paste made by a given weight of cement

powder varies with the specific gravity of the cement and the amount of water necessary in gaging. The lighter cements require more water and yield less paste for a given volume of cement than the heavier ones. To form a cubic foot of plastic paste requires usually from 80 to 95 pounds of natural cement, while from 95 to 101 pounds of Portland cement are necessary.

Table IV gives approximate quantities of materials ordinarily required for 1 cubic yard of compact plastic mortar. A barrel of cement is taken as 4 cubic feet, corresponding to a weight of 94 pounds per cubic foot for Portland cement and 70 pounds for natural cement. The sand is dry and measured loose.

TABLE IV.—MATERIALS FOR 1 CUBIC YARD OF MORTAR

Proportions.		Quantity of Sand to 1 Sack Cement.		Materials for 1 Cu. Yd. Compact, Plastic Mortar.	
Cement.	Sand.	Portland, Cu. Ft.	Natural, Cu. Ft.	Cement, Barrels.	Sand, Cu. Yds.
1	0	...	...	6.75 to 7.85	....
1	1	1.0	1.3	4.25 to 4.75	0.63 to 0.70
1	2	2.0	2.7	2.95 to 3.15	0.87 to 0.93
1	3	3.0	4.0	2.20 to 2.37	0.98 to 1.04
1	4	4.0	5.3	1.75 to 1.85	1.03 to 1.09

The differences in quantities are mainly due to variations in the fineness of the sand, in the amount of moisture contained by the sand, and in the compactness given to the mortar. Smaller amounts are required when using fine than when using coarse sand; more materials are required when the sand is moist than when it is dry. The compactness of the mortar is affected by the quantity of water used in mixing and the method of placing the mortar.

37. Mixtures of Lime and Cement.—The addition of slaked or hydrated lime to cement mortar causes the mortar to work more smoothly, and makes it easier and more economical to handle in masonry construction.

A lean cement mortar may be improved in density and strength by the addition of a small quantity of lime paste. Lime in larger quantities, or lime added to rich mortar, diminishes the strength of the mortar but may sometimes be economical, through cheapening the mortar and improving its working qualities, when high strength is not of special importance.

Lime may be used with cement either by mixing lime paste with

cement mortar, or by mixing dry hydrated lime with cement before mixing the mortar. Lime must always be thoroughly slaked before mixing with cement, as unhydrated lime in cement mortar is always a detriment. It is also essential that the mixture be very uniform, and that the mortar be worked to an even color. For this reason, the use of dry hydrated lime is to be preferred over lime paste.

In proportioning lime to cement, the method of measurement is important. Hydrated lime from nearly pure limestone contains about 75 per cent of quicklime and ordinary lime paste contains about 40 per cent of lime by weight. About 25 pounds of quicklime are required to make a cubic foot of lime paste.

Experiments upon mixtures of lime and cement show that 10 to 15 per cent of lime (measured as unslaked lime) may be substituted for an equal weight of cement in a 1 to 3 cement mortar without sensibly decreasing the strength of the mortar. In some instances when not more than 10 per cent of lime is used the strength is increased and the mortar made more dense. As the proportion of lime is increased the strength of the mortar is lessened. For mortars leaner than 1 to 3 of Portland cement the use of a small amount of lime is usually an advantage.

With some natural cements, lime may be used to replace cement to the extent of 25 to 30 per cent of the weight of the cement without appreciable loss of strength in the mortar. Cement so treated becomes slower in action and is longer in gaining strength than when used without lime. Mixtures of this kind with either Portland or natural cement are frequently used in mortar for ordinary building operations. Hydrated lime is sometimes added to cement for the purpose of rendering the mortar less permeable where water-tight work is needed, and is also sometimes added to Portland cement concrete in small quantity to make the concrete flow more readily in filling the forms.

38. Strength of Cement Mortar.—The strength of cement mortar is dependent upon the quality and proportions of cement and sand; the quantity of water used in gaging; the method of mixing and thoroughness of working; the temperature and moisture conditions under which it is kept during hardening; the age of the mortar.

The effect upon tensile strength of varying proportions of cement and sand is shown in Diagram I, which gives the relative strengths for an average Portland cement, or cement paste, and mortars with standard sand, for a period of one year after mixing. Individual cements may vary quite widely from the curves shown. Some gain strength more slowly at first and continue to gain for a longer

period. Others have greater early strength and show more loss of strength during the period of retrogression.

Nearly all Portland cements after gaining strength rapidly for a time reach a maximum and then lose strength for a period. This loss of strength is usually regained later. It seldom occurs in less than three months or more than one year after the mortar is mixed. Cement which gains strength very rapidly and has high early strength is apt to suffer greater loss of strength later than cements

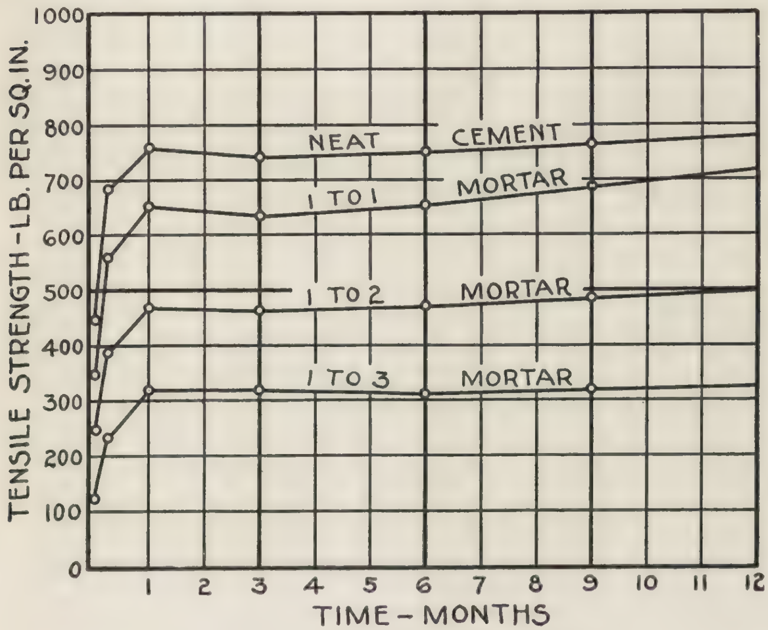


Diagram I.—Strength of Portland Cement Mortar.

of more moderate action, and less likely to regain fully the losses. Mortars usually show less of the effects of retrogression than cement paste, and frequently continue to gain strength for much longer periods.

Diagram II shows average values for good grades of natural cement. These cements vary more widely than Portlands. They gain strength much more slowly and continue to gain for a longer period.

Character of sand.—Coarse, well-graded sand usually gives higher strength in cement mortar than standard Ottawa sand, while fine or poorly graded sand may fall below the strength shown by standard sand. Sands showing less than 75 per cent of the strength

given by standard sand are poor materials and are sometimes rejected by specifications for masonry materials.

Fineness of Cement.—The fineness of the cement has an important influence upon the strength of mortar. Table V shows the results

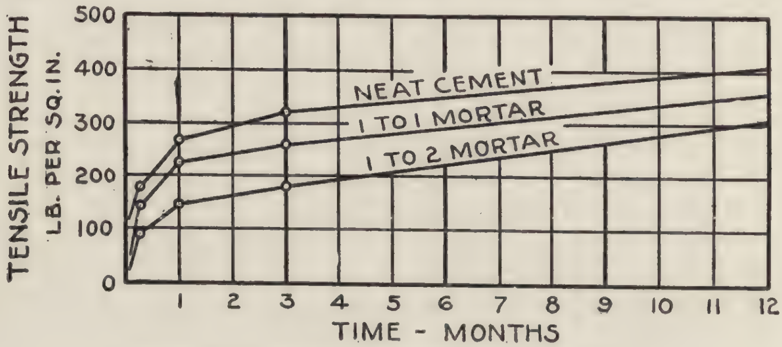


DIAGRAM II.—Strength of Natural Cement Mortars.

of a series of tests made upon Portland cement by Mr. Richard K. Meade.[4] In making these tests a bag of cement was selected and divided into five parts, and each of these ground to a different degree of fineness.

TABLE V.—STRENGTH OF THE SAME CEMENT GROUND TO VARIOUS DEGREES OF FINENESS

Tensile strength in pound per square inch.

Age, Days.	Neat or Sand.	Per Cent Passing No. 200 Sieve.				
		80	85	90	95	100
1	Neat	396	241	308	282	200
7	Neat	955	796	749	627	558
28	Neat	963	840	775	626	594
1	1 to 3 sand	235	248	351	363	382
28	1 to 3 sand	297	353	468	498	576
7	1 to 4 sand	160	204	234	247	263
28	1 to 4 sand	224	266	324	377	392

The strength of neat cement is decreased by fine grinding, while the strength of sand mortar is increased by fine grinding. The same strength may be reached in sand mortar by using less cement when the cement is finely ground than when it is coarsely ground.

[4] Proceedings American Society for Testing Materials, 1908, p. 412.

In the table it is shown that the strength of 1 to 4 mortar with cement 90 per cent fine is stronger than 1 to 3 mortar with cement 80 per cent fine.

The desirability of fine grinding depends upon the relative costs of cement ground to different degrees of fineness. Fine grinding increases the rapidity of setting very rapidly, and many Portland cements if ground so that 95 per cent passes the No. 200 sieve become so quick setting that they could not be used for ordinary work. In order to secure greater fineness, the methods of manufacture would need considerable modification.

Effect of Consistency upon Strength.—The amount of water used in mixing mortar necessarily depends upon the requirements of the use to be made of the mortar. The mortar used in concrete is usually much softer than that employed in masonry construction, or than the consistency used in testing.

For well-compacted mortar, strength decreases as the quantity of water used in mixing increases. The extent of this effect varies with the character of the sand, being less for coarse than for fine sand. This difference is very considerable in short time tests, but disappears to a great extent as the age of the mortar increases. When tested after seven and after twenty-eight days, mortar of standard consistency may have nearly double the strength of that mixed with 50 per cent more water.

Cement mortar hardens more rapidly and attains greater strength if kept moist during setting and the first period of hardening than if it be exposed at that time to dry air.

ART. 9. GYPSUM PLASTERS

39. Classification.—Pure gypsum is a hydrous lime sulphate ($CaSO_4+2H_2O$). It occurs in nature as a massive rock, or sometimes as gypsum sand or earth. Native gypsum usually contains small amounts of silica, alumina, iron oxide, and calcium carbonate. Alabaster is a massive rock of nearly pure white gypsum.

Native gypsum is ground and used as a dressing for certain soils. It is also used quite extensively as an adulterant in the manufacture of Portland cement. (See Section 13.)

Gypsum plasters are made by calcining gypsum sufficiently to drive off part or all of the water of combination. When this dehydration is accomplished at a temperature below 190° C., three-fourths of the water is driven off and the resultant product is called *plaster of paris* ($CaSO_4+\frac{1}{2}H_2O$). At a temperature above 190° C. all of

the water is driven off and the product is known as *flooring plaster* ($CaSO_4$). These products are modified by adding certain substances to the gypsum before calcining, or by the use of impure gypsum.

The following classification of gypsum plasters is given by E. C. Eckel in his " Cements, Limes and Plasters ":

Classification of Plasters

(*a*) Produced by the incomplete dehydration of gypsum, the calcination being carried on at a temperature not exceeding 190° C.

(1) *Plaster of paris*, produced by the calcination of a pure gypsum, no foreign materials being added either during or after calcination.

(2) *Cement plaster* (often called *patent* or *hard wall plaster*) produced by the calcination of a gypsum containing certain natural impurities, or by the addition to a calcined pure gypsum of certain materials which serve to retard the set or render more plastic the product.

(*b*) Produced by the complete dehydration of gypsum, the calcination being carried on at temperatures exceeding 190° C.

(3) *Flooring plaster*, produced by the calcination of a pure gypsum.

(4) *Hard finish plaster*, produced by the calcination at a red heat or over, of gypsum to which certain substances (usually alum or borax) have been added.

Plaster of paris and cement plaster are usually burned at temperatures from 140° to 180° C., the difference between them being due to the substances added to the gypsum in making the cement plaster. Flooring plaster and hard-finish plaster are burned at 400° to 500° C. for three or four hours. If the heat be too high or too prolonged, the plaster may be injured, becoming very slow in action, and is called dead-burnt plaster.

Keene's Cement is a well-known hard-finish plaster made by the double calcination of gypsum, alum being added between the two heatings.

Cement plaster, after being calcined, requires the addition of some material as a retarder to decrease the rapidity of set. This is usually a very small quantity (0.1 to 0.2 per cent) of organic matter such as blood or glue. Hydrated lime or clay is usually added to gypsum wall plasters to increase their plasticity and make them work better. With plasters made from gypsum earth containing clay this is unnecessary.

40. Properties and Uses.—Gypsum plasters when mixed with water set and harden through the combination of the water with the plaster to again form gypsum. The setting of plaster of paris is rapid, requiring from about five to fifteen minutes. Cement

plaster sets more slowly, requiring from one to three hours. Floor plaster and hard-finish plaster are slow setting

Very few data are available concerning the strength of gypsum plasters, which usually gain strength rapidly for a few days, reaching a maximum in three or four weeks, and then suffer retrogression in strength for a time. A series of tests made by Professor Marston of Iowa State College on hard wall plasters indicate a strength for neat plaster of 300 to 500 lbs./in.^{2} one month after mixing. About 80 per cent as much for 1 to 1 mortar and 50 per cent as much for 1 to 2 mortar with sand. These strengths would not be reached under the conditions of ordinary use. The strength is much less when the mortar is kept damp during the period of hardening.

Plaster of paris sets too rapidly for use in construction, although it is used to some extent combined with other materials, as in hard finish, composed of plaster of paris, lime putty, and marble dust. It is commonly employed for casting plaster, where quick set is desired.

Cement and hard wall plasters are used for making various wall plasters, being usually mixed with hair, asbestos, or wood fiber, and clay or hydrated lime. They are received upon the work ready for use and do not require the time or space for preparation needed for lime plaster, but are not so plastic and smooth to work.

Hard-finish plasters are used in a number of ways in making solid or hollow blocks and tiles for use in construction of partitions and in finishing floors and ceilings. Mixed with sawdust, blocks are formed which may be nailed into place. Blocks reinforced with steel are now being made for use in supporting roofs. (See Art. 18.)

CHAPTER III

STONE MASONRY

ART. 10. BUILDING STONE

41. Qualities for Building Stone.—The choice of stone for use in important structures is always a matter of moment, and frequently involves considerable difficulty, on account of the wide variation in the characteristics of the stones commonly used. Stones belonging to the same classes frequently differ greatly in their physical properties and much care needs to be exercised in securing materials of proper strength and endurance.

The qualities which are of importance in the selection of stone for structural uses are strength, durability, appearance, and cost. The relative importance of these in any particular structure depends upon the location of the structure and the purpose for which it is intended. For ordinary masonry, the most important quality of the stone is usually its durability. The element of strength is commonly of minor consequence, except in portions of a structure where the conditions are such as to bring severe stresses upon the masonry. A pleasing appearance is always desirable, but any stone possessing proper structural qualities may usually be so employed as to produce a good effect, where the purpose of the structure is not distinctly artistic. In architectural and monumental work, the appearance of the stone may be of first importance, while the strength of the stone, or its cost, is of less consequence. Stone for such uses, when in exposed situations, must possess durability in order to preserve the beauty of the structure and prevent disfigurement or discoloration of its surfaces.

The cost of the stone is always a matter of importance, commonly limiting the choice, and frequently being the determining factor in selection of stone. The cost of stone depends mainly upon the ease with which it may be quarried and worked, and the distance and means of transportation to the place where it is to be used. The equipment of a quarry for handling and working stone often determines its availability for a particular use. The kind of finish

to be given the surfaces, and the suitability of the material to the proposed treatment are also important in their effects upon cost.

A good building stone should be dense and uniform in structure, and should have no seams or crow-foots filled with material which may disintegrate and form cracks upon exposure. The fracture should be clean and sharp, and the surface free from earthy appearance.

Hardness and toughness are important properties in a stone which is to be subjected to wear or abrasion of any kind. Stones lacking in toughness and easily abraded have sometimes been seriously defaced by dust and sand particles carried by strong winds. The hardness of the stone depends both upon the hardness of the minerals of which it is composed and upon the firmness with which they are bound together. Toughness depends upon the resistance to separation of the mineral grains. Rocks of hard material may be lacking in toughness and easily worked when weakly cemented.

42. Classification of Building Stones.—All rocks are aggregations of various mineral constituents, more or less firmly held together. Geologically, as to their mode of occurrence, they are divided into three groups, as follows:

(1) *Igneous Rocks.*—Those which have been forced up in a molten condition from unknown depths and subsequently cooled. When the molten rock has cooled and solidified below the surface of the ground it is known as *plutonic*, when above ground as *volcanic*.

(2) *Sedimentary or Stratified Rocks.*—Rocks formed by being deposited as sediment in layers, and consequently showing bedding lines and stratifications. Limestones and sandstones belong in this class.

(3) *Metamorphic Rocks*, formed by subjecting igneous or stratified rocks to great heat or pressure or to both.

Building stones may also be classified according to their chemical and physical properties into three groups:

Crystalline, siliceous rocks, including granites, gneisses, traps, etc.

Calcareous rocks, including limestones and marbles.

Fragmental rocks, including sandstones and slates.

Granite is a crystalline siliceous rock of igneous origin. It consists essentially of quartz, with some feldspar and usually mica. It is readily quarried into blocks of regular shape, but is very hard and tough and is expensive to cut for ornamental work. It is the strongest and most durable of our building stones in common use, and is very generally employed in important work where these qualities are of special importance. Heavy foundations, base courses,

water tables, and columns in important buildings are very commonly of granite.

The color of granite is usually gray, but pink, red, and black granites are found. It is largely used in monumental work and in architecture for exterior work where the most beautiful and durable results are desired. The use of machinery for working the stone has made this use economically feasible.

Syenite is a rock similar to granite, but composed mainly of feldspar instead of quartz. It has much the same qualities as granite and is usually classed as granite when used.

Diorite and *Gabbro* are rock of the same general character as granite but differing in mineral composition. They are usually classed commercially as granites.

Gneiss is a metamorphic rock of the same composition as granite. It is metamorphosed granite or syenite, and usually classed as granite, being often called stratified or bastard granite. Gneiss differs from granite in having a somewhat laminated structure which causes it to split in parallel layers. It is often used for flagging and paving blocks on this account.

Granites are found quite widely distributed in the mountain regions of the United States. The main supply comes from the New England States, where large quarries are in operation and have gained wide reputation. Commercial granites of good quality are found in the South Atlantic States, in Wisconsin and Missouri, while Montana, Wyoming, Colorado, California, and Washington are plentifully supplied with granite which is comparatively undeveloped.

Limestones are sedimentary calcareous rocks, consisting mainly of the mineral calcite, which is composed of calcium carbonate ($CaCO_3$). They also usually contain small amounts of iron oxide, silica, and clay. Magnesia is also commonly present in the pure limestones in very small amounts, and varying—through the magnesian limestones, in which 10 per cent or more of magnesia is present—to dolomite, which consists mainly of the mineral dolomite $(CaMg)CO_3$.

Limestone varies from stone soft enough to cut with a saw to hard material which works with difficulty. Some of the soft stones harden on exposure and are durable in use, the Topeka stone used in Kansas being of this character. Many of the fine-grained, light-colored limestones form excellent building material; they are hard and tough and show good durability in use, although inferior in this respect to the best sandstone and granite. Some of them are used

in ornamental work and take good polish. Stone containing pyrite is apt to show poor weathering qualities, while spots of flint found in many of these stones are objectionable on account of weathering unevenly and sometimes causing the stone to split under frost action.

The following analysis of typical limestones are given by Ries[1] to show the range of chemical composition:

	I	II	III	IV
Calcium carbonate ($CaCO_3$)	97.26	54.53	81.43	98.91
Magnesian carbonate ($MgCO_3$)	0.37	39.41	15.04	0.58
Alumina (Al_2O_3), Ferric oxide (Fe_2O_3)	0.49	0.26	0.57	0.63
Silica (SiO_2)	1.69	3.96	2.89	0.10
Water (H_2O)	.....	1.50	0.08	....

Limestones exist in large quantities through the States of the Middle West, and are locally developed in many places. The well-known Bedford, Indiana, stone is extensively used and shipped for considerable distances.

Marbles are limestones which have been subjected to metamorphic action. In composition they are identical with limestones, or dolomites, but are crystalline in texture and may be polished. The term marble is commonly used to designate any limestone capable of taking a polish.

Marbles are commonly employed for interior finish in buildings and for monumental work. The scarcity and cost of the best marbles have prevented their extensive use for ordinary building construction. Their weathering properties are similar to those of limestone, although some of the more ornamental ones are suitable for interior work only. For structural work the more dense fine-grained stone is to be preferred.

Sandstones are essentially grains of quartz cemented together. Iron oxide, silica, carbonate of lime, or clay may be the cementing medium. The character of the stone varies with that of the cement binding the sand grains.

Sometimes other minerals than quartz are present in sandstones, as feldspar, mica, or pyrite, thus modifying the character of the stone, usually rendering it less durable. Sandstones in which silica is the cementing material are usually the most durable. They are commonly light in color. When considerable silica is present, the stone

[1] Building Stones and Clay Products, New York, 1912.

is very hard and difficult to work, while some stones containing less cement work easily and remain gritty under wear.

Sandstone in which the cement is iron oxide is usually of a red or brown color. These stones usually work easily, and are often durable in use as building stones. When the cementing material is carbonate of lime, the stone usually possesses fair strength, but is not often so durable as that with silica or iron oxide. These stones are usually light colored, soft and easy to work. Clay as a cement in sandstone is usually less desirable than the others; the stone containing it is not so strong; it absorbs water and may be liable to injury from frost. When present in small amount and uniformly distributed through the stone, clay may make the stone easier to work without otherwise injuring it.

"Sandstones, as a rule, show good durability. Some of the softer ones may disintegrate under frost action. Those with clay seams are liable to split with continued freezing. Mica scales, if abundant along the bedding planes, are also likely to cause trouble, and this is aggravated if the stone is set on edge instead of on bed. A striking example of this is the Connecticut brownstone so extensively used in former years for fronts in many of the Eastern cities. In order to get a smooth surface it was rubbed parallel with the bedding, and the stone set in the building on edge. The result is that hundreds of buildings put up more than fifteen or twenty years ago are scaling badly, and in many cases the entire front has been redressed."[2]

Sandstones are of sedimentary origin and are more or less in layers. They should always be laid on their natural beds, and are apt to scale off if placed on edge. They vary in texture from grains of powdery fineness to those in which the grains are of coarse sand. The fine-grained stones are usually the strongest and most durable.

Sandstone is quite widely distributed over the United States, and is one of the most desirable and most extensively used building stones. Many quarries are in use throughout the country for local purposes, while a few quarries supply stone for wider distribution. The Berea stone of Ohio is frequently shipped to considerable distances. The brownstone of Connecticut, Medina sandstone of western New York, Kettle River sandstone of Minnesota are examples of well-known stones in common use.

Slate is a metamorphic rock produced from clay or shale. It is characterized by a tendency to split into thin sheets with smooth surfaces. The direction of this cleavage is not parallel to the bedding and has probably been caused by heavy lateral pressure. These

[2] Ries, Building Stones and Clay Products, p. 165.

sheets of slate are strong under transverse loading and quite impervious to water. They therefore make good roof covering, or may be used as flags for spanning openings. They are also commonly used for blackboards, school slates, etc. The color of slate is commonly dark blue, gray, or black, although green and red slates are also found.

Good slate should be dense and tough and not corrodible by atmospheric gases. When loaded transversely, it should bend appreciably before breaking, and should show a modulus of rupture from 7000 to 10,000 lb./in.^{2}

Most of the slate now in use comes from the New England and Middle Atlantic States, notably from Vermont and eastern Pennsylvania. Important quarries have also been opened in Arkansas and California.

43. Strength of Building Stone.—The loads brought upon masonry structures are rarely sufficient to tax the strength of the stone in compression. The strength of masonry is not directly dependent upon that of the stone used in its construction. The strength of the mortar, thickness of joints, and the care and accuracy used in bedding the stones have important effects upon the strength of the masonry. It is desirable that building stone should be strong and capable of resisting heavy loads, and tests of the strength of the stone may show whether the stone is of good quality and fit for use.

When stone is to be used to span openings and carry transverse loads, its strength is important and care should be taken in its selection. The ability of stone to resist cross-bending stresses is mainly dependent upon its tensile strength. Tests of transverse strength may serve to detect brittleness and lack of toughness or uniformity in the texture of the stone.

Tests for Compressive Strength.—The compressive strength of stone is determined by measuring the loads necessary to crush small blocks cut from the stone. The results of such tests vary with the sizes and shapes of the blocks tested and the methods of placing them in the testing machine. It is necessary in comparing the strengths of different stones to use a standard form and size of specimen and standard method of testing. It is usual to use small cubes, 2 inches on the edge. The size does not seem to very greatly affect the resistance per unit area, but it is desirable to use blocks of the same size in making comparative tests.

The shape of the block is highly important in its effect upon the results of such tests. When subjected to compression, materials

of this kind break by shearing on planes making angles of about 30° with the direction of the compressing force. The ratio of the height of the specimen to its lateral dimensions is therefore important. The strength of the flat slab is much greater than that of a cube, while a prism whose height is greater than its breadth will show less strength on the test.

The test of small specimens gives no indication of the actual strength of the stone in large masses, and tests of this kind can be of value only as indicating the quality of the material, through comparison with the results of similar tests applied to other stones.

The method of preparing the specimen may be quite important in the results of a test. When the dressing is done with hand tools, the shocks frequently have the effect of weakening the internal structure of the stone. This effect with small specimens may amount to a decrease of 30 or 40 per cent as compared with the strength of sawed blocks. The use of sawed test pieces is desirable in such work.

The manner of placing the specimen in the testing machine is also important. It is essential that the test piece be accurately centered in the machine, and that it be evenly in contact with the pressing surfaces, in order to distribute uniformly the compressing force over the area of the block. If the surfaces of the test piece be carefully ground to parallel planes, and the piece carefully centered in the machine in exact contact with the metal surfaces, the best results will be obtained. This method, however, involves considerable labor in preparation of the specimen, and is expensive. The more common method is to set the specimen in a thin bedding of plaster of paris between the plates of the machine and leave it under light pressure for a few minutes, to allow the plaster of paris to set, before applying the load. This method, if carefully handled, gives uniform results, although the strength shown is somewhat less than that obtained by using ground surfaces.

A block of stone may have much less strength in one direction than in another. Most rocks have planes of cleavage in one direction in which they split more easily than in other directions. These planes are usually parallel to the natural bed of the rock and are known as the rift of the rock. Care should be taken to place the test specimen on its natural bed, or in such position that the compression is applied in a direction normal to the rift.

Compressive Strength.—The results of tests upon building stones show a wide variation in compressive strengths of different samples

of the same classification, as well as between different classes of stone.

Hard limestones usually show crushing strengths of 8000 to 12,000 lb./in.^{2}, although softer stones of good quality may run from 3000 to 6000 lb./in.^{2}

Sandstones used in building vary in compressive strength from about 4000 to 15,000 lb./in.^{2} The better grades of stone usually reach 9000 to 12,000 lb./in.^{2}

Granites of good quality should show a crushing strength of 10,000 to 20,000 lb./in.^{2}

Transverse Strength.—Tests of transverse strength are usually made on a small bar of stone, 1 inch square in section, and the method of preparing the specimen is important in its effect upon the results of the test. Comparatively few data exist concerning the strengths of stone under transverse loadings. The following table gives approximate values which have been obtained for ordinary stone used in building.

MODULUS OF RUPTURE, LB./IN.^{2}

Granite	from 1400 to 2500
Limestone	from 500 to 3000
Sandstone	from 600 to 2000

In selecting stone for use as lintels, or where it is to carry transverse loads, it is desirable that the stone be tested in blocks of size comparable to those in which it is to be used. The results of tests upon small specimens is not of much value for this purpose.

The table on p. 68, by Herbert F. Moore, is taken from Merriman's "American Civil Engineer's Pocket Book."

44. Durability of Building Stone.—That stone should be durable under the conditions of use is evidently one of the most important points to be considered in the selection of material for use in construction. The situation in which the stone is to be placed and the climatic or other conditions which may affect the durability should therefore be carefully considered. Local conditions have frequently been overlooked in selecting stone, with disastrous results. The White House at Washington is of sandstone which requires frequent painting. The obelisk, in perfect condition after long exposure in Egypt, began to disintegrate almost immediately when set up in New York City. The Parliament House, built of stone selected with the greatest care, is not able to resist the disintegrating influences of the London atmosphere.

TABLE VI.—DATA FOR BUILDING STONES OF GOOD QUALITY

VALUES BASED MAINLY ON TEST DATA FROM THE WATERTOWN ARSENAL

Kind of Stone.	Weight, Lbs. per Cubic Feet.	Compressive Strength, Lbs. per Square Inch.	Shearing Strength, Lbs. per Square Inch.	Modulus of Rupture, Lbs. per Square Inch.	Modulus of Elasticity, Lbs. per Square Inch.	Coefficient of Expansion per Deg. F.	Absorption of Water Per Cent of Weight of Stone.
Granite, range,	160 to 170	15,000 to 26,000	1800 to 2800	1200 to 2200	5,900,000 to 9,800,000		
Average..	165	20,200	2300	1600	7,500,000	0.0000040	0.5
Sandstone, range,	135 to 150	6,700 to 19,000	1200 to 2500	500 to 2200	1,000,000 to 7,700,000		
Average..	140	12,500	1700	1500	3,300,000	0.0000055	5.0
Limestone, range,	140 to 180	3,200 to 20,000	1000 to 2200	250 to 2700	4,000,000 to 14,000,000		
Average..	160	9,000	1400	1200	8,400,000	0.0000045	7.7
Marble, range,	160 to 180	10,300 to 16,100	1000 to 1600	850 to 2300	4,000,000 to 12,600,000		
Average..	170	12,600	1300	1500	8,200,000	0.0000045	0.4
Slate, range,	170 to 180	14,000 to 30,000	....	7000 to 11,000	13,900,000 to 16,200,000		
Average..	175	15,000	......	8,500	14,000,000	0.0000058	0.5
Trap, average	185	20,000					

The range of changes in temperature, presence of moisture and gases in the atmosphere, and the action of winds and dust are the principal causes of deterioration in stones used in structures.

Expansion and contraction due to changes in temperature create an almost continual tendency to motion among the particles of the stone, an effect which is felt mainly at the exposed surfaces

where expansions are very unequal, and may cause the scaling of the surface layers. Surfaces exposed to the direct rays of the sun are most affected from this cause. In a number of instances, scaling of the surfaces on the south side of buildings has been observed, when the less exposed sides were free from it.

Frost Action.—When stone saturated with water is frozen, the expansion of the liquid in freezing causes a heavy internal pressure, which may be greater than the tenacity of the stone. In the climate of the Northern United States this is commonly one of the most active causes of disintegration of building stones, and the ability to resist frost action is of chief importance. The results of the action of frost on a stone depend upon the porosity of the stone and upon the texture and toughness of the material.

Granite usually absorbs not more than 1 per cent of water, and is not often appreciably affected by frost. Sandstones and limestones may absorb from about 2 to 12 or even 15 per cent. Ordinarily, a good stone that does not absorb more than 4 or 5 per cent of water may be expected to stand frost well. Some more porous stones have also shown well in use. A porous stone of coarse texture is more apt to resist frost action than one of fine texture. Moisture escapes more readily and the stone is less likely to be saturated when frozen.

Fire Resistance.—Any building stone may be injured if subjected to high heat as in the case of serious fires. This injury is intensified by contact of water when so heated. Unequal expansions and sudden surface contractions are likely to cause internal stresses beyond the strength of the stone.

Granites are apt to split and spall badly on the surface and usually show poor fire-resisting qualities. Limestones usually resist fire better than granite until the heat becomes sufficient to drive off the carbonic acid. At high heats they are destroyed. When suddenly cooled by water, limestone is likely to spall badly. Sandstones usually withstand fires better than other building stones, sometimes coming through severe fires without serious injury. They are, however, likely to spall and crack under the combined action of a hot fire and water.

Chemical Agencies.—Rock to be durable in use as building stone must be capable of resisting changes due to the presence of water and gases in the atmosphere.

Certain ingredients in the rock may be soluble in water carrying acids in solution; limestones commonly weather in this way, the carbonate of lime being somewhat soluble in water containing car-

bonic or sulphurous acid, hence these stones are usually liable to surface deterioration in cities. The extent of such deterioration is greater for the more absorbent stones.

Building stone may sometimes be discolored by the oxidation of pyrite or other iron compounds in its surface. This may or may not be an injury to the appearance of the structure. Pyrite is apt to cause rusty blotches which are objectionable, although when evenly and finely distributed through sandstone the result is sometimes enhances its appearance. Sandstones in which iron oxide is the cementing medium are often changed in color by oxidation. Siliceous sandstones are not affected in this manner.

Seasoning of Stone.—All stone is improved by being allowed to stand and dry out before being used in construction, the evaporation of the quarry water being accompanied by hardening of the stone, and the formation of a crust upon the surface. In most cases this indurating effect is comparatively small, but some soft limestones and sandstones, which are easily cut and weak when first quarried, soon acquire considerable hardness and strength, the supposition being that the quarry water contains a small amount of cementing material which is deposited in the pores of the stone upon the evaporation of the water. For this reason the cutting of the stone should be done before the seasoning has taken place, in order that the surface skin may not be broken. This is particularly the case where elaborate dressing or carving is to be done.

Tests for Durability.—Observations of the stone where it has been used in construction or where it has been long exposed in the quarry is the best means of determining the probable durability of a stone. Stone frequently varies considerably in character in different parts of the same quarry, and this must be taken into account in the selection.

There are no standard tests for durability. A number of tests have been proposed and sometimes applied for comparisons of stones, but there is no standard to which they may be referred.

Absorption Tests.—Tests to determine the amount of water absorbed by stone are sometimes made. A stone absorbing little water is less likely to be injured by frost or atmospheric gases than one absorbing water freely; in making this test, it is usual to dry the stone at 100° C. until it ceases to lose weight, then soak the stone for twenty-four hours in water and weigh again.

$$\text{The percentage of absorption} = \frac{\text{Weight of water absorbed} \times 100}{\text{Weight of dry stone}}.$$

The method recommended for brick (see Section 61) may also

be employed for stone, although there is no standard for comparison of the results.

Frost Tests.—Tests of the effect upon a stone sample of repeatedly freezing and thawing it, while saturated, have sometimes been made. About twenty repetitions are usual, and the loss of weight or the loss in compressive strength of samples is measured. This test requires considerable time and a means of producing low temperatures. The differences obtained are usually very small, and not easy to evaluate.

Another test intended to simulate the effects of freezing is known as the *Brard test*, which consists in boiling the specimen in a concentrated solution of sulphate of soda, then exposing it to the air, and observing the effects as the salt crystallizes in the pores of the stone. This is much more severe than the ordinary freezing test, and may be partly due to chemical action.

Acid Test.—Samples of the stone are sometimes immersed in weak solutions of hydrochloric and sulphuric acid, to determine the presence of soluble material, by noting the loss of weight after several days. Exposure to an atmosphere of carbonic acid, or oxygen, is sometimes employed and changes of color observed. None of these tests has been definitely formulated and standardized.

BOOKS ON BUILDING STONE

Complete descriptions of the various building stones of the United States, with their properties and uses may be found in the following books:

Merrill's "Stones for Building and Decoration."

Ries' "Building Stones and Clay Products."

ART. 11. STONE CUTTING

45. Tools for Stone Cutting.[3]—The kinds of finish used in dressing stone are usually defined by mentioning the tool with which the dressing is done.

The following are the names of the tools commonly employed: Double-face Hammer, Face Hammer, Cavil, Pick, Axe or Pean Hammer, Tooth Axe, Bush Hammer, Patent Hammer, Crandall, Hand Hammer, Mallet, Pitching Chisel, Point, Chisel, Tooth Chisel, Splitting Chisel, and the Plug and Feathers.

Much of the cutting, dressing, and carving of stone is now done at the quarry, and the use of compressed air has greatly reduced the time consumed in these operations.

46. Methods of Finishing the Surfaces.[3]—"All stones used in

[3] Trans. Am. Soc. C.E., Vol. VI, p. 297 et seq.

building are divided into three classes, according to the finish of the surface, viz.:

" 1. Rough stones that are used as they come from the quarry.

" 2. Stones roughly squared and dressed.

" 3. Stones accurately squared and finely dressed.

" In practice the line of separation between them is not very distinctly marked, but one class merges into the next.

" *Unsquared Stones.*—This class covers all stones which are used as they come from the quarry, without other preparation than the removal of very acute angles and excessive projections from the figure. The term *backing,* which is often applied to this class of stone, is inappropriate, as it properly designates material used in a certain relative position in the wall, whereas stones of this kind may be used in any position.

" *Squared Stones.*—This class covers all stones that are roughly squared and roughly dressed on beds and joints. The dressing is usually done with the face hammer or axe, or, in soft stones, with the tooth hammer. In gneiss, it may sometimes be necessary to use the point. The distinction between this class and the third lies in the degree of closeness of joints. Where the dressing on the joints is such that the distance between the general planes of the surfaces of adjoining stones is $\frac{1}{2}$ inch or more the stones properly belong to this class.

" Three subdivisions of this class may be made, depending on the character of the face of the stones.

"(*a*) *Quarry-faced Stones* are those whose faces are left untouched as they come from the quarry." (Squared off for joints only, with freshly split face. In distinction from rock-faced in that the latter may be weather-worn.)

"(*b*) *Pitch-faced Stones* are those on which the arris is clearly defined by a line beyond which the rock is cut away by the pitching chisel, so as to give edges that are approximately true (Fig. 15).

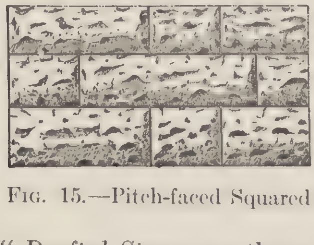


Fig. 15.—Pitch-faced Squared Stone.

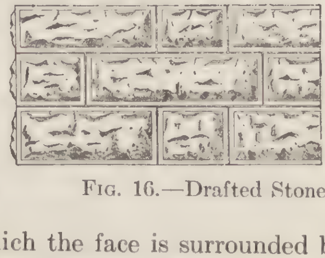


Fig. 16.—Drafted Stone.

" *Drafted Stones* are those on which the face is surrounded by a chisel draft, the space within the draft being left rough (Fig. 16).

Ordinarily, however, this is done only on stones in which the cutting of the joints is such as to exclude them from this class.

" In ordering stones of this class, the specifications should always state the width of the bed and end joints which are expected, and also how far the surface of the face may project beyond the plane of the edge. In practice, the proportion varies from 1 to 6 inches. It should also be specified whether or not the faces are to be drafted.

" *Cut Stones.*—This class covers all squared stones with smoothly dressed bed and joints. As a rule, all the edges of cut stones are drafted, and between the drafts the stone is smoothly dressed. The face, however, is often left rough where the construction is massive.

" In architecture, there are a great many ways in which the faces of cut stone may be dressed, but the following are those which will usually be encountered in engineering work.

" *Rough-pointed.*—When it is necessary to remove an inch or more from the face of a stone, it is done by the pick or heavy point until the projections vary from ½ inch to 1 inch. The stone is then said to be rough-pointed. (Fig. 17.)

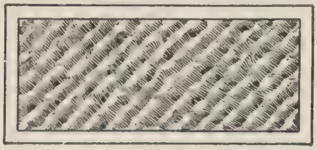


Fig. 17.—Rough-pointed.

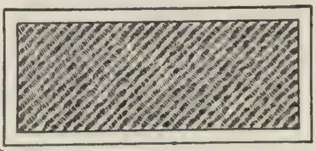


Fig. 18.—Fine-pointed.

" *Fine-pointed.*—If a smoother finish is desired, rough-pointing is followed by fine-pointing, which is done with a fine point. Fine-pointing is used only where the finish made by it is to be final, and never as a preparation for a final finish by another tool.

" *Crandalled.*—This is only a speedy method of pointing, the effect being the same as fine-pointing, except that the dots on the stone are more regular. The variations of level are about ⅛ inch and the rows are made parallel. When other rows at right angles to the first are introduced, the stone is said to be cross-crandalled.

"*Axed or Pean-Hammered and Patent-Hammered.*—These two vary only in the degree of smoothness of the surface which is produced. The number of blades in a patent hammer varies from six to twelve to the inch; and in precise specifications, the number of cuts to the inch must be stated, such as 6-cut, 8-cut, 10-cut, 12-cut. The effect of axing is to cover the surface with chisel marks, which are made parallel as far as practicable. Axing is a fine finish. (Fig. 19.)

"*Tooth-axed.*—The tooth-axe is practically a number of points, and leaves the surface of the stone in the same condition as fine-pointing. It is usually, however, only a preparation for bush-hammering, and the work is done without regard to effect, as long as the surface of the stone is sufficiently leveled.

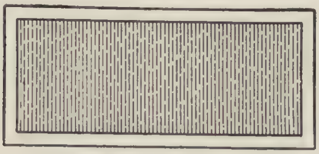


Fig. 19.—Axed or Pean-Hammered.

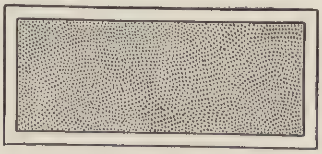


Fig. 20.—Bush-Hammered.

"*Bush-hammered.*—The roughness of the stone in pounded off by the bush hammer, and the stone is then said to be *bushed.* (Fig. 20.) This kind of finish is dangerous on sandstone, as experience has shown that sandstone thus treated is very apt to scale. In dressing limestone which is to have a bush-hammered finish, the usual sequence of operations is: (1) rough-pointing, (2) tooth-axing, (3) Bush-hammering.

"*Rubbed.*—In dressing sandstone and marble, it is very common to give the stone a plane surface at once by the use of the stone saw. Any roughnesses left by the saw are removed by rubbing with grit or sandstone. Such stones therefore have no margins. They are frequently used in architecture for string courses, lintels, door-jams, etc., and they are also well adapted for use in facing the walls of lock-chambers and in other locations where a stone surface is liable to be rubbed by vessels or other moving bodies.

"*Diamond Panels.*—Sometimes the space between the margins is sunk immediately adjoining them, and then rises gradually until the four planes form an apex at the middle of the panel. In general, such panels are called diamond panels, and the one just described is called a sunk diamond panel. When the surface of the stone rises gradually from the inner lines of the margins to the middle of the panel, it is called a raised diamond panel. Both kinds of finish are common on bridge-quoins and similar work. The details of this method should be given in the specifications."

The following classification of the surface finish for stone used in masonry is given by the American Railway Engineering Association:[4]

Dressing.—The finish given to the surface of stones or concrete.

Smooth.—Having surface the variations of which do not exceed $\frac{1}{16}$ inch from the pitch line.

[4] Manual, American Railway Engineering Association, 1921.

Fine-pointed.—Having irregular surface, the variations of which do not exceed $\frac{1}{4}$ inch from the pitch line.

Rough-pointed.—Having irregular surface, the variations of which do not exceed $\frac{1}{2}$ inch from the pitch line.

Scabbled.—Having irregular surface, the variations of which do not exceed $\frac{3}{4}$ inch from the pitch line.

Rock-faced.—Presenting irregular projecting face, without indications of tool mark. (Often used indiscriminately with quarry-faced.)

47. Cutting by Machinery.—In large yards and large building operations much of the shaping and dressing of stone is done by machinery. Portable machines using pneumatic tools are frequently employed, such as pneumatic hammers, drills, and chisels, which dress the stone in much the same manner as hand tools. The machines commonly employed also include saws adapted to all classes of stone—cutters for rough surfacing, planers for more accurate surfacing, and rubbing machines for grinding and polishing. The details of these machines and the character of the tools used with them vary with the nature of the stone to be worked.

In dimension stone and trimming work, drawings and dimensions for shaping the stones are provided, and the stones are usually cut at the yard and shipped to the point of use ready to place.

ART. 12. WALLS OF STONE MASONRY

48. Classification of Masonry.—Stone work is commonly divided into two general classes; *ashlar* and *rubble,* depending upon the degree of care exercised in cutting the stone and the closeness of the joints.

Ashlar masonry is that in which the joints are not more than $\frac{1}{2}$ inch thick. The term ashlar is also sometimes extended to include masonry of squared stones in which the joints are not so accurately dressed, but this is not usual.

FIG. 21.—Coursed Ashlar.

FIG. 22.—Random Range.

Ashlar masonry may be divided according to the arrangement of the stones into:

Coursed ashlar, sometimes called *Range masonry* (Fig. 21), arranged in courses of uniform thickness.

Random, range, in which the stones are not arranged in courses (Fig. 22.)

Broken-coursed or *Random-coursed ashlar,* in which broken ashlar work is arranged in more or less continuous courses, or masonry laid in parallel, but not continuous courses.

In the best cut-stone work, as used by architects for public buildings in the cities, the joints may not be more than $\frac{1}{4}$ inch thick, while in first-class masonry in important engineering construction joints from $\frac{1}{4}$ to $\frac{1}{2}$ inch are usually allowed. When the thickness of courses and length of stones in ashlar masonry are specified, the work is known as *dimension stone* masonry.

The exposed surfaces of ashlar masonry may be finished by any of the methods in the preceding section, and the masonry is frequently classified as pitch-faced ashlar, drafted-stone ashlar, or cut-stone masonry, which includes all of the more accurate methods of dressing; pointing, bush-hammering, axing, etc. Pitch-faced and drafted-stone work is often called *rock-faced* ashlar.

Rubble masonry is that which is not dressed or laid with sufficient accuracy to be classed as ashlar, and may include stones roughly squared or those of irregular shapes.

Rubble masonry is usually uncoursed, but sometimes is leveled off into courses at specified heights, and is then known as coursed *rubble.* Figure 23 shows the face of a wall of ordinary uncoursed rubble.

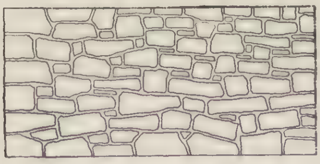


Fig. 23.—Random Rubble.

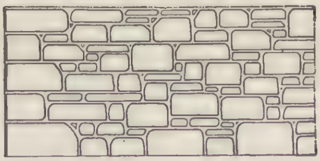


Fig. 24.—Russian Bond.

Figure 24 shows a type of rubble work sometimes used in building construction, in which hammer-dressed joints are more accurately fitted on the face of the wall. Joints $\frac{1}{2}$ to $\frac{3}{4}$ inch may be used in such work. This is sometimes called " Russian Bond " and is usually rock-faced work.

Dry Masonry.—Masonry of rough stone without the use of mortar is sometimes employed and is known as dry masonry. Such walls are frequently used for railway culverts and similar purposes. When stone is roughly placed about the bases of piers or abutments, or on the banks of streams to prevent erosion, it is commonly called *riprap.*

Squared-Stone Masonry is a term frequently used to indicate a class of masonry between ashlar and rubble. When this classification is used, it commonly includes masonry of squared stones, with joints from $\frac{1}{2}$ to 1 inch thick, and the term rubble is limited to the use of irregular and unsquared material.

Trimmings.—In architectural work, an additional classification is sometimes employed to designate stone used for special purposes, such as moldings, sills, caps, etc. These usually require cutting to specified dimensions and close joints.

49. Parts of a Masonry Wall.—The exposed surface of a masonry wall is called its *face*, while the interior surface is known as the *back* of the wall.

Batter is the slope of the surface of a wall, stated as a ratio of horizontal to vertical dimension. The walls of buildings usually have no batter. Retaining walls, bridge piers, and other heavy structures for carrying loads, are commonly given a batter on the face. This gives an appearance of strength and stability to the wall.

Coping is a course of stone on top of the wall to protect it and give a finished appearance. The coping usually projects a few inches over the surface of the wall.

Courses.—A horizontal layer of stones in the wall is called a course, the arrangement of courses in a wall being determined by the character of the material and the appearance desired. When the stone may be readily obtained in blocks of uniform thickness an arrangement in courses, with the thickest courses at the bottom, gives an appearance of stability, and is common practice in engineering structures. In architectural work, the arrangement of courses may be made to accord with other features of the design of the structure.

Facing and Backing.—The stones which form the face of the wall are called facing, while those forming the back of the wall are called backing. In the construction of walls, the facing and backing are commonly of different classes of masonry. An ashlar facing is frequently joined to a rubble or concrete backing.

In heavy walls the masonry of the interior of the walls, between the facing and backing, is known as filling, and this may sometimes be different from either the facing or backing. In constructing walls, the facing and backing should always be well bonded, so that the whole acts together in supporting loads or resisting pressures.

Headers and Stretchers.—A stone whose greatest dimension lies perpendicular to the face of the wall is called a header; one whose greatest dimension is parallel to the face of the wall is a stretcher.

The *bond* of the masonry in the wall is secured by proper arrangement of headers and stretchers. The vertical joints in adjoining courses should not be too nearly in the same plane. A stone in any course should break joints with the stones in the course below by a distance at least equal to the depth of the course. The strongest bond is obtained by using an equal number of headers and stretchers in the face of the wall, a header being placed over the middle of each stretcher as shown in Fig. 21.

In the use of coursed ashlar facing, the rubble filling and backing is usually also coursed at the same height as the ashlar. Sometimes in massive work the filling may be constructed of irregular uncoursed rubble. Usually, however, concrete would be used in such work instead of rubble.

In the walls of buildings with ashlar facings, the rubble backing is laid in courses with the ashlar and occasional headers are run through the wall as shown in Fig. 25. Brick backing is commonly used for such work and is usually preferable to rubble. (Fig. 26.)

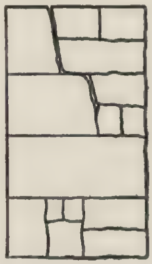


Fig. 25.—Ashlar Facing with Rubble Backing.

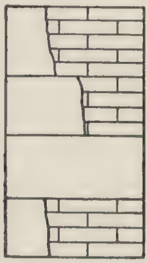


Fig. 26.—Ashlar Facing with Brick Backing.

In building work, thin ashlar, 2 to 4 inches thick, is sometimes employed as a veneer on the exterior of a wall of rubble or brick; this is frequently done in marble buildings. The veneer is tied to the backing by iron clamps, and occasional belt courses of wider stones, extending 6 or 8 inches, into the filling give support to the ashlar.

Dowels and Cramps.—For the purpose of strengthening the bond where great resistance is required, dowels or cramps are often employed. A dowel is a straight bar of iron which enters a hole in the upper side of one stone and also a hole in the lower side of the stone above. A cramp is a bar of iron with ends bent at right angles to the length of the bar, the ends entering holes in the tops of adjacent stones.

50. Setting Stonework.—The layers of mortar between stones

are called *joints.* The horizontal joints are commonly called *beds* or *bed joints.*

The kind of mortar used in stonework depends upon the character of the work. In engineering structures, 1 to 2 or 1 to 3, Portland cement mortar is usually employed. Cement mortar stains many stones and care must be used in architectural work to prevent injury to appearance of the stone surface from this cause. This may often be effected by keeping the bed and joint mortar back from the face and using non-staining mortar for pointing. The bed joint of ashlar stones should be carefully dressed to a plane surface in order that the stone may bear evenly upon the bed. These joints are sometimes cut slightly concave to make them easier to set with close joints at the surface, which brings the loads upon the edges of the stone with danger of chipping the edges.

The vertical joints in ashlar facing should be carefully dressed to a depth of several inches from the face of the wall, but do not need to be accurately dressed the full depth of the stone. The backs of the ashlar stones may be laid as rubble without cutting. The arrangement of headers and stretchers in the rubble backing should be the same as in the ashlar facing to secure good bond throughout the wall.

Placing Stone.—All stones should be set in a full bed of mortar. The mortar bed should be prepared and the stone lowered upon it without disturbing stones already set. The stone must not be slid upon the bed so as to scrape away the mortar. Stones too large to be handled by one man are placed with a derrick, and are settled in place with light blows from a hammer. No cutting or trimming of stone after placing is allowable; the stone must be fitted to its place before spreading the mortar.

In the construction of rubble masonry, less care may be taken in the exact placing of the stones, but it is highly important that all the joints be completely filled. The strength of the masonry depends upon the stone having full bearing on the mortar at all points. The interstices between large stones in rough rubble are filled by driving stone chips into the mortar.

Stratified stones should always be set upon their natural beds, and not set on edge.

Dimensions of Stones.—A rule frequently used is that the width of a stone shall not be less than its height. The length of the stone, to avoid danger from cross-breaking, in well-laid masonry, may be about three times the thickness for the weaker stones, and about five times the thickness for the stronger ones.

Pointing.—In laying masonry it is not feasible to make well-filled, smooth joints at the face of the wall. It is usual, therefore, after the masonry has been laid, to clear out the joint to a depth of about an inch and refill with special mortar. This is called pointing the masonry. If, in placing the masonry, the mortar in the joints is not brought quite to the face of the wall, the labor of pointing may be somewhat lessened.

The joint is cleared and brushed out to a depth of at least an inch and well moistened before applying the pointing. The mortar is then applied with a small trowel, squeezed in, and smoothed with a special tool called a jointer, which is provided with an edge to form the kind of finish desired. There are a number of ways of finishing joints, of which the most common are shown in Fig. 27.

Fig. 27.—Methods of Finishing Joints.

The best pointing mortar is usually composed of Portland cement and sand, 1 to 1. Coloring matter is added when needed. The mortar is used quite dry, like damp earth. When the face of the stone would be stained by Portland cement, a putty made of lime, plaster of paris, and white lead is sometimes employed. Various non-staining cements are also available.

51. Trimmings.—In the erection of masonry structures, certain special parts are ordinarily required to be of cut stone, which must be of definite form and dimension. These trimmings have to do with the ornamentation of the structure, finishing about openings, or joining different types of construction.

Water-tables with sloping surfaces are used at the top of foundation walls, where they join the narrower upper walls.

Copings, *cornices*, *window sills*, and sometimes *belt-courses* project beyond the surface of the wall; they must have sufficient width to be firmly held in the wall, and to balance on the wall in laying. The projections should also have upper surfaces which slope away from the wall, and a drip (called the wash) underneath to cause water to drop off at the outer edge, the drip being made by cutting a groove on the under side of the stone.

When cut-stone trimmings are used for a brick wall, they should be dimensioned so that they will fit into the brickwork without splitting the brick.

Window sills just the width of the opening and not built into the wall at the ends are called *slip sills*, while those extending into the walls are called *lug sills*. The ends of lug sills are rectangular, the sloping surface of the sill being made the width of the opening. Lug sills should be bedded only at the ends to prevent cross-bending stresses due to the weight of the wall.

When stone lintels are used to span openings, care must be taken in selecting the stone, and making sure that it has the transverse strength necessary to carry the load. When necessary an angle bar or I-beam may be used to support the lintel, a recess being cut into the back of the stone for this purpose.

52. Specifications for Stone Masonry.—The following general requirements for stone masonry and special requirements for bridge and retaining wall masonry are recommended by the American Railway Engineers' Association in their Manual for 1921:

General Requirements

Stone.—3. Stone shall be of the kinds designated and shall be hard and durable, of approved quality and shape, free from seams or other imperfections. Unseasoned stone shall not be used where liable to injury by frost.

Dressing.—4. Dressing shall be the best of the kind specified.

5. Beds and joints or builds shall be square with each other, and dressed true and out of wind. Hollow beds shall not be permitted.

6. Stone shall be dressed for laying on the natural bed. In all cases the bed shall not be less than the rise.

7. Marginal drafts shall be neat and accurate.

8. Pitching shall be done to true lines and exact batter.

Mortar.—9. Mortar shall be mixed in a suitable box, or in a machine mixer, preferably of the batch type, and shall be kept free from foreign matter. The size of the batch and the proportions and the consistency shall be as directed by the engineer. When mixed by hand the sand and cement shall be mixed dry, the requisite amount of water then added and the mixing continued until the cement is uniformly distributed and the mass is uniform in color and homogeneous.

Laying.—10. The arrangement of courses and bond shall be as indicated on the drawings, or as directed by the engineer. Stone shall be laid to exact lines and levels, to give the required bond and thickness of mortar in beds and joints.

11. Stone shall be cleansed and dampened before laying.

12. Stone shall be well bonded, laid on its natural bed and solidly settled, into place in a full bed of mortar.

13. Stone shall not be dropped or slid over the wall, but shall be placed without jarring stone already laid.

14. Heavy hammering shall not be allowed on the wall after a course is laid.

15. Stone becoming loose after the mortar is set shall be relaid with fresh mortar.

16. Stone shall not be laid in freezing weather, unless directed by the Engineer. If laid, it shall be freed from ice, snow, or frost by warming. The sand and water used in the mortar shall be heated.

17. With precaution, a brine may be substituted for the heating of the mortar. The brine shall consist of 1 pound of salt to 18 gallons of water, when the temperature is 32° F.; for every degree of temperature below 32° F., 1 ounce of salt shall be added.

18. Before the mortar has set in beds and joints, it shall be removed to a depth of not less than 1 inch. Pointing shall not be done until the wall is complete and mortar set; nor when frost is in the stone.

19. Mortar for pointing shall consist of equal parts of sand, sieved to meet the requirements, and Portland cement. In pointing, the joints shall be wet, and filled with mortar, pounded in with a "set-in" or calking tool and finished with a beading tool the width of the joint, used with a straight-edge.

Bridge and Retaining Wall Masonry, Ashlar Stone

Bridge and Retaining Wall Masonry, Ashlar Stone.—20. The stone shall be large and well proportioned. Courses shall not be less than 14 inches or more than 30 inches thick, thickness of courses to diminish regularly from bottom to top.

Dressing.—21. Beds and joints or builds of face stone shall be fine-pointed, so that the mortar layer shall not be more than $\frac{1}{2}$ inch thick when the stone is laid.

22. Joints in face stone shall be full to the square for a depth equal to at least one-half the height of the course, but in no case less than 12 inches.

Face or Surface.—23. Exposed surfaces of the face stone shall be rock-faced, with edges pitched to the true lines and exact batter. The face shall not project more than 3 inches beyond the pitch lines.

24. Chisel drafts $1\frac{1}{2}$ inches wide shall be cut at exterior corners.

25. Holes for stone hooks shall not be permitted to show in exposed surfaces. Stone shall be handled with clamps, keys, lewis, or dowels.

Stretchers.—26. Stretchers shall not be less than 4 feet long with at least one and a quarter times as much bed as thickness of course.

Headers.—27. Headers shall not be less than 4 feet long; shall occupy one-fifth of face of wall; shall not be less than 18 inches wide in face; and where the course is more than 18 inches high, width of face shall not be less than height of course.

28. Headers shall hold in heart of wall the same size shown in face, so arranged that a header in a superior course shall not be laid over a joint, and a joint shall not occur over a header; the same disposition shall occur in back of wall.

29. Headers in face and back of wall shall interlock when thickness of wall will admit.

30. Where the wall is 3 feet thick or less, the face stone shall pass entirely through. Backing shall not be permitted

Backing.—31*a*. Backing shall be large, well-shaped stone, roughly bedded and jointed; bed joints shall not exceed 1 inch. At least one-half of the backing stone shall be of the same size and character as the face stone and with parallel ends. The vertical joints in back of wall shall not exceed 2 inches. The interior vertical joints shall not exceed 6 inches.

Voids shall be throughly filled with concrete, or with spalls, fully bedded in cement mortar.

31*b*. Backing shall be of concrete, or of headers and stretchers, as specified in paragraphs 26 and 27, and heart of wall filled with concrete.

Paragraphs 31*a* and 31*b* are so arranged that either may be eliminated according to requirements.

32. Where the wall will not admit of such arrangement, stone not less than 4 feet long shall be placed transversely in heart of wall to bond the opposite sides.

33. Where stone is backed with two courses, neither course shall be less than 8 inches thick.

Bond.—Bond of stone in face, back, and heart of wall shall not be less than 12 inches. Backing shall be laid to break joints with the face stone and with one another.

Coping.—35. Coping stone shall be full size throughout, of dimensions indicated on the drawings.

36. Beds, joints and top shall be fine-pointed.

37. Location of joints shall be determined by the position of the bed plates as indicated on the drawings.

Locks.—38. Where required, coping stone, stone in the wings of abutments, and stone on piers, shall be secured together with iron cramps or dowels, to the position indicated on the drawings.

BRIDGE AND RETAINING WALL MASONRY, RUBBLE STONE

39. The stone shall be roughly squared and laid in irregular courses. Beds shall be parallel, roughly dressed, and the stone laid horizontal to the wall. Face joints shall not be more than 1 inch thick. Bottom stone shall be large, selected flat stone.

40. The wall shall be compactly laid, having at least one-fifth the surface of back and face headers arranged to interlock, having all voids in the heart of the wall thoroughly filled with concrete, or with suitable stones and spalls, fully bedded in cement mortar.

ART. 13. STRENGTH OF STONE MASONRY

53. Compressive Strength.—Stone masonry varies widely in strength according to the character of the construction. The accuracy with which the joints are dressed, the strength of the mortar, the bonding of the masonry and size of blocks of stone are more important than the strength of the stone itself.

No experimental data are available which show the actual strength of masonry as used. The mortar has usually much less strength than the stone, and in some experiments on brick piers, the mortar seemed to squeeze out, causing the failure of the brick in tension. The loads to which masonry is ordinarily subjected are much less than its actual strength, but when heavy loads are being carried by piers or arches, it is frequently necessary to proportion the section to the load.

When the masonry is of cut stone with thin joints and Portland cement mortar, the strength of the masonry may be proportioned to the strength of the stone. For rubble with thick joints, the strength of the stone has no material effect upon the strength of the masonry.

The loads used in practice vary quite widely according to the views of the designers. Building laws of the various cities differ considerably in the loads allowed. The following may be considered as conservative values for the limits of safe loading:

Cut Stone.—Dressed stone, with joints not more than $\frac{3}{8}$ inch in first class Portland cement mortar:

	Tons per Square Foot.
Granite	50 to 60
Hard limestone or marble	35 to 40
Sandstone	25 to 30

The siliceous sandstones may have larger values, while the soft limestones should be reduced.

For ashlar of good quality as commonly laid with $\frac{1}{2}$-inch joints in Portland cement:

	Tons per Square Foot.
Granite	40 to 45
Limestone, hard	35 to 40
Sandstone	25 to 30

Rubble.—For masonry composed of large blocks of squared stone, 1-inch joints, in Portland cement mortar:

	Tons per Square Foot.
Sandstones or limestones	10 to 20
Granite	20 to 30
Uncoursed rubble:	
In cement mortar	5 to 8
In lime mortar	3 to 5

For an ashlar pier whose height exceeds ten times, or a rubble pier whose height exceeds five times, its least lateral dimension, these figures should be reduced. Piers of small dimensions carrying heavy loads should always be of ashlar. Rubble should not be used for less thicknesses than 20 to 24 inches when it is necessary to develop the full strength of the masonry.

Failures of masonry most frequently occur through defective foundation or workmanship. Masonry, to develop its full strength,

must always be adequately supported, so that unequal pressures are not produced through settlement.

Weight of Masonry.—In determining loads, it is usually necessary to estimate the weight of masonry. This depends upon the specific gravity of the stone and the closeness of the joints. The following table gives approximate weights for the different classes of stone masonry:

	Pounds per Cubic Foot.
Limestone, ashlar	155 to 165
Limestone, squared rubble	145 to 150
Limestone, rough rubble	135 to 140
Granite, ashlar	165 to 170
Granite, squared rubble	155 to 160
Sandstone, ashlar	135 to 150
Sandstone, rubble	120 to 140

54. Capstones and Templets.—When loads are to be transferred from the ends of beams or columns to masonry walls or piers, bearing blocks may be necessary to distribute the loads properly over the surface of the masonry. When used under a column or post, these blocks are called *capstones;* when used in walls to carry the ends of beams, they are *templets.*

In placing bearing blocks, the loads should always be centered on the top of the block, if possible, so as to produce uniform pressure upon the masonry below; in all cases, the center of pressure must be within the middle third of the base to avoid a tendency to open the joint between the bearing block and the masonry.

In Fig. 28, the load F is concentrically placed on the bearing block and the pressure is evenly distributed over the supporting area.

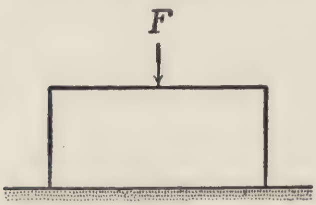


FIG. 28.—Concentric Load.

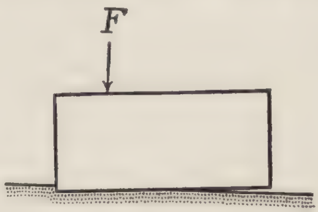


FIG. 29.—Eccentric Load.

In Fig. 29, the load F is placed near the left edge of the bearing block and the left edge sinks into the supporting area while the right edge rises above.

The theory of pressures for eccentric loading on bearings is as follows:

In Fig. 30:

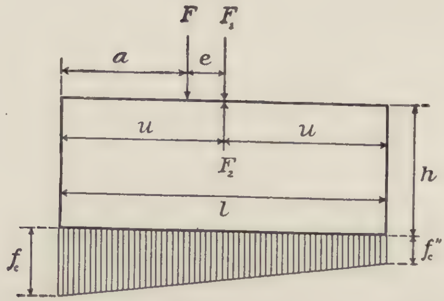


FIG. 30.—Bearing Block with Eccentric Load

Let F = the vertical load on the bearing block;

b = the breadth of the block perpendicular to the plane of the drawing;

l = the length of the block;

$u = \frac{l}{2}$, the distance from the center of the block to the edge of the block;

e = the distance from the center of the block to the center of pressure;

a = the distance from the left edge of the block to the center of pressure.

Equilibrium will not be disturbed by the introduction of two equal and opposite forces acting in the same straight line.

Let the equal and opposite forces F_1 and F_2 act at the center of the block as shown, and let each of these forces be equal to F. The result is a force and a couple replacing a single force which is permissible by the laws of mechanics.

Let f = unit pressure due to concentric load,

$$= \frac{F_1}{bl} = \frac{F}{bl}$$

and let f' = unit pressure due to the moment of the couple (Fe),

$$= \frac{Mu}{I} = \frac{12Fel}{2bl^3} = \frac{6Fe}{bl^2}.$$

Let f_c = unit pressure at left edge of block,

$$= f + f'$$

$$= \frac{F}{bl} + \frac{6Fe}{bl^2} = \frac{F}{bl^2}(l + 6e),$$

and let f_c'' = unit pressure at right edge of block,

$$=f-f'$$

$$=\frac{F}{bl}-\frac{6Fe}{bl^2}=\frac{F}{bl^2}(l-6e).$$

When $e=0,\quad f=\frac{F}{bl}.$

When $e=\frac{l}{6},$

$$f_c=\frac{F}{bl^2}\left(l+\frac{6l}{6}\right)=\frac{2F}{bl},$$ or double that due to a concentric load,

and $$f_c''=\frac{F}{bl^2}\left(l-\frac{6l}{6}\right)=0.$$

When e is greater than $\frac{l}{6}$, the load is distributed over a distance less than l and the unit pressure is correspondingly greater.

Sometimes it is convenient to have the values of f_c and f_c'' in terms of a, the distance from the toe of the wall to the center of pressure, in which case

$$f_c=\frac{F}{bl^2}(4l-6a).\quad \text{and}\quad f_c''=\frac{F}{bl^2}(6a-2l).$$

In designing a bearing block, f_c must not be greater than the safe compressive strength of the masonry. The load is commonly brought on top of a bearing block through an iron plate, which should have such area that the pressure will not be more than one-tenth to one-twelfth of the crushing strength of the stone. The bearing block must have sufficient thickness not to break under the transverse load imposed by the upward pressure of the masonry.

In designing a templet which is to be built into a wall, the weight of wall resting on the top of the templet must be included in determining the pressure on its base.

55. Lintels and Corbels.—A stone lintel is a beam of stone spanning an opening in a wall. The strength of a lintel is determined by the ordinary beam formulas. The safe modulus of rupture may be taken at about one-twelfth to one-tenth of the ultimate modulus for the stone. Mean values of the safe modulus of rupture are about as follows: granite, 180 lb./in.^{2}; limestone, 150, marble, 130; sandstone, 120 lb./in.^{2}. There are, however, certain tough sandstones, specially adapted to this use, which may be used with modulus of 250 to 300 lb./in.^{2}

Beams carrying live loads should not rest upon stone lintels. When the load upon a lintel is a solid masonry wall, it is common to assume that the masonry may arch over the opening, so that the actual weight upon the lintel is only that of a triangle whose height is about three-quarters of the span. This assumes that the lintel will yield somewhat, and be relieved of stress before reaching the maximum load. It is quite possible that in well-built masonry, with cement mortar, the lintel might be removed without the wall above yielding at all. If, however, there is no yielding of the lintel, the pressure upon its upper surface may be the same as at any other point in the same horizontal plane of the wall.

A corbel is a block of stone extending beyond the surface of a wall or pier for the purpose of carrying the end of a beam or an overhanging wall, see Fig. 31. The overhang of the corbel is a cantilever beam, which must have sufficient section at the surface of the wall to resist the bending moment due to the load. The corbel must extend sufficiently into the wall, to give a resultant pressure within the middle third of the base of the corbel ($R = P + W$) as in the case of bearing blocks.

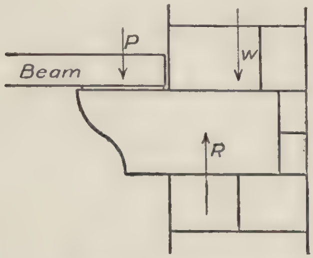


Fig. 31.—Corbels.

Double corbels may be used when necessary, each being separately treated in determining strength. When weight of wall above (W) is lacking, the corbel must be anchored to the wall below by steel ties.

ART. 14. MEASUREMENT AND COST

56. Methods of Measurement.—In engineering work it is usual to estimate stone masonry in cubic yards of actual masonry. When parts of the work are of special character, requiring cut-stone finish, special prices per cubic yard may be given, or the additional costs of the cut surfaces are paid for by the square yard.

In architectural work masonry is measured by the cubic yard or by the perch. A perch may be $16\frac{1}{2}$, 22, or 25 cubic feet, according to the custom of the locality in which the masonry is constructed. In the use of the perch as a unit, it is advisable to state the number of cubic feet to be considered a perch.

In building work it is common to take outside measurements of walls, thus including the corner masonry twice; it is also customary to measure small openings as solid wall. Commonly, openings less than 70 square feet are not deducted. In some cases allow-

ances are made for openings more than 6 feet wide. Customs differ in different parts of the country, and it is necessary to know the local usage, unless the method of measurement is stated.

57. Cost of Stone Masonry.—So many items are included in the cost of masonry and these items vary so widely in different localities that it is not feasible to give any definite values to the costs of different kinds of work. The items of cost include the price of the rough stone at the quarry, the transportation to place of use, dressing joints and faces of stone, mortar for joints, setting the stonework, and pointing the joints.

Rough stones at the quarry are commonly classified into rubble or small stone and dimension stone. Rubble stone includes the more irregular stones and blocks suitable for small ashlar. Dimension stone includes all stone required to be of particular sizes and blocks of large dimensions and definite thicknesses, as required for coursed ashlar. These classes vary according to the kinds of stone in the quarry and the specifications to be met by the stone.

Rubble stone is commonly sold by the ton free on board cars at point of delivery. Prices for rubble stone delivered have varied in various localities from $0.50 to $2 per ton, when wages of quarry men were about $4.50 per day and common labor $1.50. The cost is largely a matter of locality. A ton of rubble stone may lay from about 16 to 22 cubic feet of masonry.

Dimension stone and ashlar in the rough may cost from $0.50 to $1.25 per cubic foot for limestone or sandstone and $0.75 to $1.50 per cubic foot for granite, according to quality and location.

Cost of Stone Cutting.—The cost of cutting ashlar depends upon the hardness of the stone and the shape in which the blocks are received. Some stratified stones require almost no dressing on the bed joints, while other stones need every joint dressed from an irregular surface. With wages of stone cutters at $5 per day, the following may be considered average costs per square foot for cutting to ½-inch joints; granites, 27 to 35 cents; hard sandstones and limestones, 20 to 30 cents; soft stones, 16 to 22 cents. Costs of peculiar face cuttings and of trimmings are so special to particular stones that they are of little value for general use. Sills, lintels, water-tables, and copings are usually sold by the lineal foot.

The cost of sawing and machine dressing is usually much less than that for hand dressing, and varies with the way the stone is handled and the organization of the yard.

NOTE.—For late information on the cost of building stone, see "Marketing of Indiana Limestone," by Mr. H. S. Brightly in Engineering and Mining Journal-Press, Feb. 7, 1925, p. 237.

Mortar Required.—The amount of mortar needed in rubble masonry may vary from about 15 to 35 per cent of the volume of the masonry. Rubble of squared stones with joints 1 inch thick will ordinarily require 15 to 20 per cent, according to the sizes of the stones. For random rubble, stratified stones with flat beds require less than irregular stones. In the use of irregular rubble stones, the careful use of spalls in the larger joints reduces the amount of mortar materially, with saving in cost.

The amount of mortar needed for ashlar work depends upon the sizes of the stones. Ordinary ashlar with $\frac{1}{2}$-inch joints in courses 12 to 20 inches thick requires 4 to 7 per cent of mortar; random ashlar with smaller stones will require more, while with large blocks and thinner joints less will be required.

Cost of Laying Masonry.—The cost of setting stone varies with the size of the job, the organization of the work, and the skill of the masons, as well as with the character of the work itself. In ordinary rubble or square-stone work, such as cellar walls or light retaining walls, a mason should lay a cubic yard of masonry in three or four hours. A helper to two masons or a helper to each mason, according to convenience of work, being required to supply stone and mortar. With masons at 50 cents an hour and helpers at 20 cents, this would cost from \$1.80 to \$2.80 per cubic yard. In large work, where stone is handled by derricks, and rubble constructed of large blocks, the cost of placing the stone is frequently reduced to \$0.85 to \$1.25 per cubic yard. The cost of setting ordinary ashlar varies from about \$3 to \$5 per cubic yard for limestone and sandstone, and from \$6 to \$9 for granite.

The total cost of masonry in place, made up by so many varying items, necessarily varies within wide limits. Ordinary rubble at prices which have existed within the past few years (previous to the War), averages in cost from \$5 to \$7 per cubic yard. Rubble in heavy construction, usually granite, where the stone was quarried on the work and handled by machinery, has run from \$5 to \$11 per cubic yard. Sandstone and limestone bridge masonry, with ashlar facings and rubble backing and filling, usually varies from about \$8 to \$14 per cubic yard.

Gillette's "Handbook of Cost Data" gives a number of detailed statements of costs of stone masonry. Such costs vary in about the same ratio as the pay of labor employed. The unsettled state of prices and labor costs since the War make it impracticable to give costs based upon present prices.

CHAPTER IV

BRICK AND BLOCK MASONRY

ART. 15. BUILDING BRICKS

58. Clay and Shale Bricks.—The cheapness, ease of construction, and durable qualities of good brick masonry make it one of the most desirable materials for general structural work. It is not as largely used in engineering work as stone or concrete, but in building construction it is very extensively employed. The qualities of clay bricks vary widely according to the character of the clay and methods of manufacture, and care must be taken in selection of material in order to secure good results.

Composition of Clay Bricks.—Clay consists primarily of silicate of alumina. Common clays also usually contain certain percentages of iron oxide, magnesia, lime, and alkalies. These are known as fluxes, having the effect, when in considerable quantities, of making the clay fusible. Fire clays contain a low percentage of fluxes, and withstand a high degree of heat without fusing.

Sandy clays contain high proportions of silica in an uncombined state, a factor which, if not in excess, is of value, tending to give stability to the form of the brick. Sand is commonly added to plastic clays for this purpose.

The color of the brick is mainly dependent upon the amount of iron oxide present in the clay. The color varies from white, through buff to red as the percentage of iron oxide increases. The presence of iron oxide is also of value in adding strength and hardness to the brick.

Lime, when present in appreciable quantities, must be finely divided and uniformly distributed through the clay. If in lumps, the slaking of the lime, subsequent to burning, may cause the brick to become distorted and cracked. When in excess, lime neutralizes the color effect of the iron oxide, making the bricks lighter in color, buff or yellow colors being sometimes due to this cause.

Excess of alumina usually makes the clay very plastic and causes it to shrink and crack in drying.

Physical Properties.—The physical properties of clay are of more importance than the chemical composition.

Plasticity is one of the important properties of clay for brick making, as it permits the clay to be worked into a plastic mass, and to be molded into the desired form. Clay shrinks in drying and also in burning, very plastic clay shrinking more than that less plastic. Sand is frequently mixed with clay to reduce excessive shrinkage. The degree of plasticity is sometimes controlled by mixing clays which differ in this respect.

When subjected to high heat, clay gradually becomes soft and fuses together, and as the heat is increased the softening and shrinkage progresses until the material finally melts sufficiently to lose its shape. The temperature required for burning varies widely with different clays, and the degree of burning given to brick depends upon the kind of product desired and the fusibility of the clay.

Manufacture.—There are three methods in use for forming the brick. They are known as the *soft-mud*, the *stiff-mud*, and the *dry-press* methods.

The *soft-mud* process consists in pulverizing the clay or shale and tempering it with water to the consistency of soft mud. This paste is then pressed into wooden molds, which are usually sanded on the surface to prevent the clay sticking, thus giving the brick five sanded surfaces.

The *stiff-mud* process consists in mixing the pulverized clay or shale with sufficient water to form a stiff paste, capable of retaining its form, which is then forced through a die, resulting in a bar of the section of the brick. The bar is then cut into bricks by wires. These bricks may be either side cut or end cut.

Dry-press bricks are made by pressing pulverized clay containing a small amount of moisture into steel molds, a method used to secure bricks with smooth faces and sharp edges for face brick.

Repressed bricks are made by putting bricks made by the soft-mud or stiff-mud methods into presses and subjecting them to high pressure. The purpose is to give the brick more perfect form and sometimes to imprint a design upon the surface.

Bricks made by the wet method must be dried before being placed in the kiln. In some yards this is accomplished by exposing the molded bricks to the air on floors or racks, while in the larger plants the drying is done more rapidly in dryers using artificial heat.

The burning is accomplished either in temporary kilns, built of the brick to be burned, or in permanent kilns arranged usually with fire boxes on the outside and a downdraft and intended to give

uniform heat throughout the kiln. This cannot be fully accomplished and not all of the brick will be perfectly burned. The degree of burning received by brick in temporary kilns depends upon the position in the kiln. They must be sorted after burning into shades, varying from the light underburned to the dark arch brick.

Good bricks may be made by any of the methods of manufacture, provided the material is carefully handled and the burning properly regulated. The differences due to method used are mainly those of the form and appearance of the brick. Dry-press brick are usually somewhat softer and weaker than stiff-mud brick of equally good material.

Classification of Brick

Bricks used in structural work may be classified as follows:

Common bricks are those used for ordinary brickwork, where appearance is not of special importance. They are burned at moderate temperatures. The best, well-burned common bricks are known as *hard* or *cherry* bricks, or sometimes as *stock* bricks. Those next the fire and heavily burned are known as *clinker* or *arch* bricks. Those from the underburned portion of the kiln are known as *salmon*, *pale* or *soft* bricks. The relative proportions of each kind in a kiln vary with the material and the skill used in burning.

Pressed, *face* or *front* bricks are those made with greater care, so as to secure uniformity of form and color. They are used for facing walls of common brick and where appearance is important, and are usually dry-pressed or re-pressed brick.

Vitrified bricks are made from a more refractory clay and burned at a high heat to the point of vitrification, so that considerable softening and shrinkage occurs, though the brick still hold its shape. These bricks are commonly made in larger sizes than common bricks, called *paving blocks*, and are used in street pavements. They are also frequently used in building construction, where obtainable at moderate prices. Blocks too lightly burned for use in pavements often make good material for building construction.

Fire bricks are made from clay which is lacking in fluxing ingredients. They are usually light in color, on account of the absence of iron oxide, and are used when high temperatures are to be resisted.

Enameled bricks are made by coating the surface of pressed or re-pressed bricks before burning with a slip, which will burn to the proper color, and covering with a glaze. The enamel is usually applied to a single surface of brick, except corner bricks, which have enamel applied to one side and one end.

The following designations are also frequently employed:

Sewer bricks are those common bricks which are so hard burned as to be practically non-absorbent of moisture, and are commonly used for lining sewers.

Compass bricks are shorter on one edge than the other, for use in circular walls.

Feather-edge bricks are made wedge shaped, for use in arches.

Furring bricks are those having a surface grooved for plastering.

Ornamental bricks are those having designs stamped in relief upon their faces, or bricks of special forms intended for use in making an ornamental surface design.

Properties of Clay and Shale Brick

Good building brick should show a uniform compact structure without laminations. They should have plane, parallel faces and sharp edges, and should not show kiln marks on their edges.

The dry-pressed and re-pressed bricks are usually smoother and more accurate in shape than those made by the soft-mud or stiff-mud processes, their density and strength being largely dependent upon the degree of burning and the shrinkage in the kiln. The under-burned, salmon bricks are porous and weak, and are usually employed only where strength is not important and in unexposed positions. The well-burned cherry or hard bricks are the best building brick. The overburned clinker bricks are more dense and absorb less water, but may be brittle, and are frequently distorted in shape. The overburned and distorted bricks are sometimes used by architects for special exterior designs with very good effect.

Vitrified bricks, as manufactured for use in paving, are superior in strength and density to common bricks. They frequently show kiln marks on one side, due to softening in the kiln. A clay for making vitrified brick must burn at high temperatures and have considerable range of temperature between the point of incipient fusion and the point of vitrification. It is difficult to maintain the temperature uniformly, so as to burn a large portion of the bricks to the right degree, unless the range of temperature is considerable.

59. Sand-lime Bricks.—Bricks made of sand cemented with lime have been used in a small way for many years. These bricks, as formerly made, were molded and allowed to harden by standing in the air, or in an atmosphere rich in carbon dioxide (CO_2). Bricks of this kind are virtually composed of ordinary lime mortar, but with less lime, and are called *mortar* bricks. They depend, like

lime mortar, upon the formation of carbonate of lime for their hardening, and are weak and of little value as brick, although some structures of such materials have proven substantial and durable.

In 1881 Dr. Michaelis of Berlin patented a process of hardening mixtures of lime and sand by the use of steam at high pressure. He discovered that, in the presence of steam at high temperature, the lime combines with a portion of the silica of the sand, forming a silicate of lime, which acts as a cementing medium. This silicate is formed upon the surfaces of the grains of sand and binds the sand into a single hard block.

About fifteen years after Michaelis took out his patent, the manufacture of sand-lime bricks was begun in Germany on a commercial scale, and soon developed into a considerable industry. In 1901 the first plant was opened in the United States, and the growth of the industry in this country was also very rapid.

Manufacture.—In the manufacture of sand-lime bricks, four operations are essential:

(1) The lime must be completely slaked.

(2) A very uniform mixture of the lime and sand must be obtained.

(3) The material must be formed into bricks under high pressure.

(4) The bricks must be subjected to the action of steam at high pressure for several hours.

The methods employed in different plants for performing these operations vary considerably, depending upon the character and condition of the materials used.

Hydrated-lime Process.—In this process the lime is first slaked to a powder, or a putty, and then mixed with the sand and pressed. The lime may be slaked by any of the methods ordinarily employed in the manufacture of hydrated lime, or it may be reduced to a paste by the use of an excess of water. It is easier to obtain a uniform mixture of the lime and sand when dry hydrated lime and dry sand are used and the necessary water added afterward. It may, however, be advantageous sometimes to use wet materials, and good results may be obtained by either method if the mixing be thorough and the lime uniformly incorporated in the sand.

Caustic Lime Process.—Caustic lime is sometimes pulverized and mixed with the sand before slaking. Enough water is then added to slake the lime and reduce the mixture to proper consistency for pressing. High-calcium lime, which slakes quickly, is necessary when this method is used, as sufficient time must be given for the complete slaking to take place before the mixture goes to the press. In some plants the mixture is placed in a silo and allowed to stand

for a few hours before pressing, in order to insure that no unslaked lime is left in the mixture when the brick is formed.

The caustic lime process is sometimes modified by grinding the lime with a portion of the sand to a fine powder, which is mixed with the remainder of the sand, and water added to slake the lime and wet the mixture. This is then placed in a silo for a sufficient period to allow the lime to become completely slaked before pressing. It is claimed that grinding the sand and lime together produces an intimate mixture and insures the complete combination during the steaming into silicate which forms the cementing medium of the brick. Grinding the lime and sand together reduces the lime to very fine condition, minimizes the danger from any unslaked particles of lime left in the mixture, and also fills the voids in the sand more completely, making a more dense brick.

Molding.—The bricks are formed in molds similar to those used for dry clay bricks, and are subjected to high pressure in molding.

Hardening.—After molding, the bricks are loaded upon cars and run into the steaming cylinders, where they are subjected to steam pressure of from 100 to 110 pounds per square inch for a period of six to ten hours, resulting in the combination of the lime with the silica into the cementing substance and binding the sand into a solid block. The bricks continue to harden and gain in strength for a time after their removal from the steaming cylinder, as they gradually dry out.

Materials.—High-calcium lime seems preferable for this use, on account of this rapid action and the fine subdivision of its particles. Any good lime may, however, be used for the purpose if care be taken to insure that it be completely slaked.

The requirements for sand to be used in making sand-lime bricks are not essentially different from those for sand to be used in cement mortar. The graduation of sizes to give a dense material is desirable. The presence of more fine material seems to be needed, however, in order to secure a smooth and compact mixture, and to lessen the wear upon the molds, which may become an important item of cost. Coarse sands seem to give stronger brick, but fine sand produces brick with smoother surfaces.

Properties of Sand-lime Brick.—In strength and durability, sand-lime bricks do not differ materially from good average clay bricks. When of good quality they possess sufficient strengths for all the purposes for which building brick are ordinarily employed, and are

usually more dense, and absorb less water than common clay bricks.

Sand-lime bricks are usually very uniform in size and shape, and are commonly gray in color, the shade depending upon the sand used in manufacturing them, unless artificially colored.

60. Cement Bricks.—Bricks made of cement mortar or concrete are used in a number of localities. They are commonly made of mortar, about one part Portland cement to four parts of sand, or sometimes of a richer mortar, 1 to $2\frac{1}{2}$ or 1 to 3, mixed with about an equal quantity of coarser material, varying from $\frac{1}{4}$ to $\frac{1}{2}$ inch in diameter.

These bricks are made by pressing in hand or power presses, a mixture as wet as is feasible to shape well in the press. About two weeks are required for hardening before the bricks can be used. The materials need to be carefully selected, and require the same properties as do those used for mortar in masonry or concrete. The strength may vary considerably with the grading of the aggregate, the compression given to the blocks, and the moisture conditions under which the bricks are kept during the period of hardening, the greatest strength will result when they are kept warm and thoroughly dampened. The compressive strength at twenty-eight days should not be less than 1000 lb./in.^{2}, and the absorption not more than 15 per cent.

Cement bricks are usually employed as face bricks. The appearance will depend upon the texture of the aggregates used and the method of finishing, which may be smooth or roughened by the use of brushes or acids. Color may be given to the bricks by the use of various mortar colors.

61. Tests for Building Brick.—In determining the suitability of a brick for structural work, examination is commonly made of the material as to form and texture with reference to the particular needs of the work in hand, and sometimes durability tests are called for.

Form.—For neat work, the bricks should be uniform in size with plane faces and sharp edges. Care in sorting is usually necessary with clay brick to secure uniformity of color and dimension in particular work.

Texture.—Good bricks should be uniform and compact in structure should be sound and free from cracks, and the broken surfaces should be free from flaws or lumps. Clay brick should be thoroughly burned, and when struck with a trowel or another brick should give a clear ringing sound. Bricks which meet these requirements are usually suitable for all ordinary work.

In ordinary building work little care is usually given to inspection of the materials, and defective work frequently results from the use of poor bricks. Seriously defective bricks are so easily detected by inspection that there is usually no excuse for their inclusion in brickwork of good character.

Durability Tests.—The durability of bricks under difficult weather conditions is one of their most valuable qualities. Tests are sometimes made of the effect upon bricks of freezing while in a saturated condition. These tests have been made in various ways, usually by immersing the brick in water, then freezing and thawing it repeatedly, commonly twenty repetitions, and determining the loss of weight or of strength. Very soft, porous bricks may be disintegrated by such treatment; those of low absorption and good strength usually show but slight effect.

The Committee of the American Society for Testing Materials, in 1913, suggested a method for making this test. They have not, however, found it of sufficient value to include in their later specifications.

A test in which the brick is saturated with a solution of sodium sulphate, which is then allowed to crystallize in the pores of the brick, has sometimes been used, the results of this action being similar to those of freezing, but much more rapid and severe. A study of this method has been made for the Committee by Professor Edward Orton, Jr.[1] and it may become a standard method of testing brick. It has not yet been definitely formulated for use in specifications.

After giving the subject careful consideration, the American Society for Testing Materials in 1920 adopted the following Standard Specifications for Building Brick. (Serial Designation: C 21-20.):

I. Standard Size.

Standard Size.—1. The standard size of building brick shall be $2\frac{1}{4}$ by $3\frac{3}{4}$ by 8 in.

II. Sampling.

Sampling.—2. For the purpose of tests, bricks shall be selected by an experienced person so as to represent the commercial product. All bricks shall be carefully examined and their condition noted before being subjected to any kind of test. For the purpose of the tests ten bricks will be required; they shall be

[1] Proceedings, American Society for Testing Materials, 1919, Part 1.

thoroughly dried to constant weight in a suitable oven at a temperature of from 225° F. (107° C.) to 250° F. (121° C.).

III. Physical Tests.

Absorption.—3. (*a*) At least five dry bricks shall be weighed and completely submerged in water at a temperature between 60 and 80° F. (15 and 27° C.). The water shall be heated to boiling within one hour, boiled continuously for five hours and then allowed to cool to a temperature between 60 and 80° F. (15 and 27° C.). The bricks shall then be removed, the surface water wiped off with a damp cloth and the bricks quickly weighed.

(*b*) The percentage of absorption shall be calculated on the dry weight, according to the relation:

$$\text{Percentage of absorption} = \frac{100(B-A)}{A},$$

where A = weight of dry brick and B = weight of saturated brick.

Compression Tests.—4. (*a*) Compression tests shall be made on at least five half bricks, previously dried, each taken from a different brick. The half brick shall be prepared either by sawing or cutting upon a yielding bed with a sharp mason's chisel, which shall be the full width of the brick. The specimen shall be tested on edge. To secure a uniform bearing in the testing machine the edge surfaces shall be bedded on a thin coat of calcined gypsum (plaster of Paris) spread upon plate glass previously coated with a film of oil. Before applying the calcined gypsum (plaster of Paris), the bearing surfaces of the brick shall receive a coating of shellac. The brick shall be pressed firmly upon the surface, making the layer as thin as possible, and be permitted to remain undisturbed until set. The depressions of recessed or paneled bricks shall be filled with neat Portland-cement mortar, which shall stand at least 24 hours before testing.

Transverse Tests.—5. (*a*) At least five bricks, previously dried, shall be tested, laid flat-wise, with a span of 7 inches, and with the load applied at midspan. The knife edges shall be slightly curved in the direction of their length. Steel-bearing plates, about $\frac{1}{4}$ inch thick by $1\frac{1}{2}$ inches wide, may be placed between the knife edges and the brick. The use of a wooden base block, slightly rounded transversely across its top, upon which to rest the lower knife edges, is recommended. (See A. S. T. M. Standards—1924 for alternate.)

(*b*) The modulus of rupture shall be computed in pounds per square inch by the following formula:

$$R = \frac{3Wl}{2bd^2},$$

in which l = the distance between the supports in inches;
b = the breadth;
d = the depth of the brick in inches, and
W = the load in pounds at which the brick failed.

Record of Test Results.—6. In recording the results of the test, the type of brick shall be defined, whether stiff mud, soft mud, dry pressed, repressed, sand lime or other types. It is recommended that the data obtained be recorded on the "Laboratory Record" as shown in A. S. T. M. Standards—1924, pp. 668 and 669.

IV. Classification of Bricks

Classification of Bricks. 7. (*a*) According to the results of the physical tests, the bricks shall be classified as vitrified, hard, medium, and soft bricks on the basis of the following requirements:

Name of Grade.	Absorption Limits, Per Cent.		Compressive Strength, (On Edge) Lb. per Sq. In.		Modulus of Rupture, Lb. per Sq. In.	
	Mean of 5 Tests.	Individual Maximum.	Mean of 5 Tests.	Individual Minimum.	Mean of 5 Tests.	Individual Minimum.
Vitrified Brick.	5 or less	6.0	5000 or over	4000	1200 or over	800
Hard Brick....	5 to 12	15.0	3500 or over	2500	600 or over	400
Medium Brick..	12 to 20	24.0	2000 or over	1500	450 or over	300
Soft Brick.....	20 or over	No Limit	1000 or over	800	300 or over	200

(*b*) The standing of any set of bricks shall be determined by that one of the three requirements in which it is lowest.

ART. 16. BRICK MASONRY

62. Joints in Brickwork.—In the construction of brick masonry, it is necessary that the joints between the bricks be filled with mortar, the purpose of which is to give a firm and even bearing to the bricks, so that the pressure upon them will be uniformly distributed. The mortar should also adhere to the bricks and bind them into a monolithic mass.

As thick joints usually make weaker masonry than those that are thin, it is desirable that the joints be made as thin as practicable. When attempts are made at too thin joints, they are apt to be imperfectly filled, and thus weaken the masonry. Joints in wall masonry of common brick, as used in building construction, are usually from $\frac{1}{4}$ to $\frac{3}{8}$ inch thick. It is common to specify the thickness of joints by stating the thickness for eight courses of brick. It is frequently required that the thickness of eight courses of brick masonry shall not exceed the thickness of eight courses of dry bricks by more than 2 inches. When pressed bricks are used for the face of a wall, the joints in the face are usually from $\frac{1}{8}$ to $\frac{3}{16}$ inch thick. Pressed bricks, being smoother, may be laid to thinner joints with good effect. In heavy masonry as sometimes used in engineering work, the joints usually of cement mortar—are often $\frac{1}{2}$ inch thick.

Mortar for Brickwork.—Lime mortar is more extensively used for ordinary brickwork in building construction than any other.

Mixtures of lime and cement mortars in about equal quantities are coming largely into use. The cement materially increases the strength of the mortar and its adhesion to the brick, while the smoothness of the lime mortar is maintained. In important structures, where considerable strength is needed, it is common to use cement mortar with addition of 10 to 15 per cent of hydrated lime—a mixture which retains the strength of the cement, but makes the mortar easier to work, and usually secures better work than would result from the use of cement alone. In engineering work, cement mortar is usually employed, but the mixture of hydrated lime with the cement is rapidly coming into use.

Laying the Brick.—In the construction of a brick wall the two outer courses are laid first, by spreading a bed of mortar where the brick is to be placed, and against the surface of the last brick laid, then shoving the brick horizontally into place so as to squeeze the mortar into the bottom of the vertical joint between the bricks. A bed of mortar is placed between the outside bricks and the filling bricks are shoved and pressed into place. Mortar is then slushed or thrown with some force into the upper part of the vertical joints to fill them completely.

Bricks should be thoroughly wet before being laid, in order to prevent the water being absorbed from the mortar by the brick. Good adhesion cannot be had between mortar and dry, porous bricks.

In finishing joints upon the face of the wall, a flush joint may be made by pressing back the mortar with the flat edge of the trowel. This is usually done upon interior walls. A weather joint may be made, as shown in Fig. 32, by using the point of the trowel held obliquely.

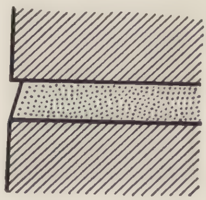


Fig. 32.—Weather Joint.

63. Bond of Brickwork.—Brickwork is always laid in horizontal courses, and lateral bond is secured by several different arrangements of the brick in courses.

Common Bond is the bond most commonly used in the United States, for walls of common brick. In this bond, one course of headers is used to four to six courses of stretchers on the face of the wall, as shown in Fig. 33.

In *Flemish Bond* (Fig. 34), alternate headers and stretchers are used in each course, each header being placed over the middle of the stretcher in the course below. Small closers are introduced next to the headers at the corners.

English Bond consists of alternate layers of headers and stretchers (Fig. 35). This construction, like the Flemish bond, makes very

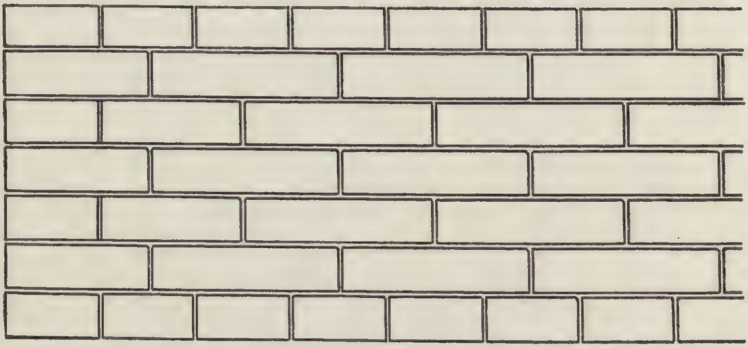


FIG. 33.—Common Bond.

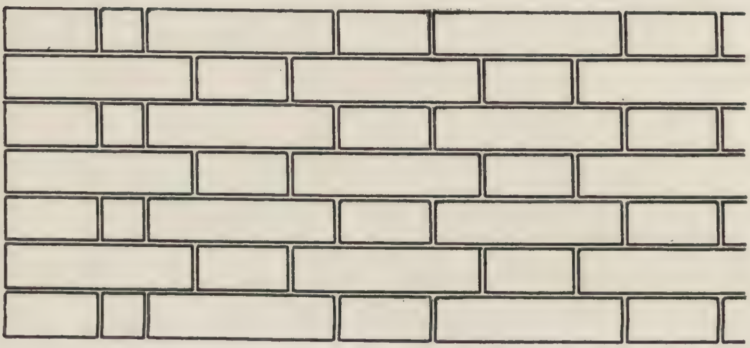


FIG. 34.—Flemish Bond.

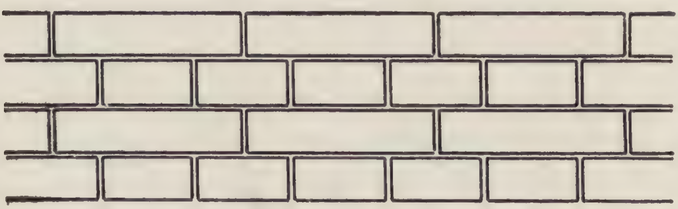


FIG. 35.—English Bond.

strong work. English bond in which the alternate courses of stretchers break joints with each other is called *Cross English Bond*.

In Fig. 36 is shown the cross-sections of 8-, 12-, and 16-inch walls with proper arrangements of headers.

Face Bricks.—In applying a facing of pressed bricks or other special bricks to a wall of common bricks, it is quite usual to lay all of the face bricks as stretchers or to use alternate stretchers and false

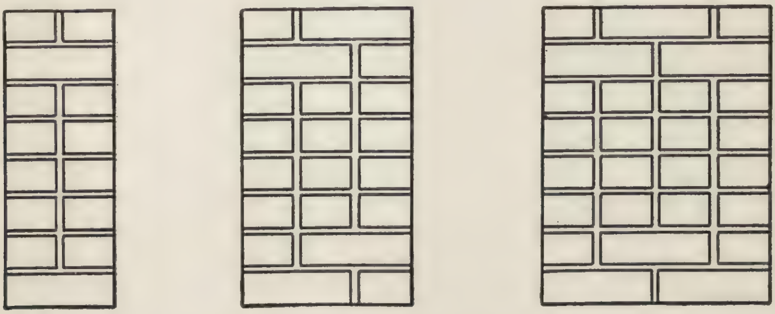


Fig. 36.—Arrangements of Headers.

headers (bats) simulating the Flemish bond. When this is done, the face may be tied to the body of the wall by means of *diagonal bond* or by the use of metal wall ties.

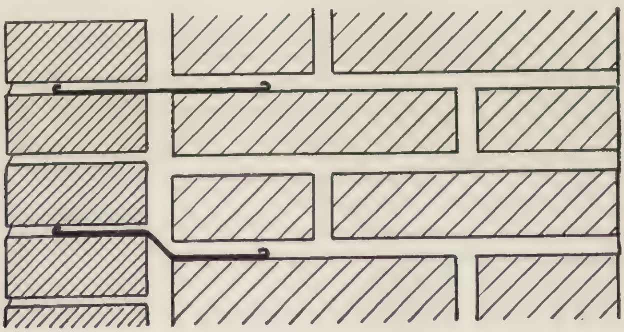


Fig. 37.—Metal Ties for Face Brick.

Diagonal Bond consists in breaking off the back corners of face bricks and inserting bricks diagonally to bond with the face bricks. These bonds are not very strong, and the face bricks are not considered as adding to the strength of the wall or carrying any load. Because of the labor involved and the accompanying waste this practice is losing favor.

Metal Wall Ties.—Several types of metal wall ties have been introduced. Those of crimped galvanized iron of No. 22 gage seem

to give satisfaction. If the face bricks differ in thickness from those in the body of the wall, the metal wall ties should be used in those joints where the face bricks and body bricks come to a common level, not more than six courses apart.

Figure 37 shows the use of metal wall ties for bonding face bricks. The upper tie is correctly used. Metal wall ties should not be bent from one level to another as shown by the lower tie.

Hollow Brick Walls.—For the purpose of providing air space in a wall to prevent the passing of moisture or changes of temperature through it, hollow construction is sometimes adopted. This consists of building a double wall with a narrow air space between the outer and inner portions.

It is necessary for the proper strength that the two portions of the wall be bonded in some way, either by occasional headers which span the opening or by metal ties. The headers constitute a connection between the masonry of the two walls, and are sometimes objected to as likely to cause moisture to pass from one wall to the other. The metal ties may be provided with a drip at the middle which insures the complete isolation of the walls from each other. Such walls require more careful work and are more expensive to construct than solid walls. When loads are to be carried, one of the walls must be capable of bearing them, or proper pilasters must be provided.

The Ideal Wall.—In 1921 the Common Brick Manufacturers' Association of America, after a series of tests, suggested a form of

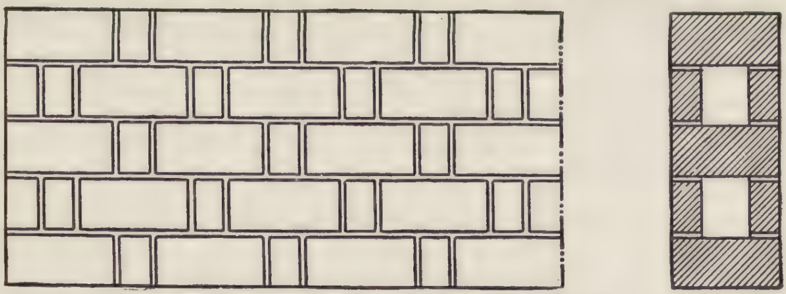


Fig. 38.—The Ideal Wall—All-Rolok Type.

brick work called "The Ideal Wall," which produces a substantial and well-insulated wall of any thickness from 8 inches up, and at a lower cost of construction. It is claimed that The Ideal Wall has the great advantage of considerable saving in both materials and labor, and that it combines the advantages of the solid brick and hollow types of wall at a lower cost than either.

There are two types of this construction. In the Ideal all-rolok wall all the bricks are laid on edge with the Flemish bond, with a $3\frac{1}{2}$-inch air space. In the Ideal rolok-back wall the outer 4-inch course bricks are laid on their flat beds, so that the wall has the usual brick appearance; the backing bricks are laid on edge and header courses may be run at every third or sixth course and any bond may be used; the air space is 2 inches.

The thickness of the Ideal wall may be increased by adding courses of brick laid in the regular manner either inside or outside with proper bond. The "All-Rolok" type is shown in Fig. 38.

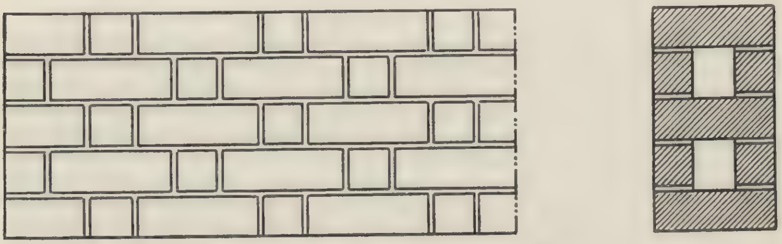


FIG. 39.—The Three-Three-Nine Wall.

The Three-Three-Nine Wall.—A similar hollow wall built of clay, shale, or concrete bricks, with dimensions 3 × 3 × 9 inches, is here suggested and is shown in Fig. 39. The thickness of this wall may also be increased in the manner mentioned above.

64. Surface Patterns.—The object of bond in brickwork is to tie the parts of the wall together and secure adequate strength, but it has been observed that some bonds give more pleasing effects than others and architects take advantage of the opportunity for ornamentation and variety. The common and English bonds give satisfactory results from the standpoint of strength, but they are not especially pleasing in appearance and are used mostly in factory construction or for walls that are to be covered up. The English Cross or Dutch bond gives a surface in which the bricks seem to take the form of a St. Andrew's cross. The Flemish bond gives an esthetic effect with little extra labor. Sometimes bats are substituted for some of the headers in order to save expense when more costly face bricks are used. The appearance of the Flemish bond is somewhat enhanced by the use of headers of a color slightly different from that of the stretchers, though the effect is spoiled if the contrast is too great.

In some instances sills and bands are made of bricks laid on edge in "rowlock" courses and sometimes of bricks stood on end in

"soldier" courses, but these handlings should be used with discretion to avoid the appearance of weakness.

In Europe, and to some extent in the United States, bricks are sorted for light and dark shades and laid in surface patterns or diaper work. Effects of great variety and beauty are obtained by experts in this kind of work. In some cases more than two shades of brick are used and sometimes the color of the mortar is varied to obtain desired results.

Architects are ever striving for novel and pleasing textural surface effects, and are aided by the manufacturers, who furnish bricks

FIG. 40.—Skintled Brickwork.*

in great variety, including wire-cut, matt surface, glazed, and enameled, as well as numerous colors and tints for tapestry brickwork.

Skintled Brickwork is that in which the bricks are laid in such a way that they are not brought to a plane surface on the face of the wall but project or recede in a very irregular and artistic manner. In this type of construction the mortar joints are also rough and irregular. When properly handled this work gives extremely pleasing results. Skintled brickwork seems to lend itself especially to the English half-timber style of architecture. Figure 40 illustrates one of the many effects that may be obtained.

* Courtesy of The Common Brick Manufacturers' Association of America.

65. Strength of Brick Masonry.—In tests which have been made on the crushing strength of brick piers, failure occurred by the lateral bulging of the piers. When pressure is applied longitudinally upon the pier, a lateral expansion normal to the direction of pressure results. This causes tension upon the brickwork and the pier yields through the breaking of the bricks in tension and the pulling apart of joints. The transverse strength of the bricks may also be called into play when they are not bedded with perfect evenness—a fact proven by a series of tests on brick piers at the Watertown arsenal in 1907,

TABLE VII

SUMMARY OF TESTS OF BRICK COLUMN

Average Values

Ref.	Characteristics of Columns.	Average Unit Load, lb. per sq. in.	Ratio of Strength of Column to Strength of Brick	Ratio of Strength of Column to Strength of "A"	Crushing Strength of 6-in. Mortar Cubes, lb. per sq. in.	Ratio of Strength of Column to Strength of Cubes
	Shale Building Brick					
A	Well laid, 1:3 Portland cement mortar, 67 days.	3365	.31	1.00	2870*	**1.17**
B	Well laid, 1:3 Portland cement mortar, 6 months	3950	.37	1.18	....	....
C	Well laid, 1:3 Portland cement mortar, eccentrically loaded, 68 days.	2800	.26	.83	....	....
D	Poorly laid, 1:3 Portland cement mortar, 67 days.	2920	.27	.87	2870*	**1.05**
E	Well laid, 1:5 Portland cement mortar, 65 days.	2225	.21	.66	1710	**1.30**
F	Well laid, 1:3 natural cement mortar, 67 days.	1750	.16	.52	305	5.75
G	Well laid, 1:2 lime mortar, 66 days.	1450	.14	.43	....	....
	Underburned Clay Brick					
H	Well laid, 1:3 Portland cement mortar, 63 days.	1060	.27	.31	2870*	**.37**

* Average value based on 1:3 tests of 1:3 Portland cement mortar cubes sixty days old.

in which bricks set on edge gave somewhat higher strengths than when laid flat. Piers in which the joints were broken at every third or sixth course gave slightly better results than those breaking joints at every course, as was also observed in piers tested in 1884.

The strength of brickwork depends upon the bond as well as upon the adhesion of the mortar and the strength of the bricks. In masonry to be subjected to heavy loads, careful attention should be given to the bonding of the work and to the complete filling of the vertical joints in laying the masonry.

The advantage of using strong mortar in such work is demonstrated by many tests made at Watertown arsenal and reported by the Ordnance Department of the United States Army in "Tests of Metals, etc." That the strength of brick masonry in piers is somewhat proportional to the strength of the bricks is also demonstrated by these tests.

A series of tests made by A. N. Talbot and D. A. Abrams at the University of Illinois Experiment Station in 1908 shows very interesting results. A summary of these results is given in Table VII.

In the testing of brick piers it has been found that the initial yielding of the pier usually occurs at about one-half the breaking load. The safe load should be taken at not more than one-tenth to one-twelfth of the breaking load, on account of the many elements of uncertainty concerning the actual strength, chances for defective work, etc.

In 1908 a committee of engineers and architects made a special study of safe working pressures for brick masonry in building construction and made recommendations to the City of Chicago.

The building code of the City of Chicago for 1924 gives the following allowable compression in pounds per square inch on brick masonry:

	Lb./in.^{2}
No. 1 paving brick, 1 part Portland cement, 3 parts torpedo sand	350
No. 2 pressed brick and sewer brick, mortar same as referred to above	250
No. 3 hard common select brick, Portland cement mortar, same as referred to above	200
No. 4 hard common select brick, 1 part Portland, 1 lime, 3 sand, as referred to above	175
No. 5 common brick, all grades, Portland cement mortar	175
No. 6 common brick, all grades, good lime and cement mortar	125
No. 7 common brick, all grades, natural cement mortar	150
No. 8 common brick, all grades, good lime mortar	100

The building code of the City of St. Louis, in 1924, gives the following allowable compression on brick masonry:

	Lb./in.^{2}
Vitrified paving brick, one part Portland cement, three parts sand	300
Strictly hard pressed brick, one part Portland cement, three parts sand	250
Ordinary hard and red brick, one part Portland cement, three parts sand	200
Ordinary hard and red brick, one part Portland cement, one lime, three sand	175
Merchantable brick, good lime mortar	100

Vitrified paving brick and strictly hard brick shall not crush at less than five thousand (5000) pounds pressure per square inch. Ordinary hard and red brick shall not crush at less than two thousand and three hundred (2300) pounds pressure per square inch. Merchantable brick shall not crush at less than one thousand and eight hundred (1800) pounds pressure per square inch.

66. Efflorescence.—The appearance of brick masonry is sometimes marred by a white coating which exudes from the masonry and is deposited upon its surface. This is called efflorescence, and is caused by soluble salts in the brick or the mortar, usually the latter, which are dissolved by water when the wall is wet and deposited on the surface as the water evaporates. Such deposits usually consist of salts of soda, potash, or magnesia contained in the lime or cement, or of sulphate of lime or magnesia from the brick.

Efflorescence may be prevented by keeping the wall dry. The use of impervious materials, and the making of the masonry itself impermeable, render the appearance of efflorescence improbable. When a wall is in a damp situation, a damp-proof course at the base of the wall to prevent moisture rising in the masonry is desirable. If the masonry is permeable and is dampened by rain, some waterproof coating may be applied to the surface of the wall. There are various patented preparations for this purpose, and the Sylvester process is sometimes successfully used. This consists in applying first a wash of aluminum sulphate (1 pound to 1 gallon of water), and then a soap solution (2.2 pounds of hard soap per gallon of water). These applications are made twenty-four hours apart. The soap solution is applied at boiling temperature. The walls must be dry and clean, and the air temperature should not be below about 50° F., when the application is made.

Efflorescence may usually be removed by scrubbing with a weak solution of hydrochloric acid.

67. Measurement and Cost.—Measurement of brickwork is usually made by estimating the number of thousand bricks. It may be assumed that an 8- or 9-inch wall contains 13 bricks per square foot of surface; a 13-inch wall, 19½ bricks, a 17- or 18-inch

wall, 26 bricks, etc. These figures are for the new standard size brick ($2\frac{1}{4} \times 3\frac{3}{4} \times 8$ inches) laid with $\frac{3}{8}$-inch joints. Proper allowance for waste should be made in estimating.

The methods of estimating are sometimes rather complicated and are subject to rules established by custom. The plain wall is the standard of measurement, openings less than 80 square feet are usually not deducted; larger openings are measured 2 feet less in width than they actually are. Hollow walls and chimneys are measured solid.

A pier is sometimes measured as a wall whose length is the circumference and whose thickness is the width of the pier. Sometimes one-half the circumference is taken as the length.

Stone trimmings are not deducted from the brickwork measurements. Various rather complicated rules are used in estimating footings, pilasters, detached chimneys, etc.

Having estimated the work in thousands of brick by these rules, a price per thousand, suited to the plain wall, is used for the entire job. When pressed brick facing is used, the area of such facing is separately estimated. If an ashlar facing be used, its thickness is not included in that of the brick wall.

In engineering work, brickwork is usually measured, like stone masonry, by the cubic yard of actual masonry.

Number of Bricks Required.—The actual number of bricks needed for the construction of masonry varies with the size of the bricks and the thickness of joints. For ordinary brickwork, with common bricks of the usual ($2\frac{1}{4} \times 3\frac{3}{4} \times 8$ inches) size, and joints $\frac{3}{8}$ inch thick, 1000 bricks will lay about 1.9 cubic yards of masonry.

With common bricks of standard size in masonry walls, 6.5 bricks will usually be required per square foot of wall surface for each width of brick in the thickness of the wall. For ordinary pressed-brick fronts, 6 to $6\frac{1}{2}$ bricks are required per square foot of actual wall surface. In average building construction, deductions for openings will reduce the number by about one-third of those required for solid wall.

Mortar Required.—For ordinary building construction with $\frac{3}{8}$-inch joints, 0.5 to 0.6 cubic yard of mortar is required per 1000 bricks. This needs for 1 to 3 Portland cement mortar, about 1.5 barrels of cement and 0.6 cubic yard of sand; for lime mortar about 200 pounds ($2\frac{1}{2}$ bushels) of lime and 0.6 cubic yard of sand.

In heavy masonry with joints $\frac{1}{2}$ to $\frac{5}{8}$ inch, about 0.35 to 0.40 cubic yard of mortar per cubic yard of masonry, or approximately one barrel of cement and 0.4 cubic yard of sand for 1 to 3 Portland cement mortar.

Labor of Laying Bricks.—A bricklayer on ordinary work may lay from about 125 to 175 common bricks per hour, according to the skill of the workman and the organization of the work. He should place somewhat less than half as many face bricks. The number of bricks laid may be somewhat less with cement mortar than with lime mortar. On thin walls, with careful work, one helper may be needed for two bricklayers. On common brickwork, in building construction, one helper may be needed for each mason.

In recent work (1924) with masons at 60 to 70 cents per hour, helpers at 30 to 35 cents per hour, lime at 40 to 50 cents per bushel, the cost of laying common bricks in the walls of buildings has run from $5 to $8 per 1000 bricks. Costs for scaffolding, for machinery and labor in erection of brickwork necessarily vary materially with the conditions under which the work must be done.

At prices which have existed since the World War, these figures would be largely increased. Costs have varied widely in different localities and are now very unstable.

ART. 17. TERRA-COTTA CONSTRUCTION

68. Structural Tiling.—Hollow tiling for use in building construction is made in many different forms. It is employed either as the main structural material or as fireproof covering for other materials.

The materials of which the tiles are made are similar to those used in making bricks, but requiring usually higher grade and more refractory materials. Shales or semi-fire clays, similar to those used for paving bricks, are frequently employed for this purpose, or sometimes fire clays are mixed with plastic clays to prevent fluxing at moderate temperatures. Tiling for use in construction may be made either dense or porous according to the qualities desired.

Dense Tiling is made from materials which vitrify at high temperatures (above 2000° F.) and is burned to the point of vitrification like paving bricks. This material when of good quality possesses high strength and is practically non-absorbent. It is used in outer walls of buildings, or for floor and wall construction when strength is needed.

Hollow blocks as made for ordinary wall construction are not usually vitrified, but are burned to a less degree than the best dense tiling. They must be hard burned to be of value. In the rapid growth of the tile industry, attempts have been made to produce hollow tiling from inferior materials, and soft tiles lacking in strength

and durability have sometimes been offered. Care must be exercised in selecting tiling to make sure of its quality.

Porous Tiling, or *Terra-Cotta Lumber*, is made from refractory plastic clays by mixing sawdust with the clay in forming the blocks, and burning at high temperature. The sawdust burning out leaves the material light in weight and porous. These blocks may be cut with a saw, and nails or screws may be driven into them without

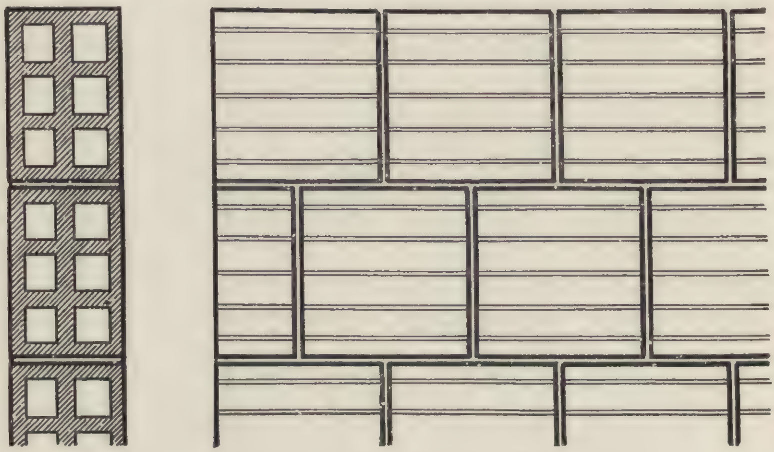


FIG. 41.—Hollow Rectangular Blocks.

difficulty. This tiling does not possess the strength of good dense tiling, but is tough and less brittle, and is largely used in fireproofing and for interior walls and partitions.

Tiling of less porosity but possessing somewhat the character of the terra-cotta lumber is sometimes made by mixing ground coal with the clay before burning. It is claimed that this makes a better fireproofing than the dense tiling. These blocks are sometimes known as *semi-porous tiling*.

The *forms and sizes* of hollow blocks depend upon the uses to be made of them. For walls or partitions, the blocks are usually in 12-inch lengths, and of rectangular or interlocking sections.

Rectangular blocks are made in various sizes—12-inch widths may be had from 2 inches to 8 inches thick. Widths of 6 and 8 inches are made in thicknesses from 2 to 5 inches. They are divided by webs into cells, as shown in Fig. 41. In the heavier tiling, intended for use where loads are to be carried, and in outside walls, the shells are at least 1 inch and the webs at least $\frac{3}{4}$-inch in thickness, and the cells not more than $3\frac{1}{2}$ or 4 inches in width. In lighter tiling, used

as filler in concrete work or for light partitions, the webs are $\frac{3}{8}$ to $\frac{1}{2}$ inch, and cell openings may be 5 or 6 inches.

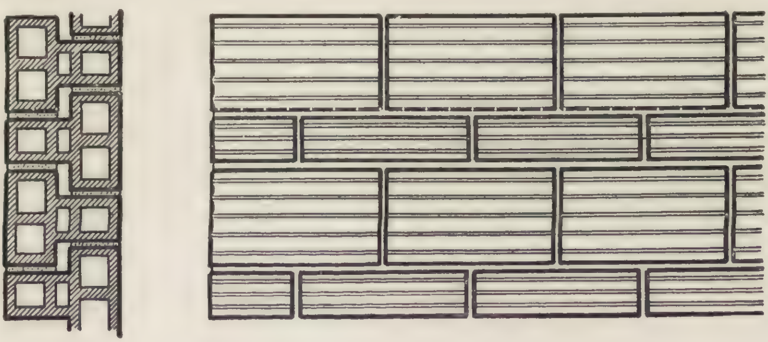
Interlocking blocks are made in various shapes, with the object of improving the bond of the wall, and eliminating joints extending through the wall. These blocks are often used in outside walls to prevent moisture passing through the wall and provide air spaces in all parts of the wall. Figure 42 shows one of the common forms of interlocking tile.

Hollow blocks for use in fire protection are made in many shapes to fit around structural members of other materials. They are also made to fit together in round or flat arches to support floors between steel beams.

Good tiling must be well burned, true in form and free from checks or cracks, and should give a ringing sound when struck with metal.

The following requirements for hollow tile are given in the Building code of the City of St. Louis for 1924:

All hollow tile used in the construction of walls or partitions shall be hollow shale or terra cotta, well manufactured and free from checks and cracks, each piece or block to be molded square and true and to be hard burned so as to give



42.—Denison Interlocking Tile.

a good clear ring when struck, and not to absorb more than twelve (12) per cent of its own weight in moisture. Each of said blocks shall develop an ultimate crushing strength of not less than three thousand (3000) pounds per square inch of available section of web area, and shall not be loaded when in the wall more then eighty (80) pounds per square inch of effective bearing area. Tiles shall have outer shells or walls not less than three-quarters ($\frac{3}{4}$) of an inch thick and shall be additionally reinforced by continuous interior walls or webs which shall not be less than one-half ($\frac{1}{2}$) inch thick, and so arranged that no void shall exceed four (4) inches in cross-section at any point. It is further provided that the building commissioner may require a test to be made of such blocks before allowing the same to be placed in the wall, if, in his judgment, there be any doubt as to whether such blocks, proposed to be used, meet the requirements above specified.

69. Block Construction.—In the construction of walls of ordinary hollow rectangular blocks, the blocks are usually laid so as to break joints and extend through the walls. They should be so

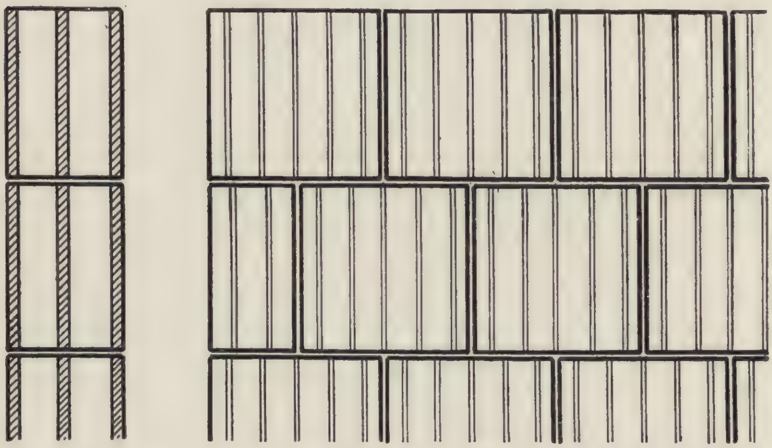


FIG. 43.—Walls of Natco Hollow Blocks.

placed that the vertical webs in each course are directly above those in the course below. Such construction is shown in Fig. 41.

In using tile with horizontal cells, jamb blocks and corner blocks are made with the cells vertical. When very light walls are used, longitudinal reinforcement, consisting of thin band iron or of special forms of wire mesh, is placed in the joints. This is necessary for 2-inch partitions or for 3-inch partitions more than 10 feet high.

Tiles with vertical cell openings are made by some makers. Figure 43 shows construction with standard tiling of this type.

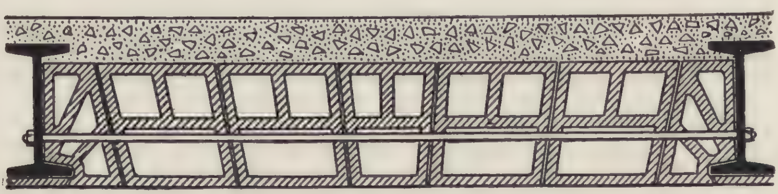


FIG. 44.—Flat Arch Floor Construction.

Portland cement mortar, or mortar of lime and cement, is used in laying hollow blocks. In walls which are to carry considerable loads, Portland cement with 10 to 15 per cent of hydrated lime by volume (4 to 6 per cent by weight) should be used in 1 to 3 mortar

with well-graded sand. For walls which are not to carry loads, a larger amount (equal volumes) of lime may be used. The surfaces of tiles are often grooved to aid the adhesion of the mortar in the joints. When the finish of the wall is to be plaster or stucco, the surface of the tile is grooved to hold the plaster. If brick veneer is to be applied, or if the surface of the tile is to be used for exterior finish, a smooth finish may be desirable.

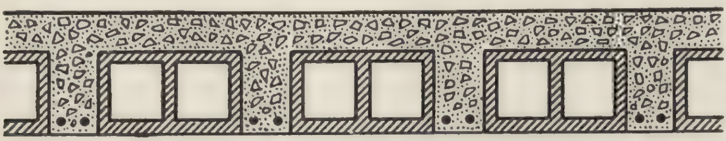


Fig. 45.—Hollow Block Fillers in Concrete Floors.

Floor Construction.—The method of using hollow blocks in flat arch floor construction is shown in Fig. 44. These arches vary from about 3 to 6 feet in span and from 6 to 12 inches in depth. The blocks required consist of the skewback, the fillers and the key-block. The skewbacks are usually made of such form as to enclose the bottom of the I-beam for fire protection.

Such arches are now commonly made by the end-construction method in which the cell openings run lengthwise of the arch. The blocks do not break joints, but form a series of independent arches side by side. A number of different shapes are offered for these arches by different makers, lighter weight being obtained than with side-construction arches for the same strength.

Hollow blocks are frequently used as fillers in reinforced concrete floors, the blocks filling spaces between the webs of the T-beams of concrete, as shown in Fig. 45. Blocks 12 inches wide are usually employed for this purpose, the depth depending upon the span and loading of the floor.

Strength of Block Masonry.—Comparatively few data are available upon the strength of constructions of terra-cotta blocks. A very carefully constructed wall of natco tile (see Fig. 43) was tested by R. W. Hunt & Company. The wall was $36\frac{5}{8}$ inches long, 8 inches thick, and 12 feet $2\frac{1}{2}$ inches high, and was twenty-eight days old when tested. It failed under a load of 436,000 pounds, giving a compression of 3110 lb./in.^{2} on the net section of the web, or about 1500 lb./in.^{2} of gross area. Tests of a wall of Denison tile (see Fig. 42) faced with brick, forty-two days old, was made at the laboratory of the Bureau of Standards. This wall was 5 feet, 1 inch in

length, $12\frac{1}{2}$ inches thick, and 31 feet high. It carried a load of 686,000 pounds, or about 900 lb./in.^{2} of gross area.

Good dense tiling should have a crushing strength of 3000 to 6000 lb./in.^{2} of net section. When laid in masonry the allowable load is usually not more than one-fifteenth of the ultimate strength of the block. Carefully laid masonry of good quality hollow blocks may be allowed to carry a load of 200 lb./in.^{2} of net section of block, or in general about 5 tons per square foot of gross area.

70. Architectural Terra-cotta.—Terra-cotta for exterior finish or ornamental work is usually made from a mixture of clays, carefully selected to secure the desired qualities. The clay is ground, mixed, tempered, and worked to a proper condition of plasticity. It is then formed into the desired shapes in plaster molds or by hand, modeled as may be necessary, and dried. After drying, it is given a surface treatment, by spraying with a liquid. This determines both the kind of finish to be given in burning and its color.

The blocks of terra-cotta may have a length up to 30 inches, and depth of 6 to 10 inches, with height according to the requirements of the work. They are constructed as hollow shells with webs about $1\frac{1}{4}$ inches thick, and cells 6 inches or less in width. These blocks are built into the body of the wall by bonding the masonry into and filling the cells.

Several kinds of surface finish are used for terra-cotta. Standard terra-cotta is that in which no special finish is applied, leaving the block somewhat porous. Vitreous terra-cotta has a spray applied to the surface which causes the surface material to vitrify during burning, making the material non-absorbent. Glazed terra-cotta has an impervious coating of glaze upon the surface. When the glaze is deadened, it is called mat-glazed. A variety of colors are available for use with this material, and makes its use possible in a wide range of artistic designs.

Terra-cotta of good quality is one of the most durable materials for use in trimming and ornamentation of masonry structures. Being practically non-absorbent, it is not affected by frost, or by the gases in the atmosphere. The facility with which it may be worked into desired forms makes it a desirable material for artistic design.

ART. 18. GYPSUM AND CEMENT BLOCK CONCRETE

71. Gypsum Wall Blocks.—Blocks made by mixing gypsum plaster (see Section 39) with wood fiber or similar materials are used for partition walls in fireproof building construction. They are made

30 inches long, 12 inches high, and from 3 to 8 inches thick, with tapering openings through the block.

They are laid in the wall to break joints and cemented with mortar composed of gypsum cement plaster and sand, usually 1 to 3. They are not used for walls bearing loads, but form very light partitions, and have good soundproof and fireproof qualities.

The 3-inch blocks are used to height of wall of about 12 feet, the 4-inch to 17 feet, and the 6-inch to 24 feet. The material may be cut with a saw, and plaster is applied directly to their surfaces. The weights of walls of hollow gypsum blocks are approximately as follows:

Thickness of block, inches	3	4	5	6	8
Weight of wall, per sq. ft	10	13	16	20	26

Three pounds per square foot is added for plaster upon each side of the wall.

72. Roofing and Floor Blocks.—Blocks of gypsum, similar in composition to the partition blocks, and reinforced with wire mesh, are made both in solid and hollow form for use in roof construction. They are usually 3 or 4 feet in length and are used to span the openings between purlins and form a solid deck upon which the roof covering may be placed. They are made with beveled edges, and are set with their lower edges in contact and the triangular openings between them filled with a grout of cement plaster. Blocks with heavier reinforcement for openings up to 10 feet in span are also now offered.

FIG. 46.—Pyrobar Gypsum Floor Tile.

Floor blocks, to be used as fillers in reinforced-concrete floor construction, are now available. These are designed to act as forms for the concrete, and require support at the ends of the blocks, which are 2 feet long. A spacer is placed between two adjoining blocks to hold the concrete for the web of the beam, forming a smooth surface on the under side upon which plaster may be placed. A section of floor constructed with these blocks is shown in Fig. 46.

73. Concrete Blocks.—Hollow building blocks of Portland cement concrete are frequently employed in building construction in the same manner as in solid concrete construction, given in Chapter V, and the concrete is proportioned and mixed in the same manner.

The blocks are usually made to set in the wall with the webs in a vertical position. Several patented forms are on the market which make blocks to bond in the wall in different ways, giving air spaces more or less effective as insulation against moisture and heat. Such blocks, when well made and properly set, make a substantial and durable building, and may be used in such manner as to give a pleasing appearance. The color of the blocks may be regulated by choice of the aggregate used upon their exposed faces. The use of coloring matter in the concrete has not usually been very successful, although there are mineral colors available which may be used without material injury to the concrete.

Metal molds are commonly employed, and concrete of rather dry consistency is compressed into them by tamping or by hydraulic pressure. This yields concrete of greatest strength and also makes a block which may be quickly removed from the mold. For ornamental work, sand molds are frequently employed, a wooden pattern being used in forming the mold, and the concrete poured in a wet mixture.

The curing of the blocks is important in its effect upon the strength and durability of the concrete, which must not dry out during the period of hardening. After the blocks are removed from the molds, they are allowed to stand in the air until the cement has set, when they may be transferred to a steam chamber, where they are subjected to an atmosphere charged with steam at a temperature about 110° to 130° F. After two or three days in the steam, they may be removed to the open air, but should be sprinkled often enough to keep them continually damp for ten or twelve days. When a steam chamber is not employed, the blocks are cured in the open air, but should be kept wet for a longer period to give time for complete hardening. The temperature to which they are subjected during hardening should never go lower than about 50° F.

CHAPTER V

PLAIN CONCRETE

ART. 19. AGGREGATES FOR CONCRETE

74. Materials Used for Aggregates.—Concrete as used in construction is essentially a mixture of cement mortar with broken stone, gravel, or other coarse material. The mortar serves to fill the voids in the stone and the whole is bound together into a solid monolith by the setting and hardening of the cement.

An ideal concrete may be said to be composed of a suitable broken stone, clean, dry, and thoroughly compacted, with sufficient ideal Portland cement mortar (Sec. 34) to coat thoroughly each piece of stone and completely fill the interstices among the pieces; the mass to be mixed vigorously for the length of time necessary to produce proper plasticity, and then thoroughly compacted.

The coating of each piece of stone with mortar will increase the sizes of the pieces and the volume of the resulting concrete will be greater than that of the original compacted broken stone. To insure the complete filling of the voids, a quantity of mortar somewhat in excess of the minimum may be required. For the sake of economy in chuting the concrete into place, the volume of water may have to be increased. It should be kept in mind that the addition of water beyond that required for the hydration of the cement tends to weaken the concrete, and the volume of the water used should be kept as low as possible consistent with results to be obtained.

The materials mixed with the cement in forming concretes are known as aggregates. The sand or stone chips in the mortar is called the *fine aggregate* and the coarser gravel or broken stone is the *coarse aggregate*. In the manufacture of good concrete it is essential that each of the materials be of proper quality, and that they be properly proportioned and incorporated into the mixture.

Fine Aggregate.—Material which will pass a $\frac{1}{4}$-inch screen and be held on a No. 100 screen is usually included under the term fine aggregate, or sand. The requirements for sand and its use in mortar have been discussed in Chapter II. Ordinarily, the sand which makes the strongest and most dense mortar will also give the best results in concrete, though this may not always be the case. The grading of the

sand should be such as to reach maximum density when combined in proper proportions with the coarse aggregate to be used in the concrete.

Coarse Aggregate.—This may consist of any hard mineral substance broken to proper size—usually broken stone or gravel, although sometimes broken slag, cinders, or broken brick is used.

The value of stone as an aggregate depends upon much the same qualities as are needed for building stone. For high-class concrete work, it is important that the stone should possess strength, and absorb but little water. Stones breaking to cubical shapes give better results than those of shaly or slaty character, while rounded pieces pack closer and show fewer voids than those with sharp corners.

Trap and granite are usually the best of concrete materials. When the concrete is to be subjected to abrasive wear, trap is a superior material. For resistance to direct compression, good granite is to be preferred. Limestones and sandstones vary greatly in their values as concrete materials, hard limestones and some of the more compact sandstones being desirable materials, while the softer varieties are not generally suitable for first-class concrete work. Gravel, when of flint or other hard material, may make excellent concrete.

Sizes for Broken Stone.—The sizes to which concrete stone should be broken depends upon the use to which the concrete is to be put. In heavy walls or massive work, the upper limit of size may be 2 or 3 inches in diameter. It is desirable to have the stones as large as can be easily incorporated into the mixture. In reinforced work, where the concrete must pass between and under the reinforcing bars, it may not be feasible to use stone of more than 1 inch diameter.

In stone or gravel for coarse aggregate, as in sand for mortar, the grading of sizes should be such as to give maximum density. For a given stone, the strongest concrete will ordinarily be made by that arrangement of sizes which requires the least mortar to completely fill the voids in the stone, as a surplus of mortar beyond that required for completely filling the voids is an element of weakness in the concrete, as well as a waste of the more expensive materials. Stone as ordinarily used in concrete contains all sizes, from the largest allowed to the size of the largest sand. All material retained on a $\frac{1}{4}$- or $\frac{3}{8}$-inch screen is commonly regarded as coarse aggregate, and stone is used as it comes from the crusher with all the sizes included, only the chips being screened out.

Gravel containing sand is sometimes used without screening by mixing with cement. This is not desirable practice. The sand is seldom in proper quantity or uniformly distributed through the

gravel. It should be screened out and proportioned properly to the cement and gravel.

In concrete work it is usually necessary to use the materials available in the locality of the work, but where important work is to be done, careful attention should be given to the character of these materials and of the concrete made from them. The design of concrete structures should be based upon full information concerning the properties of the concrete to be used, and this is largely a question of aggregates. Poor concrete work has much more frequently resulted from the use of poor aggregates than from the use of inferior cement.

In many cases it may be feasible and desirable to use materials of low grade in concrete work. Local materials may be of poor quality, but usable by taking proper precautions and designing the work in accordance with the character of the concrete. Failures have sometimes resulted from the use of low-grade materials without investigation of their qualities. Many users of concrete have failed to recognize the importance of the quality of the aggregates and seem to have regarded any stone broken to proper size as good enough for concrete.

Cinders as Coarse Aggregate.—Some designers favor the use of cinders as coarse aggregate in those situations where lightness is desired and where low strength is permissible. A 1 : 2 : 4 (by volume) cinder concrete will weigh about 90 lb./ft.^{3} and has an ultimate strength of about 700 lb./in.^{2} Cinder concrete is used mostly as a filler, but is permitted for use in reinforced concrete slabs (not in beams or girders) by some building ordinances. The allowed compressive unit stress is usually taken at about one-half of that for 2000 lb. concrete, and the ratio of the modulus of elasticity of the steel to that of the concrete is taken at about 30. The quality of cinders is generally limited to "clean, thoroughly burnt, steam boiler cinders, free from matter other than cinders," and it is commonly stipulated that "cinder concrete piers or walls shall not be permitted to carry loads or be given any credit therefor."

75.—Tests for Coarse Aggregates.—The Joint Committee of the Engineering Societies on Concrete and Reinforced Concrete in its report of August, 14, 1924, has suggested the following for coarse aggregate:

13.—*General Requirements.*—Coarse aggregates shall consist of crushed stone, gravel or other approved inert materials with similar characteristics or combinations thereof, having clean, hard, strong, durable, uncoated particles free from

injurious amounts of soft, friable, thin, elongated or laminated pieces, alkali, organic, or other deleterious matter.

14.—*Grading.*—Coarse aggregate† shall range in size from fine to coarse within the limits given in table.

SIZE AND GRADING OF COARSE AGGREGATE

Passing.	Percentage by Weight.
....* in. sieve (maximum size)........................	Not less than 95
....* in. sieve (maximum size)........................	Not less than 95
....* in. sieve (intermediate size)....................	Not less than ..* Not less than ..*
No. 4 sieve....................................	Not more than 10
No. 8 sieve....................................	Not more than 5

* The Engineer must insert in these blanks the sizes and percentages required with regard to materials available. The following table indicates desirable gradings for coarse aggregates of certain nominal sizes:

Nominal Maximum Size of Aggregate in Inches	Percentage by Weight Passing through Standard Sieves with Square Openings.						Percentage Passing Not More than:	
	3 in.	2 in.	1½ in.	1 in.	¾ in.	½ in.	No. 4 Sieve	No. 8 Sieve
3	95	.....	40–75	.....	.....	.....	10	5
2	.....	95	.....	40–75	.....	.....	10	5
1½	.....	.....	95	.....	40–75	.....	10	5
1	.....	.....	.....	95	.....	40–75	10	5
¾	.....	.....	.....	.....	95	.....	10	5
½	.....	.....	.....	.....	.....	95	10	5

15.—*Sieve Sizes.*—The test for size and grading of aggregates shall be made in accordance with the "Standard Method of Test for Sieve Analysis of Aggregates for Concrete" (Serial Designation: C41–24) of the American Society for Testing Materials for 1924, pp. 767 and 768.

16.—*Permissible Variations.*—Coarse aggregate‡ which does not conform to the above requirements, may be used only when approved by the Engineer and then in such proportions as he may require.

Apparent Specific Gravity.—The weight of a given volume of the solid material of which the aggregate is composed is often of importance in the determination of voids, or in proportioning concrete, a result obtained by determining the apparent specific gravity. The

† On work of considerable magnitude where several suitable coarse aggregates are available, an investigation of the relative economy of each is advisable.

‡ Requirements for the quality of coarse aggregate for special purposes should be inserted.

term "apparent specific gravity" as here used refers to the material as it exists, and includes the voids in the block of material tested; it may be somewhat less than the true specific gravity. For this purpose, the water to which it is referred need not be distilled, and determinations at ordinary air temperatures are sufficiently accurate.

The following is a very simple method of determining the apparent specific gravity of coarse aggregate:

First, weigh about 400 g. of the sample to be tested which has passed the $1\frac{1}{2}$-inch sieve and which has been washed and oven dried. Record this as weight A.

Second, measure 300 cc. (300 g.) of water in a 500-cc. cylinder and record this as weight C.

Third, after soaking the dried and weighed pieces of the specimen in water for twenty-four hours and wiping the individual pieces free from surface moisture with a towel or blotting paper, introduce the pieces of aggregate into the water in the cylinder. Care must be exercised not to entrain any air in the operation, and the amount of aggregate introduced must be such as to leave the final water level above the aggregate. Record the final elevation of water in cubic centimeters as the weight in grams B.

Then, $$\textit{Apparent Specific Gravity} = \frac{A}{B-C}.$$

It is understood that the apparent specific gravity includes the voids in the specimen and is therefore always less than or equal to, but never greater than the true specific gravity of the material.

TABLE VIII.—SPECIFIC GRAVITIES AND APPROXIMATE WEIGHTS PER CUBIC FOOT OF MATERIALS COMMONLY USED FOR AGGREGATES

	Specific Gravity.	Weight per Cubic Foot.
Gravel	2.65	165
Trap	2.85–3.00	178–187
Granite	2.65–2.80	165–175
Limestone	2.50–2.75	155–170
Compact sandstone	2.45–2.70	153–168
Porous sandstone	2.10–2.40	130–150
Cinders	1.40–1.60	90–100

Determination of Voids.—The voids in coarse material, such as gravel or broken stone not containing sand or other fine material,

may be obtained by filling a measure of known volume with the material, and pouring in water until the measure is full.

$$\text{Then, the percentage of voids} = \frac{\text{The volume of water}}{\text{The total volume}} \times 100.$$

When the specific gravity of the material is known, the voids may be obtained by weighing a measured volume of the broken stone, subtracting this weight from the weight of an equal volume of the solid material, and dividing by the solid weight.

The apparatus and method of filling the measure specified in the Standard Method of Test for Unit Weight of Aggregate for Concrete (Serial Designation: C 29-21) given in the A. S. T. M. 1924, pages 759 and 760 are recommended for the voids test for coarse aggregate.

If the aggregate contains fine material, the methods used for sand as given in Art. 7 must be used.

It is evident that the percentage of voids in a mass of broken material is not a fixed quantity, but varies with the arrangement of the pieces. If the material were composed of equal cubes, it would be possible to place them side by side so as to leave no voids which could be filled by smaller material. Poured loosely into a measure, such cubes would probably show at least 45 per cent of voids, which would be somewhat modified by shaking down and compacting the mass.

When the aggregate contains small pieces which may lie in the voids of the larger ones, the tendency to variation in results according to arrangement is greatly reduced, but the method of filling the measure, and amount of shaking that is given, will somewhat affect the results. Commonly, the material is shoveled into the measure and lightly shaken to get what may be a fair estimate of the voids in the material as it is to be used.

When fine material is introduced into a coarse aggregate to fill the voids, particles of the fine material get between the larger pieces and hold them apart so that the voids to be filled in the larger material are increased, and cannot be completely filled. This is shown by the fact that the volume of the mixture is greater than that of the coarse aggregate even though the volume of fine aggregate used is much less than the volume of voids in the larger material.

76. Rubble and Cyclopean Aggregate.—The Joint Committee in its Report of August, 1924, recommends the following:

17. *Rubble Aggregate.*—Rubble aggregate shall consist of clean, hard, durable stone or gravel larger than 3 inches and weighing not more than 100 lb.

18. *Cyclopean Aggregate.*—Cyclopean aggregate shall consist of clean, hard, durable stone weighing more than 100 lb.

77. Storage of Aggregate.—In the same report the following is recommended:

19. *Storage of Aggregate.*—Aggregate shall be so stored as to avoid the inclusion of foreign materials. Frozen aggregate or aggregate containing lumps of frozen material shall be thawed before using.

78. Water.—Also in the same report the following is recommended:

20.—*General Requirements.*—Water for concrete shall be clean and free from injurious amounts of oil, acid, alkali, organic matter, or other deleterious matter.

ART. 20. PROPORTIONING CONCRETE

79. Arbitrary Proportions.—The common method of proportioning concrete is by assuming ratios between the volumes of cement, sand, and coarse aggregate. These proportions are varied according to the character of the work, and sometimes are adjusted to the qualities of the materials. A formula of definite proportions does not always lead to the same result unless the method of measuring the materials is the same, as cement measured loose may vary considerably in weight for the same volume. A barrel of cement may measure from 3.5 to 5 cubic feet, according to its degree of compactness. It is desirable to follow the recommendation of the Joint Committee on Concrete and take one sack (94 pounds) of cement as a cubic foot, or a barrel as 4 cubic feet in measuring the materials.

Specific fixed proportions have to a certain extent become standard in ordinary practice for various kinds of work. For reinforced concrete in building construction and where it is necessary to develop considerable strength, the proportions of 1 part cement, 2 parts sand, and 4 parts broken stone are commonly employed. For positions where strength is of special importance, as in column construction, or work in light superstructures of buildings, the proportions $1 : 1\frac{1}{2} : 3$, or sometimes $1 : 1 : 2$, are used. In more massive work and where only compressions are to be carried with ample sections, the proportions $1 : 2\frac{1}{2} : 5$ and sometimes $1 : 3 : 6$ are employed.

The common proportions are based upon the requirement that the volume of fine aggregates shall be one-half that of the coarse aggregate. For materials commonly used, this gives a quantity of mortar sufficient to fill compactly the interstices in the coarse

aggregate. The quality of the mortar is varied by changing the ratio of cement to fine aggregate, and the strength of the concrete varies accordingly. The ratios between fine and coarse aggregates are often varied when the coarse aggregates contain more or fewer voids than is usual, and 1 : 2 : 3, 1 : 3 : 5, 1 : 2 : 5 or 1 : 3 : 7 concrete is frequently used.

While good results have no doubt been secured in some individual cases in practice by this method of proportioning, equally good results with more economy will be obtained by a more careful adjustment of the ingredients.

80. Proportioning by Voids.—A method of proportioning sometimes followed is to determine the voids in the aggregates, and use enough cement to fill the voids in the fine aggregate and enough mortar to fill the voids in the coarse aggregate. A small excess of fine materials is used in each case on account of inequalities of mixing. If the fine materials would all lie in the voids of the larger materials, this method would always give the desired result, and produce the concrete of maximum density and greatest strength. In practice, however, the voids cannot be completely filled, the volumes of the larger materials are increased by the smaller particles lying between them, and the distribution of fine material through the mass is not uniform.

Usually a volume of mortar 5 to 10 per cent in excess of the voids most nearly fills the voids without leaving appreciable excess of mortar. More mortar than this swells the volume of the concrete without increasing density, and has the effect of weakening the concrete. If for instance for a cubic yard of concrete, broken stone containing 50 per cent voids is used with sand containing 40 per cent voids, and 8 per cent each is allowed for the "bulking" of the stone and sand, the quantities would be: broken stone, 0.92 cubic yard, and mortar, 0.5 cubic yard; the mortar would be composed of sand, 0.46 cubic yard, and cement, 0.2 cubic yard or 1.35 bbl. This would correspond to an arbitrary assignment ratio of 1 : 2.3 : 4.6. The ratio of the cement to the combined aggregates would be about 1 : 4.96.

This method of proportioning is an improvement over that of arbitrary assignments of ratios, and usually gives approximately the most desirable proportions. Variations in the relative sizes of the materials, however, may change considerably the proportions necessary to give the most dense concrete. A certain sand may easily work into the voids of a given broken stone without materially increasing its volume, while with another stone containing the same

percentage of voids but of different sizes, the same sand may produce quite different results, and to secure greatest density would need to be differently proportioned. The object should be to get the greatest density in the final mixture of fine and coarse aggregates.

The inaccuracies involved in proportioning cement to sand by determining the voids in the sand is explained in Art. 7. When determining the ratio of fine to coarse aggregates by the method of voids, it is usual to proportion cement to sand by adopting an arbitrary ratio between the two, although some users of concrete have adopted the void method for this purpose also.

81. Proportioning by Trial.—A method which still finds favor with some engineers is that of proportioning by trial. The object is to secure that mixture of fine and coarse aggregates which when mixed with cement and water will result in a concrete of the greatest density. The procedure consists in mixing various batches made up of different proportions and comparing the densities of the resulting concrete.

For making these tests, it is convenient to use a cylindrical measure 8 or 10 inches in diameter and 12 or 15 inches high. A batch of concrete is mixed in assumed proportions to the consistency to be used on the work, and the height to which it fills the cylindrical measure is noted. Other batches are prepared with the same total weight of materials, but differing in proportions of aggregates, and measured in the same manner. The greatest density is that which occupies the least volume for the same weight. It is necessary to use a uniform method of filling the cylinders, and it is usually desirable to compact the mass by introducing the concrete in layers and "rodding" each layer as recommended by the Joint Committee and described under 4. *Procedure*, on p. 139.

Amount of Cement.—In this method of proportioning, the object is to determine the proper proportions of aggregates to give the most dense concrete. In each batch, the weight of cement to be used is assumed as a definite ratio to the total weight of the aggregates. This ratio depends upon the character of the work and the need for strength in the concrete. In important work, it is desirable to test the strength as well as the density of the concrete from time to time and modify the proportion of the cement to meet the requirements.

More cement is usually required to produce the same strength when the sizes of the coarse aggregates are small than when larger aggregates are used. Stone broken to pass the $\frac{3}{4}$-inch screen may

require from 20 to 25 per cent more cement for the same strength than the same stone broken to pass the $1\frac{1}{2}$-inch screen.

82. Proportioning by Fineness Modulus.—After extensive investigations at Lewis Institute, Chicago, Professor Duff A. Abrams presented before the annual meeting of the Portland Cement Association in December, 1918, a paper entitled, "Design of Concrete Mixtures." His paper was based on the results of about 50,000 tests, and he concluded that proportioning of concrete based on arbitrary assignment of ratios, density of aggregates, density of concrete, and surface areas of aggregates did not give satisfactory results; a grading coarser than that giving maximum density is necessary for highest strength. He proposed as a substitute a method of proportioning based on what he calls the "fineness modulus" of the aggregates.

To determine the fineness modulus of a given aggregate, a sieve analysis is made using the Tyler series: 100, 48, 28, 14, 8, 4-mesh, $\frac{3}{8}$ inch, $\frac{3}{4}$ inch, and $1\frac{1}{2}$ inch, each of which has an opening twice the width of the preceding one. The percentage of aggregates coarser than each sieve is determined and the fineness modulus is the sum of these percentages divided by 100. The fineness modulus of three sands is given in Table I, page 45, and the fineness modulus of each of three varieties of pebbles and of a concrete aggregate is given in Table IX.

TABLE IX.—FINENESS MODULUS OF AGGREGATES

Sieve Size.	Pebbles.			Concrete Aggregate.
	Fine. (D)	Medium. (E)	Coarse. (F)	(G)
100-mesh	100	100	100	98
48-mesh	100	100	100	92
28-mesh	100	100	100	86
14-mesh	100	100	100	81
8-mesh	100	100	100	78
4-mesh	86	95	100	71
$\frac{3}{8}$-in	51	66	86	49
$\frac{3}{4}$-in	9	25	50	19
$1\frac{1}{2}$-in	0	0	0	0
Fineness modulus	6.46	6.86	7.36	5.74

Note.—Sieves Nos. 50, 30, and 16 have been substituted for Nos. 48, 28, and 14 respectively without change of separation.

Concrete aggregate "G" is made up of 25 per cent of sand "B" of Table I, p. 45, and 75 per cent of pebbles "E" in Table IX.

"A well graded torpedo sand up to No. 4 sieve will give a fineness modulus of about 3.00; a coarse aggregate graded from No. 4 to $1\frac{1}{2}$ inch will give a fineness modulus of about 7.00; a mixture of these two in proper proportions for a mortar mix of 1 : 4 will have a fineness modulus of about 5.80. A fine sand such as drift sand may have a fineness modulus as low as 1.50."

"There is an intimate relation between the sieve analysis curve for the aggregate and the fineness modulus."

"Many different series of tests have shown that for a given plastic condition of concrete and the same mix there is an intimate relation between the fineness modulus of the aggregate and the strength and other properties of the concrete."

Diagram III, p. 130, gives the results of certain compression tests which bring out the relation between the strength of the concrete and the fineness modulus of the aggregate. It will be noted in the diagram that a separate curve may be drawn for each mix. In each case there is a steady increase in the compressive strength of the concrete as the fineness modulus of the aggregate increases, until a certain value is reached which corresponds to a maximum point. It will be noted also that this maximum point corresponds to higher and higher values of fineness modulus as the quantity of cement is increased. In other words, the maximum strength comes at a fineness modulus of about 5.80 for the 1 : 9 mix and about 6.40 for the 1 : 4 mix. In these tests the different values of the fineness modulus were secured by using a preponderance of the coarser sizes, but in all cases maintaining the same limiting size, that is, 0 to $1\frac{1}{4}$ inches. Table X gives the fineness modulus for each of the aggregates used in Diagram III.

In Diagram IV, p. 131, is found a similar relation between the strength and the fineness modulus, except that no maximum point is reached. This condition arises from the fact that the maximum size of aggregate is increasing without changing the type of the sieve analysis curve, consequently the fineness modulus curve continues to rise indefinitely. The height to which the curve rises is limited only by the size of the aggregate which may be used. It is to be noted that there is no conflict between Diagrams III and IV. Table XI gives the fineness modulus for each of the aggregates used in Diagram IV.

That aggregates with the *same fineness modulus*, even though they have the widest variation in the grading of the particles, will have practically the same strength is shown in Table XII, p. 132.

The same table also shows that the surface areas of the aggregates do not serve as a good guide for proportioning concrete.

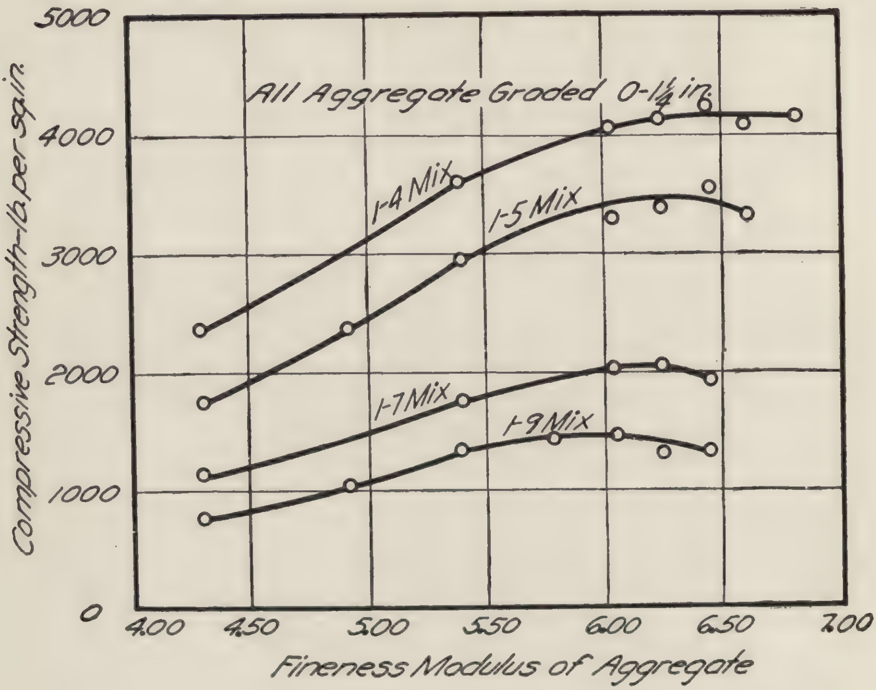


Diagram III.—Relation between Fineness Modulus of Aggregate and Strength of Concrete.
(Courtesy of Prof. Duff A. Abrams.)

TABLE X.—SIEVE ANALYSIS OF AGGREGATES IN DIAGRAM III

Sand and pebble aggregate graded 0–1¼ inch; 28-day compression tests of 6 by 12-inch cylinders. (Series 78.)

Range in Size.	Fineness Modulus.	Per Cent Coarser than Each Sieve.									
		100	48	28	14	8	4	⅜	¾	1½	2
0–1¼	4.30	89	82	72	62	51	38	25	11	0	..
	4.93	95	89	82	73	61	47	32	14	0	..
	5.40	98	94	88	80	69	55	38	18	0	..
	6.04	99	98	95	90	81	68	49	24	0	..
	6.25	100	99	97	92	85	72	53	27	0	..
	6.45	100	99	98	95	88	77	58	30	0	..
	6.60	100	100	99	96	91	80	62	32	0	..
	6.82	100	100	99	98	94	86	68	37	0	..

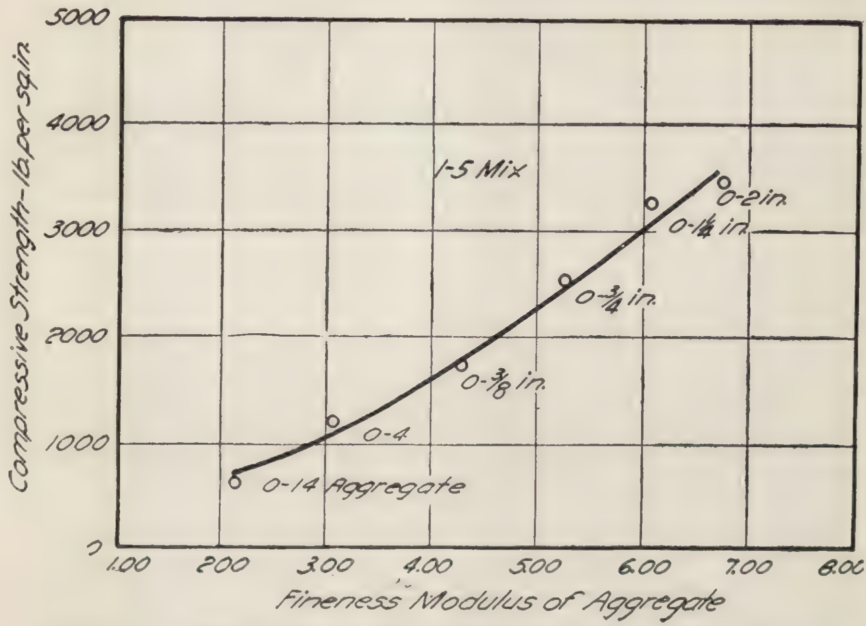


Diagram IV.—Relation between Fineness Modulus of Aggregate and Strength of Concrete.
(Courtesy of Prof. Duff A. Abrams.)

TABLE XI.—SIEVE ANALYSES OF AGGREGATES IN DIAGRAM IV

Twenty-eight-day compression tests of 6 by 12 inch cylinders. (Series 78). Sand and pebble aggregate graded to sizes shown. The contrast between the relation shown by these tests and those in Diagram III should be noted.

Range in Size.	Fineness Modulus.	Per Cent Coarser than Each Sieve.									
		100	48	28	14	8	4	⅜	¾	1½	2
0–14	2.16	95	84	37	0						
0– 4	3.06	96	90	62	40	18	0				
0– ⅜	4.26	93	91	83	71	31	0				
0– ¾	5.24	98	93	91	83	71	54	31	0		
0– 1¼	6.04	99	98	95	90	81	68	49	24	0	
0– 2	6.72	100	99	97	94	87	77	62	42	14	0

Maximum Permissible Values of Fineness Modulus of Aggregates.—Since a maximum practicable value of fineness modulus is found for each size of aggregate and mix, it is necessary to place certain limits on the value which may be used for proportioning materials for concrete mixes. Table XII gives limits which will be found prac-

TABLE XII.—EFFECT OF GRADING OF AGGREGATES ON THE STRENGTH OF CONCRETE.

Compression tests of 6 by 12-inch-concrete cylinders.

Mix 1 : 5 by volume; age at test, 28 days; stored in damp sand; tested damp.

Aggregates—sand and pebbles from Elgin, Ill. Aggregates were screened to different sizes and recombined to conform to predetermined sieve analyses.

The same quantity of water was used in all specimens of a given consistency. The 110 per cent consistency contains 10 per cent more water than the 100 per cent.

Each specimen was made from a separate batch.

Each value in the strength tests is the average from 5 tests made on different days. (From Series 78.)

Ref. No.	Sieve Analysis of Aggregate Per Cent Coarser than Each Sieve.											Fineness Modulus of Aggregate.	Surface Area of Aggregate Square Inch.		Compressive Strength of Concrete at 28 Days (Lb. per Sq. In.)	
	100	48	28	14	8	4	$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{1}{2}$	2		Per Pound of Aggregate.	Per Gram Cement.	100% Consistency.	110% Consistency.
40	99	98	95	90	81	68	49	24	0	..	..	6.04	602	8.8	3300	2890
259	99	98	96	92	84	67	46	22	0	..	..	6.04	569	8.2	2950	2650
260	98	97	93	88	80	67	52	29	0	..	..	6.04	764	11.4	3120	2760
261	97	94	91	85	77	67	58	35	0	..	..	6.04	999	15.2	3140	2790
262	95	92	87	82	75	67	67	39	0	..	..	6.04	1292	20.1	3100	2800
263	95	90	84	78	73	67	62	55	0	..	..	6.04	1451	23.0	2830	2740
264	95	89	82	75	67	67	67	62	0	..	..	6.04	1565	25.2	2680	2580
265	100	97	91	79	72	67	58	40	0	..	..	6.04	761	11.9	3070	2690
266	100	97	93	88	83	67	50	27	7	0	..	6.04	616	9.0	3080	2790
267	99	97	94	86	77	67	47	27	16	..	0	6.04	709	10.5	3150	2710
268	98	95	90	83	83	83	50	22	0	..	..	6.04	834	12.6	3080	2500
269	98	94	90	86	83	80	55	18	0	..	..	6.04	898	13.3	3050	2550
270	96	90	80	80	80	80	60	39	0	..	..	6.04	1391	21.5	2970	2550
271	100	96	92	87	81	75	50	23	0	..	..	6.04	672	10.0	2930	2710
272	95	91	87	82	77	73	59	40	0	..	..	6.04	1315	20.2	3000	2580
273	99	95	88	80	76	73	61	32	0	..	..	6.04	911	13.9	2950	2740
274	90	85	81	78	75	73	66	56	0	..	..	6.04	1992	31.3	2680	2440
275	100	93	82	73	73	73	63	47	0	..	..	6.04	1076	16.7	2820	2620
276	100	100	100	92	81	60	45	26	0	..	..	6.04	390	5.6	3040	2780
277	100	98	95	90	80	60	50	31	0	..	..	6.04	557	8.3	2900	2770
278	100	99	96	92	84	55	50	28	0	..	..	6.04	483	7.0	2940	2750
279	100	99	96	91	80	50	50	38	0	..	..	6.04	514	7.6	3080	2750
280	98	84	84	84	84	57	57	57	0	..	..	6.04	1276	19.7	3000	2780
281	99	98	91	86	80	76	38	38	0	..	..	6.04	701	10.4	2940	2700
282	99	98	91	86	80	76	46	30	0	..	..	6.04	697	10.2	3020	2660
283	99	98	91	86	80	76	61	15	0	..	..	6.04	689	10.1	2930	2670
284	99	98	91	85	80	76	67	8	0	..	..	6.04	685	9.9	2970	2630
Average												6.04	904	13.8	2990	2690
Minimum Value												....	390	5.6	2680	2440
Maximum Value												....	1992	31.3	3300	2890
Mean Variation from Average—per cent												....	34.4	37.2	3.41	3.04

ticable. Subsequent experience may dictate certain modifications in the details.

The purpose of Table XII is to avoid the attempt to secure an aggregate grading which is too coarse for its maximum size and for

the amount of cement used. It is also useful in prohibiting attempts to use sands which are too coarse for best results in concrete mixtures. For instance, it would be found from this table that the use of a sand of the nature of standard Ottawa sand is not permitted except in mixes of 1 : 2 or richer.

The curves in Diagram V, p. 134, are platted directly from the values given for the standard sieves in Table XIII.

TABLE XIII.—MAXIMUM PERMISSIBLE VALUES OF FINENESS MODULUS OF AGGREGATES

Mix Cement Aggregate.	Size of Aggregate													
	0 to 28	0 to 14	0 to 8	0 to 4	0 to 3*	0 to $\frac{3}{8}$	0 to $\frac{1}{2}$*	0 to $\frac{3}{4}$	0 to 1* in.	0 to $1\frac{1}{2}$ in.	0 to 2.1* in.	0 to 3 in.	0 to $4\frac{1}{2}$* in.	0 to 6 in.
1–12	1.20	1.80	2.40	2.95	3.35	3.80	4.20	4.60	5.00	5.35	5.75	6.20	6.60	7.00
1–9	1.30	1.85	2.45	3.05	3.45	3.85	4.25	4.65	5.00	5.40	5.80	6.25	6.65	7.05
1–7	1.40	1.95	2.55	3.20	3.55	3.95	4.35	4.75	5.15	5.55	5.95	6.40	6.80	7.20
1–6	1.50	2.05	2.65	3.30	3.65	4.05	4.45	4.85	5.25	5.65	6.05	6.50	6.90	7.30
1–5	1.60	2.15	2.75	3.45	3.80	4.20	4.60	5.00	5.40	5.80	6.20	6.60	7.00	7.45
1–4	1.70	2.30	2.90	3.60	4.00	4.40	4.80	5.20	5.60	6.00	6.40	6.85	7.25	7.65
1–3	1.85	2.50	3.10	3.90	4.30	4.70	5.10	5.50	5.90	6.30	6.70	7.15	7.55	8.00
1–2	2.00	2.70	3.40	4.20	4.60	5.05	5.45	5.90	6.30	6.70	7.10	7.55	7.95	8.40
1–1	2.25	3.00	3.80	4.75	5.25	5.60	6.05	6.50	6.90	7.35	7.75	8.20	8.65	9.10

* Considered as "half-size" sieves; not used in computing fineness modulus.

For *mixes* other than those given in the table, use the values for the next leaner mix.

For *maximum sizes* of aggregate other than those given in the table, use the values for the next smaller size.

Fine aggregate includes all material finer than No. 4 sieve; *coarse aggregate* includes all material coarser than the No. 4 sieve. *Mortar* is a mixture of cement, water and fine aggregate.

This table is based on the requirements for *sand-and-pebble* or *gravel* aggregate composed of approximately spherical particles, in ordinary uses of concrete in reinforced concrete structures. For other materials and in other classes of work the maximum permissible values of fineness modulus for an aggregate of a given size is subject to the following corrections:

(1) If *crushed stone* or *slag* is used as coarse aggregate, *reduce* values in Table by 0.25. For crushed material consisting of unusually flat or elongated particles, *reduce* values by 0.40.

(2) For *pebbles* consisting of *flat particles, reduce* values by 0.25.

(3) If *stone screenings* are used as fine aggregate, *reduce* values by 0.25.

(4) For the top course in *concrete roads*, *reduce* the values by 0.25. If finishing is done by *mechanical means*, this reduction need not be made.

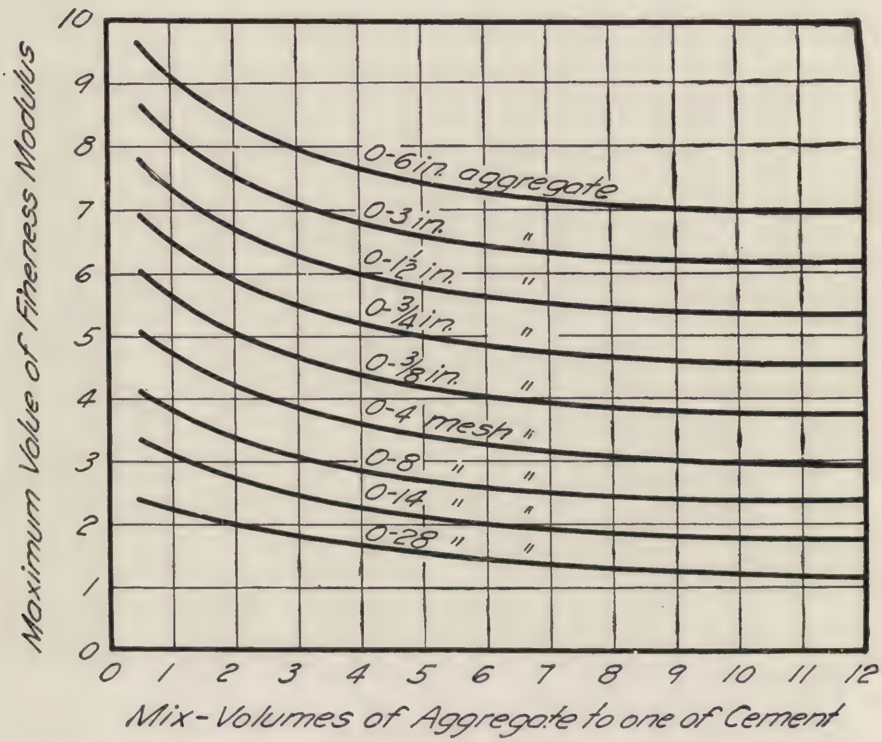


DIAGRAM V.—Maximum Permissible Values of Fineness Modulus of Aggregate.

Graphical reproduction of Table XII. These curves are based on the requirements of sand and pebble aggregate. For crushed stone aggregate the values must be reduced as noted in the table.

(Courtesy of Prof. Duff A. Abrams.)

(5) In work of *massive proportions*, such that the smallest dimension is larger than 10 times the maximum size of the coarse aggregate, *additions may be made* to the values in the table as follows: for ¾-inch aggregate 0.10; for 1½-inch 0.20; for 3-inch 0.30; for 6-inch 0.40.

Sand with fineness modulus lower than 1.50 is undesirable as a fine aggregate in ordinary concrete mixes. Natural sands of such fineness are seldom found.

Sand or screenings used for fine aggregate in concrete must not have a higher fineness modulus than that permitted for mortars of the same mix. Mortar mixes are covered by the table and by (3) above.

Crushed stone mixed with both finer sand and coarser pebbles requires no reduction in fineness modulus provided the quantity of crushed stone is less than 30 per cent of the total volume of the aggregate.

83. Consistency of Concrete.—Too much fine aggregate in concrete will produce porosity and weakness; too much coarse aggregate will result in honey-combing and weakness. The careful adjustment of the ratios of fine and coarse aggregates has received considerable attention, and the ready comparison of the combined aggregates seems to have been reached in the fineness modulus method.

The quantity of water entering into the mixture has not until recently received deserved attention. Workers in the laboratory have noticed that if an average Portland cement is casually mixed with 24 per cent of its weight of water, a comparatively stiff mass will result, whereas if the same mixture is vigorously worked, a fairly plastic mass will be obtained, that is, a mass of about " normal consistency of cement." A slightly smaller percentage of water will be sufficient to hydrate the cement, while a somewhat larger percentage will be required to produce a mixture which will flow down a plane inclined at an angle of 20° with the horizontal.

If correct proportions of cement, sand, and broken stone are used with just enough water to hydrate the cement, the concrete will have to be placed in layers from 6 to 8 inches thick and each layer thoroughly tamped to prevent honey-combing. For concrete which is to be distributed over a comparatively large area, this is uneconomical. The practice of transporting concrete in chutes is common on large contracts. The inclination of the chutes is generally limited to not less than 20°. Water to the extent of approximately 58 per cent of the weight of the cement when the mixture is properly worked will give the desired flow. Sometimes the attempt is made to obtain flow by the addition of water rather than by the use of the proper amount of water accompanied by proper vigorous working. This results in a decided weakening of the concrete. It is not uncommon to specify the time and number of revolutions per minute for machine mixing as a check to such practice.

The effect of the quantity of mixing water on the strength and other qualities of concrete has been worked out in the Structural Materials Laboratory, Chicago, through the coöperation of the Portland Cement Association.

Water Ratio Theory.—To quote from the booklet published by the Portland Cement Association entitled, "Design and Control of Concrete Mixtures": "The water ratio theory is that, for given materials and conditions of manipulation, the strength of concrete is determined by the ratio of the volume of mixing water to the volume of cement (water ratio) so long as workable mixtures are obtained. In other words, if one cubic foot of water is used for each cubic foot of cement in a concrete mixture the strength at a given age is fixed regardless of what quantities of other materials are used, so long as the mixture is plastic and workable, and the aggregates are clean and made up of sound, durable particles. The significance of this important conclusion is more readily appreciated when the cement paste is thought of as a glue binding the aggregates together. The addition of excess mixing water serves only to dilute the glue and reduce the strength. Less than $2\frac{1}{2}$ gallons of water is sufficient to hydrate one sack of cement. While it is necessary to use more than this amount to produce a workable mix of concrete, any water in excess of that required to hydrate the cement reduces the strength and makes a more porous concrete

"The quantity of cement, the plastic condition or workability of the concrete (whether it is wet or dry) and the size and grading of the aggregate affect the strength of concrete only in so far as they affect the quantity of water required in the mix. The important influence of these factors has long been recognized, but the part played by each was not clearly understood until their relation to the quantity of mixing water was demonstrated.

"It is common knowledge that, so long as other conditions are maintained constant, the strength of concrete is increased by (1) mixing the concrete drier, (2) increasing the cement content, or (3) using coarser aggregate.

"Each of these changes or combination of changes in the mixture reduces the water ratio and the strength of the concrete is increased as the water ratio is decreased. The limit to which these changes may be carried is determined by the plasticity of the concrete The amount of mixing water used must be sufficient to produce a plastic mixture and the aggregate must not be too coarse for the amount of cement used.

"More than 100,000 tests made at the Structural Materials Research Laboratory and numerous tests made at other laboratories have demonstrated the accuracy of the water ratio theory. For concrete made under average conditions and cured in the presence

of moisture, the compressive strength may be expressed by the equation:

$$S=\frac{14,000}{7^x}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

in which

S = compressive strength of concrete at 28 days, pound per square inch;

x = water ratio $\left(\frac{W}{C}\right)$ (an exponent).

From the above equation it will be seen that the compressive strength (S) increases as the water ratio $\left(x=\frac{W}{C}\right)$ decreases. It should be pointed out that these constants were determined for definite conditions of test. It would not be expected that exactly the same constants would be found for other materials and other curve B, conditions of test.

"In Diagram VI, p. 138, curve A represents this equation, and curve B, the equation

$$S=\frac{14,000}{9^x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

"This latter equation differs from the first only in that the constant in the denominator is 9 instead of 7. It gives strengths approximately 500 lb. per sq. in lower than the first, and may be considered as a reliable indication of the minimum strength to be expected where concreting operations are not under rigid control. As an example in the use of this diagram it will be seen that if concrete is mixed with a water ratio of 1.00 ($7\frac{1}{2}$ gallons of water per sack of cement) the strength at twenty-eight days which would be expected under average conditions is 2000 lb. per sq. in. (curve A) and that the minimum strength is 1500 lb. per sq. in. (curve B.)"

Consistency.—The 1924 Report of the Joint Committee says:

"30. *Consistency.*—The quantity of water used shall be the minimum necessary to produce concrete of a workability required by the Engineer. The consistency of the concrete shall be measured by the slump test——."

Slump Test.—In order to reach a comparison of the consistency or workability of concrete, the *slump test* has been suggested and is published as a "Tentative Method of Test for Consistency of Portland Cement Concrete for Pavements or for Pavement Base," (Serial Designation: D 138-22T) of the American Society for Test-

ing Materials. This Tentative Standard is published for the purpose of soliciting criticism and suggestions. It is noted that this test is

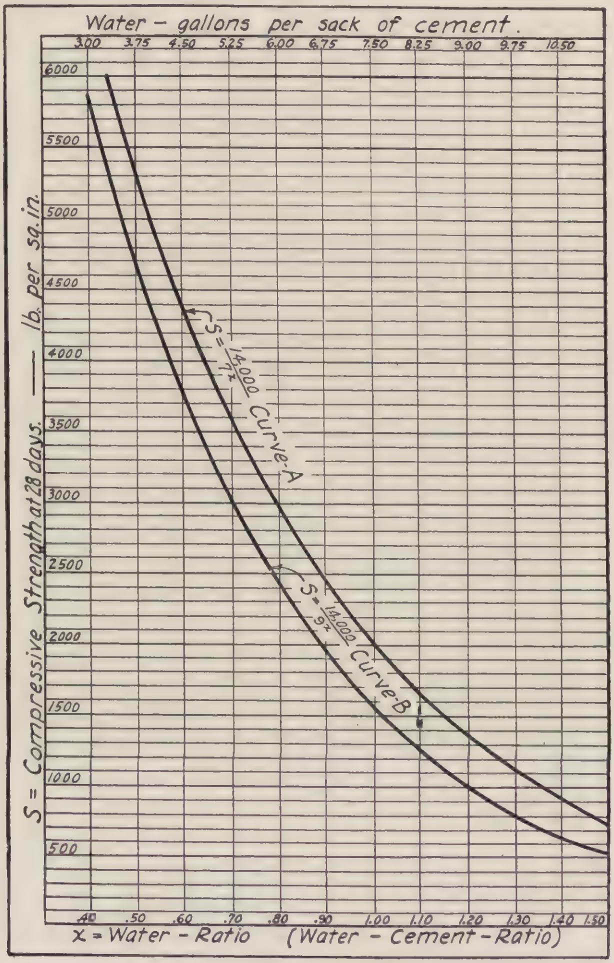


Diagram VI.—Effect of quantity of mixing water on the strenth of concrete. Curves are based on average values from nine different series of tests made over a period of four years. Tests made on about 50,000 6 by 12-inch-cylinders at 28 days.

(Courtesy of the Portland Cement Association.)

not considered applicable where there is a considerable quantity of coarse aggregate more than 2 inches in size in the concrete.

The following is a description of the method of procedure from the source mentioned:

1. *Scope.*—This test covers the method to be used both in the laboratory and in the field for determining consistency of concrete to be used for concrete pavements or for concrete base for pavements.

2. *Apparatus.*—The test specimen shall be formed in a mold of No. 16 gauge galvanized metal in the form of the lateral surface of the frustum of a cone with the base, 8 inches in diameter, the upper surface, 4 inches in diameter, and the altitude, 12 inches. The base and the top shall be open and parallel to each

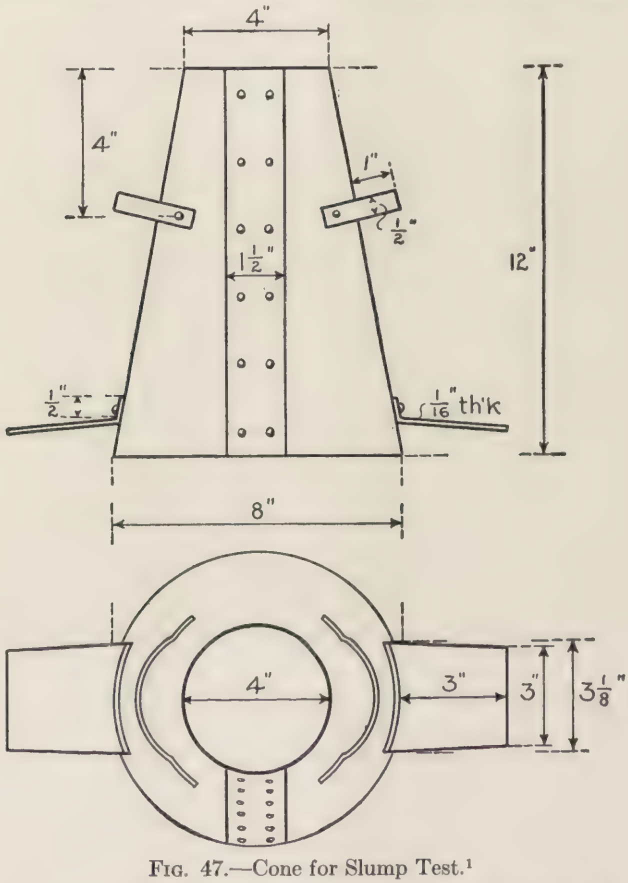


FIG. 47.—Cone for Slump Test.[1]

other and at right angles to the axis of the cone. The mold shall be provided with foot pieces and handles as shown in cut. (Fig. 47.)

3. *Sample.*—When the test is made at the mixer, the sample shall be taken from the pile of concrete immediately after the entire batch has been discharged. When testing concrete that has been hauled from a central mixing plant, the sample shall be taken from the concrete immediately after it has been dumped on the sub-grade.

4. *Procedure.*—The mold shall be placed on a flat, non-absorbent surface, such as a smooth plank or a slab of concrete, and the operator shall hold the form

[1] From Joint Committee Report of 1924. Appendix XI (Fig. 27).

firmly in place, while it is being filled, by standing on the foot pieces. The mould shall be filled to about one-fourth its height with the concrete which shall then be puddled, using 20 to 30 strokes of a $\frac{1}{2}$-inch rod pointed at the lower end. The filling shall be completed in successive layers similar to the first and the top struck off so that the mould is exactly filled. The mould shall then be removed by being raised vertically, and at exactly three minutes after being filled. The moulded concrete shall then be allowed to subside until quiescent and the height of the specimen measured.

5. *Slump.*—The consistency shall be recorded in terms of inches of subsidence of the specimen during the test, which shall be known as the slump. Slump = 12 inches of height after subsidence.

The slump-test requirement is intended to insure concrete mixed with the minimum quantity of water required to produce a plastic mixture.

The following table indicates the maximum slump desirable for the various types of concrete, based on average aggregate and proportions:

TABLE XIV.—MAXIMUM SLUMPS FOR CONCRETE

Types of Concrete and Mortar.	Maximum Slump, in Inches.
Mass concrete	3
Reinforced concrete:	
(*a*) Thin vertical sections and columns	6
(*b*) Heavy sections	3
(*c*) Thin confined horizontal sections	8
Roads and pavements:	
(*a*) Hand-finished	3
(*b*) Machine-finished	1
Mortar for floor finish	2

The consistency shall be checked from time to time during the progress of the work.

Attention is called to the fact that increased workability may be obtained by decreasing the quantity of coarse aggregate in the batch, without increasing the quantity of mixing water.

Figure 48, p. 141, shows man withdrawing the mold in making the slump test.

The relation between the consistency and the slump is shown in Fig. 49, p. 142. The tags on the piles indicate the relative consistency and the scale at the side shows the slump in inches.

"Concrete having a slump of $\frac{1}{2}$ inch to 1 inch will contain only a little more water than is necessary for maximum strength, but will be too stiff for most construction work. Such concrete is said to have a relative consistency of 1.00. Concrete containing 10 per cent more water is said to have a relative consistency of 1.10

and will give a slump of 3 to 4 inches; 25 per cent more water gives a relative consistency of 1.25 with a slump of from 6 to 7 inches; 50 per cent more water gives a relative consistency of 1.50 with a slump of 8 to 10 inches."

FIG. 48.—Slump Test-lifting Form.
(Courtesy of the Portland Cement Association.)

The table on page 143 is from "Concrete Data for Engineers and Architects" by courtesy of the Portland Cement Association.

Inter-relation of Consistency, Grading, Mix, and Strength.—The inter-relation of consistency, grading, mix, and strength is shown in Diagram VII, p. 144, taken from "Design and Control of Concrete Mixtures." The consistency is represented by the slump. Four sets of diagrams are shown including ranges of slump from $\frac{1}{2}$ inch to 10 inches. The curves are based on the water-ratio-strength-relation of

$$S = \frac{14{,}000}{9^x},$$

given in Sec. 83, p. 135.

Fig. 49.—Comparative Slumps for Different Consistencies of Concrete.

(Courtesy of the Portland Cement Association.)

TABLE XV.—APPROXIMATE QUANTITY OF MIXING WATER REQUIRED FOR CONCRETE

True Mix (by Volume).		Approximate Field Mix by Volume (Damp Aggregates Measured Loose).			Approximate Total Quantity of Water.
Volume of Cement 1 Cu. Ft. = 94 Lbs.	Volume of Combined Dry Aggregate (Rodded).	Cement.	Aggregate.		Gallons per Sack of Cement.
			Fine.	Coarse.	
1	$2\frac{3}{4}$	1	$1\frac{1}{4}$	$2\frac{1}{2}$	5 to $5\frac{1}{2}$
1	$3\frac{1}{4}$	1	$1\frac{1}{2}$	3	$5\frac{1}{2}$ to 6
1	$3\frac{3}{4}$	1	2	3	$5\frac{3}{4}$ to $6\frac{1}{4}$
1	$4\frac{1}{2}$	1	2	4	6 to $6\frac{1}{2}$
1	$5\frac{1}{2}$	1	$2\frac{1}{2}$	5	$7\frac{1}{4}$ to $7\frac{3}{4}$
1	$6\frac{3}{4}$	1	3	6	$8\frac{1}{4}$ to $8\frac{3}{4}$

Measurement of the Ingredients of Concrete.—The ingredients of concrete are usually measured by volume. A sack of Portland cement contains one cubic foot. The amount of material contained in a given volume of sand depends upon its compactness and moisture content. A volume of dry compacted standard sand may "bulk" as much as 20 to 25 per cent after being mixed with 2 or 3 per cent of its weight of water. The American Society for Testing Materials has suggested a method of uniform measurement by compacting the dry aggregate by "rodding" as described on page 140. Sand can be reduced to its most compact state by flooding or inundating, and the method of measuring sand by the "Inundation Method"[2] has been proposed and is being used to some extent.

The amount of material contained in a given volume of broken stone depends upon its compactness and to a slight extent upon its moisture content. The method of the A. S. T. M. for uniform measurement of aggregate mentioned above also applies to the coarse aggregate. Broken stone for use in concrete is sold largely by the ton and there is a tendency toward proportioning coarse aggregate by weight. The quantity of mixing water is usually stated in gallons, and mechanical mixers are now provided with

[2] "Inundation Method for Measurements of Sand in Making Concrete," by G. A. Smith and W. A. Slater, Proc. of the American Concrete Institute Vol. XIX (1923), p. 222, together with discussion by Stanton Walker.

"The Volume Moisture Relation in Sand and a Method of Determining Surface Areas Thereon," by R. B. Young and W. D. Walcott, Proc. American Society for Testing Materials, Vol. XX (1920), p. 137.

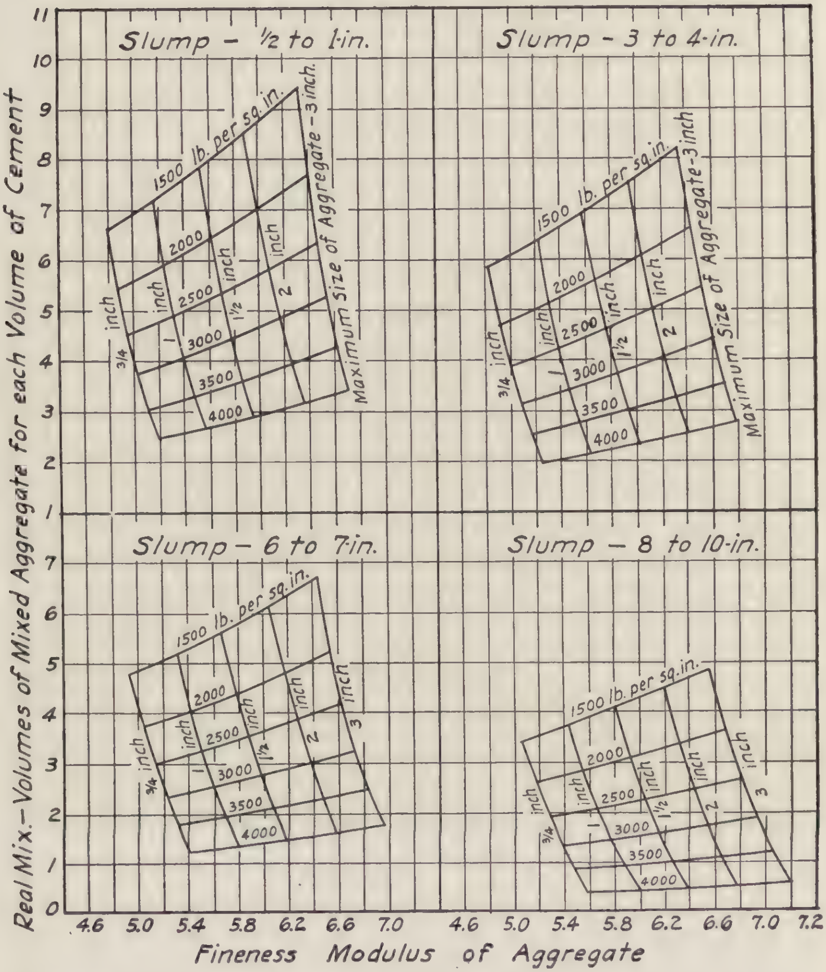


Diagram VII.—Relation of size and grading of aggregate and quantity of cement to strength or concrete. This figure is based on the relation between strength and quantity of mixing water shown by Curve B in Diagram VI.
(Courtesy of the Portland Cement Association.)

measuring devices for water and these devices are so constructed that the quantity of water can be regulated in accordance with the moisture content for the aggregates.

Combined Aggregates.—It is seldom that the fine and coarse aggregates occur in Nature so mixed as to give a desired fineness modulus. The booklet, "Design and Control of Concrete Mixtures" published by the Portland Cement Association, gives formulas to be

used in designing the mix and numerous examples of special applications.

84. Yield of Concrete.—The quantities of materials needed for a cubic yard of concrete vary with the amount of voids in the aggregates and the proportions in which they are combined. The sizes of the aggregates and the quantity of water used in mixing also influence the yield of concrete.

Concrete is made up of a mixture of cement, fine aggregate, coarse aggregate, and water, or it is a mixture of cement mortar with coarse aggregate. The volume of the concrete is the sum of the volumes of the mortar, the solid material in the coarse aggregate and the unfilled voids in the coarse aggregate.

Let C = Volume of cement in cubic feet (bags of 94 pounds each);
S = Volume of fine aggregate in cubic feet;
R = Volume of coarse aggregate in cubic feet;
V = Volume of voids in coarse aggregate in cubic feet;
s = Ratio of sand to cement $= S/C$;
r = Ratio of coarse aggregate to cement $= R/C$;
v = Ratio of voids to total volume of coarse aggregate, V/R.

The quantities of ingredients necessary to produce given volumes of cement mortars, and the variations for different materials, are discussed in Section 34, and while these quantities vary considerably with different materials, the volume of mortar produced by the mixture of different proportions of cement and sand is fairly well expressed by the expression:

$$\text{Volume of mortar} = aC + bS,$$

in which a and b are coefficients depending upon the character of the sand. The volume of concrete from given quantities of cement, sand and stone may then be expressed by the formula:

$$Q = aC + bS + c(R - V),$$

in which c is a coefficient depending upon the amount of unfilled voids in the stone. For ordinary fairly coarse sands commonly used for concrete, a may be taken .67 and b .77. For well-compacted, plastic concrete of ordinary materials, c is about 1.10. With these values of the coefficients, the formula becomes:

$$Q = .67C + .77S + 1.1(R - V),$$

or

$$Q = C[.67 + .77s + 1.1r(1 - v)].$$

The volume of cement in cubic feet required to make a cubic yard of concrete is:

$$C = \frac{27}{.67 + .77s + 1.1r(1-v)}.$$

The number of barrels of cement $= C/4$.

Cubic yards of sand $= Cs/27$,

cubic yard of stone $= Cr/27$.

Table XVI, p. 147, gives approximate quantities of materials required for 1 cubic yard of plastic concrete, using stone with differing percentages of voids. Average crusher run stone, with chips removed,

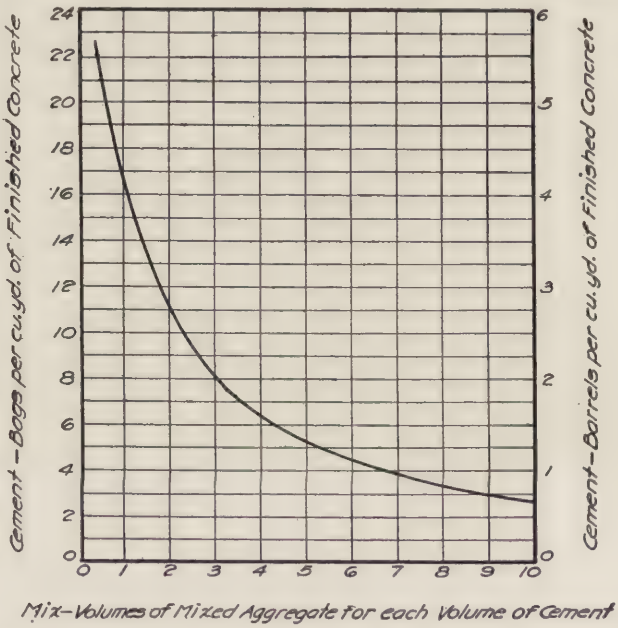


DIAGRAM VIII.—Cement for one cubic yard of finished concrete. Values are net quantities with no allowance for waste.
(Courtesy of the Portland Cement Association.)

has about 40 to 45 per cent voids; good natural gravel, screened, may have 35 to 40 per cent voids; mixed stone and gravel often runs from 30 to 35 per cent voids, while carefully grades materials may have voids reduced to from 20 to 30 per cent.

Variations in the characters of the materials used, and in the methods of handling and placing the concrete may vary considerably the quantities of materials required, Dry concrete, if thoroughly

TABLE XVI. INGREDIENTS REQUIRED FOR ONE CUBIC YARD OF COMPACT, PLASTIC CONCRETE[3]

Proportions by Volumes.			50 Per Cent Voids.			40 Per Cent Voids.			30 Per Cent Voids.			20 Per Cent Voids.		
Cement.	Sand.	Stone.	Cement, Bbl.	Sand, Cu. Yd.	Stone, Cu. Yd.	Cement, Bbl.	Sand, Cu. Yd.	Stone, Cu. Yd.	Cement, Bbl.	Sand, Cu. Yd.	Stone, Cu. Yd.	Cement, Bbl.	Sand, Cu. Yd.	Stone, Cu. Yd.
1	1	2	2.66	0.39	0.78	2.45	0.36	0.72	2.27	0.33	0.66	2.11	0.31	0.62
1	1½	3	1.95	0.43	0.86	1.78	0.39	0.78	1.63	0.36	0.72	1.51	0.33	0.67
1	1½	4	1.68	0.37	1.00	1.52	0.34	0.90	1.38	0.31	0.82	1.26	0.28	0.75
1	2	3	1.75	0.52	0.78	1.61	0.48	0.72	1.49	0.44	0.66	1.39	0.41	0.62
1	2	4	1.53	0.45	0.91	1.40	0.41	0.83	1.28	0.38	0.76	1.18	0.35	0.70
1	2	5	1.36	0.40	1.01	1.23	0.36	0.91	1.12	0.33	0.83	1.02	0.30	0.76
1	2½	4	1.41	0.52	0.84	1.29	0.48	0.76	1.19	0.44	0.70	1.10	0.41	0.65
1	2½	5	1.26	0.47	0.94	1.15	0.42	0.85	1.05	0.39	0.77	0.97	0.36	0.71
1	2½	6	1.15	0.43	1.02	1.04	0.38	0.92	0.94	0.34	0.83	0.86	0.32	0.76
1	3	5	1.18	0.52	0.87	1.08	0.48	0.80	0.99	0.44	0.73	0.91	0.41	0.68
1	3	6	1.07	0.48	0.96	0.97	0.43	0.87	0.89	0.39	0.79	0.82	0.36	0.73
1	3	7	0.99	0.44	1.02	0.89	0.40	0.92	0.81	0.36	0.84	0.74	0.33	0.77
1	3	8	0.91	0.41	1.08	0.82	0.36	0.97	0.74	0.33	0.88	0.67	0.30	0.80
1	4	7	0.89	0.52	0.92	0.81	0.48	0.84	0.74	0.44	0.77	0.68	0.40	0.71
1	4	8	0.83	0.49	0.98	0.75	0.44	0.89	0.68	0.40	0.81	0.63	0.37	0.74
1	4	10	0.73	0.43	1.08	0.65	0.38	0.96	0.59	0.35	0.87	0.54	0.32	0.80

[3] One bag (94 pounds) of Portland cement is taken as a cubic foot, or one barrel of cement as four cubic feet.

compacted by ramming, is more dense and occupies less space than plastic or wet concrete, but as ordinarily placed is more porous and occupies more space. Fine sand swells more when mixed with cement and water, and fills more space in plastic concrete, than coarse sand. Coarse broken stone compacts in concrete so as to leave fewer unfilled voids than smaller stone with the same per cent of voids. Poor work, such as irregular mixing or imperfect compacting, results in more porous concrete and requires less materials.

Tests of the yield of concrete may easily be made by mixing a batch in the proportions to be used and measuring the resulting concrete. In cases where accurate estimates of quantities are important and data concerning the particular materials are not at hand, such tests should be made.

Quantity of Cement.—The quantity of cement required for one cubic yard of concrete depending on the ratio of the volume of the cement in the dry compacted volume of the combined aggregates is shown in Diagram VIII, taken from the booklet "Design and Control of Concrete Mixtures."

ART. 21. MIXING CONCRETE

85. Preparation of Materials.—In making concrete, the materials should be properly prepared and conveniently placed for use, and the labor cost of concrete work is largely a matter of arrangements for handling materials. The work should be systematized so that it goes forward smoothly, without loss of time in any of its parts.

Broken stone can usually be obtained within such range of sizes as may be desired. In preparing crushed stone, the crusher is set to the maximum size allowed, and the product varies from this size to dust. This product is then passed through rotary screens inclined at a small angle to the horizontal, which are made in sections of different sizes of openings, and admit of screening the stone into several sizes at one operation. When large quantities of materials are being used, the cost of handling the stone may not be materially increased by using several sizes and grading the aggregate. In any case where the aggregates available are badly graded, the advantage to be gained by grading them should be carefully considered.

The screened stone usually drops from the screens into bins which are arranged so that the contents may be drawn off through chutes into cars or wagons for transportation to the work. The sizes and arrangement of the bins depend upon the need for storage and the kind of transportation. It is usually desirable that the bins

be of sufficient size to equalize variations in the rate of use, or short delays in the crushing plant, so that work may proceed continuously.

Gravel and sand nearly always need to be screened. When large quantities are to be handled, and power for operating the screen is available, rotary screens are desirable, giving the most economical handling of the material, and admitting of divisions into required sizes.

In small work it is usual to employ hand screens, which are set up in an inclined position, and the material thrown against them with a shovel, the finer material passing through and the coarser sliding down to the foot of the screens. Sometimes two or more inclined screens are placed so that the material which passes one falls upon the one below, each being hinged so that its inclination to the horizontal may be adjusted.

Sand and gravel frequently require washing to remove dirt and fine material, which is often accomplished by supplying water in the chutes leading to the screens, the dirt being washed through a fine screen which retains the aggregate. Sometimes the material is washed down a sloping trough, with a fine screen set in its bottom to permit the dirt to pass through. Portable plants for screening and washing are available in a number of forms, and often provide the most economical means of handling work of this kind. Wetting the material while in a pile, for the purpose of cleaning it, is useless.

Some storage of materials where the work is to be done is usually necessary, in order to have a supply which permits work to proceed continuously. The location of the materials with reference to the mixer, or mixing platform, should be carefully considered, as their convenience to the work affects the cost of mixing the concrete. The amount of storage should be as small as is consistent with assuring a continuous supply to the mixers.

86. Hand Mixing.—Concrete may be mixed by any method which will produce a homogeneous mass of uniform consistency. The arrangement of the work and methods of manipulating the materials in hand mixing vary greatly with the character of the construction and the ideas of the men in charge. The costs vary as widely as the methods.

Measuring the Materials.—Bottomless boxes are sometimes used for measuring the aggregates, the box being placed on the mixing platform, filled and then removed, leaving the material on the platform—an accurate means of measuring, and desirable when it can be employed without materially increasing the cost of handling the aggregates.

Measuring in wheelbarrows is commonly employed, and frequently results in very irregular proportioning, as the barrow may not always be equally filled, unless special attention be given to the loading. When this method is employed, it is desirable to have barrows of such forms that they may be evenly filled to level surface. When ordinary barrows are used, a bottomless box may be placed in the barrow, filled and removed, before starting with the load. It is worth while to use a method that will give accurate measurement, even at a small extra cost for labor.

Cement is measured by counting bags.

The mixing of concrete by hand should be done upon a watertight platform. The cement and sand should first be mixed dry, being turned by shovels, or worked by hoes, until the mixture has uniform color. Water may then be added and the mixture worked into a rather soft mortar, after which the stone is wheeled or shoveled on top of the mortar and the whole turned with shovels until thoroughly mixed. When this method is followed the stone should be wet, to prevent taking the water from the mortar.

Economy in hand mixing depends upon the work being so organized that it goes smoothly in all it parts, every man having his regular duties, and the number of men at each kind of work being such that one set of men does not have to stand idle waiting for others.

87. Machine Mixing.—Machinery is now used for mixing in practically all large concrete construction, and it is rapidly replacing hand mixing in some of the smaller work. Portable plants, which may readily be moved from place to place, are making this economically feasible. There are various mixers on the market, differing more or less in their methods of mixing the materials or in their mechanical appliances for handling materials.

Rotating batch mixers are either cubical boxes mounted to rotate about horizontal axes passing through opposite corners, or cylindrical or conical drums rotating about their geometrical axes. The interior of the mixer is usually provided with blades, causing the materials to be thrown from one part of the mixer to another by the rotation. In using these mixers, the materials in proper proportions to form a batch of concrete are put into the hopper of the machine, and charged into the mixer at one time. The mixer is then run for a sufficient length of time (usually about one minute) to mix the ingredients and the concrete is drawn off through the outlet. The amount of water required to give the specified slump should be determined, and measured for each batch. Figure 50 shows the Ransome 7-S Standard Building Mixer.

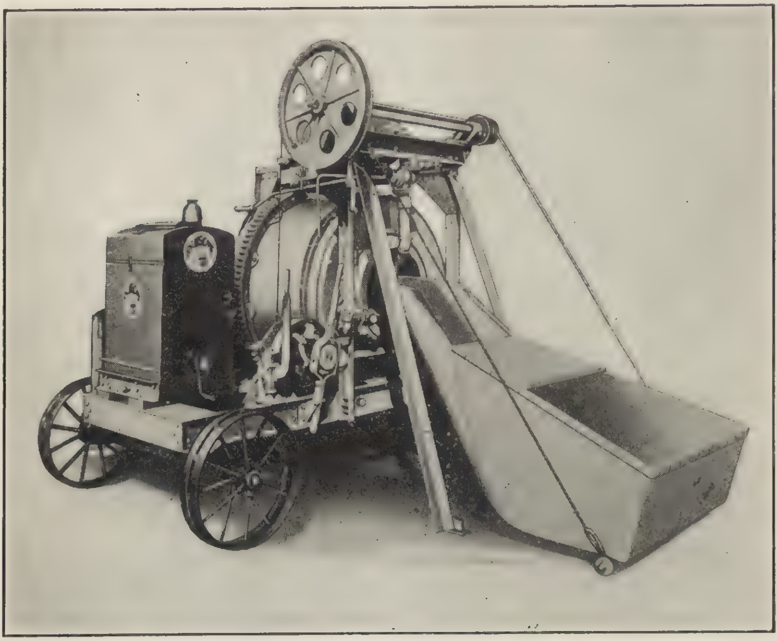


FIG. 50.—Ransome 7-S Standard Building Mixer.

The Joint Committee Report for 1924 recommends that:

The mixing of concrete, unless otherwise authorized by the Engineer, shall be done in a batch mixer of approved type which will insure a uniform distribution of the materials throughout the mass, so that the mixture is uniform in color and homogeneous. The mixer shall be equipped with suitable charging hopper, water storage, and a water-measuring device controlled from a case which can be kept locked and so constructed that water can be discharged only while the mixer is being charged. It shall also be equipped with an attachment for automatically locking the discharge lever until the batch has been mixed the required time after all the materials are in the mixer. The entire contents of the drum shall be discharged before recharging. The mixer shall be cleaned at frequent intervals while in use. The volume of the mixed material per batch shall not exceed the manufacturer's rated capacity of the mixer.

The mixing of each batch shall continue not less than 1 minute after all the materials are in the mixer, during which time the mixer shall rotate at a peripheral speed of about 200 feet per minute.

Other methods of mixing concrete have been used, such as those employing continuous mixers of the screw or paddle type as well as gravity mixers, but these methods are not now used to any great extent.

The cost of machine mixing depends largely upon the appliances

used in conveying the materials to and from the mixer and the method of feeding the mixer. The arrangement of a mixing plant must depend upon the character and amount of work to be done and the topography of the site. When the work is large and concentrated within a small area, a plant of permanent character may be erected with derricks or other mechanical appliances for handling the materials. Sometimes a plant of this kind is made semi-portable by erecting it on a framework resting upon wheels or rollers, which permit it being moved as the work progresses.

It is very common to have the mixer set so that it may discharge into barrows or carts on the ground, the materials being supplied by wheelbarrows from piles on the ground near the mixer. For this purpose, the rotary mixer, with movable hopper which may be let down to the ground for filling, is often used, portable plants mounted on wheels being very commonly of this type. Wheelbarrows with the wheel under the body of the barrow, so that the barrow may easily be dumped over the wheel, are convenient for this kind of work.

In building construction, the mixer is commonly at the surface of the ground and supplied by barrows, the concrete being delivered at required elevations by bucket hoists.

Retempering.—The retempering of concrete or mortar which has partly hardened, that is, remixing with or without additional cement, aggregate, or water is not countenanced by the Joint Committee.

ART. 22. PLACING CONCRETE

88. Transporting Concrete.—The methods of handling concrete from the mixer to its final location vary with the size of the work and the consistency of the concrete.

For small work and short distances, wheelbarrows are commonly used. Ordinary contractors' barrows carry from about 1.8 to 2.0 cubic feet at a load. For longer hauls, two-wheeled barrows, carrying about 6 cubic feet, are more economical. On large work, cars running on temporary tracks are frequently employed, or when the work is within a short radius, derricks may be used. In building operations concrete is frequently raised by a bucket hoist to the required elevation and distributed by barrows to the various parts of the work.

These methods of handling, in which a small bulk of the concrete is held together in transportation and dumped at once into place, offer little opportunity for the ingredients of the concrete to separate.

Well-mixed concrete of any proper consistency may be transported to considerable distances without being injured. Concrete that is so dry as to lack cohesion, and concrete that is so wet that the mortar is soft enough to run away from the stone, both show a tendency to separation in handling, and these consistencies should not be used.

Transportation in Chutes.—The distribution of concrete is sometimes affected by elevating it sufficiently to permit it to flow in a trough or chute to its destination, and by arranging a system of movable chutes it is often possible to distribute over considerable area from a single hoisting tower. When the mixer can be set above

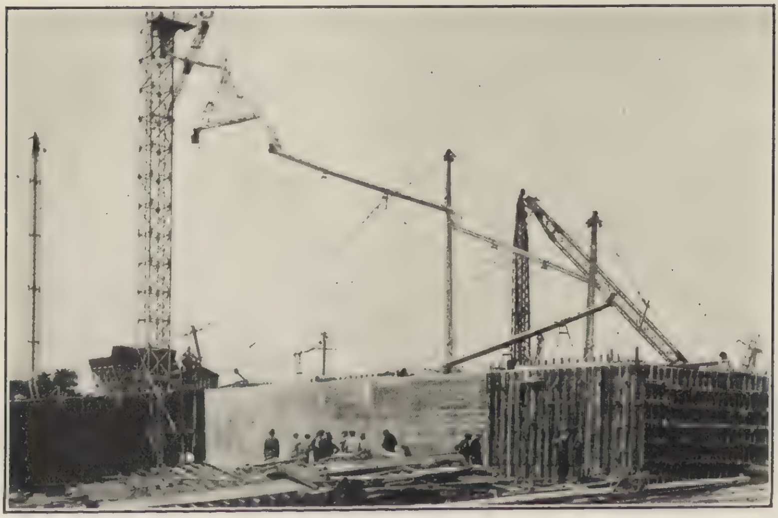


FIG. 51.—Chuting Concrete into Place.
(Courtesy of The Ransome Concrete Machinery Company of Dunellen, N. J.)

the work, as in foundations or sometimes in dams and similar structures, the concrete may be transported wholly by gravity.

To flow in chutes, rather soft, mushy concrete is necessary, unless the chutes are quite steep. When the slope of the chute is very flat, the concrete must be made very wet, and does not result in first-class work, while the extra water necessary to make the concrete flow on a flat slope causes the mortar to separate from the stone, and frequently washes portions of the cement from the mortar. The practice of adding water in the chutes to assist the flow is always detrimental.

Experience indicates that concrete may be made to flow readily in chutes on slopes from about 20° to 35° to the horizontal; for

any slope less than about 20°, the concrete must be made too wet. The mass of concrete should slide along the chute as a whole, the stone and mortar traveling together at common velocity. For ordinary mushy concrete, as commonly used in reinforced work, a slope of two horizontal to one vertical is found most efficient. Figure 51 shows a common arrangement of equipment for chuting concrete into place.

Pneumatic Transportation, by forcing the concrete through pipes by compressed air, has been used in some instances—a method available on congested work, where space is lacking for other means of transport, as in tunnel and subway work.[4]

Whatever method of transportation is used, hardened concrete and foreign materials must be removed from the inner surfaces of the mixing and conveying equipment before work is started, and the space to be occupied by the concrete must be cleared of all debris and properly prepared to receive the concrete. In reinforced concrete work the forms must be thoroughly wet (except in freezing weather) or oiled, and the reinforcement firmly secured in place and approved by the Engineer. The concrete must be handled from the mixer to the place of final deposit as rapidly as practicable without the separation or loss of the ingredients, and must be deposited so as to maintain until the completion of the unit a plastic surface approximately horizontal. Under no circumstances may concrete that has partly hardened be deposited in the work.

89. Depositing Concrete.—When concrete is mixed dry (the consistency of damp earth) and placed in mass construction, it is usually placed in layers about 6 inches deep and each layer tamped until the mortar flushes to the surface. Concrete so mixed and placed attains greater strength than if mixed with more water. If dry concrete is not tamped so as to be thoroughly compacted, it is more porous and has less strength than wet concrete; the labor required in properly placing dry concrete is considerable and the work strenuous, so that for ordinary uses dry concrete is not commonly employed. Poor work has frequently resulted from the use of dry concrete not properly compacted.

In ordinary practice concrete is mixed either to a rather stiff plastic condition or to a softer mushy consistency. Plastic concrete, when in massive work, should be spread in layers not more than 10 or 12 inches deep and lightly rammed; the mortar should readily flush to the surface and the mass quake like jelly under the

[4] See Engineering and Contracting, March 17, 1915, or Engineering News, March 16, 1916.

ramming. The rammer is usually a flat piece of iron about 6 inches square, with a vertical handle, and weighing 15 to 20 pounds. Smaller tapering rammers are also used for compacting next to the forms.

Mushy concrete may be deposited in somewhat thicker layers, being lightly tamped or worked with rammers of small section, usually about 2×3 inches for the purpose of eliminating air bubbles, and making sure that there are no open spaces unfilled with mortar. A flat spade is commonly run down next to the form to bring the mortar to the surface and prevent voids which often occur where the stones of the concrete are in contact with the form.

Forms for walls or other thin sections of considerable height should be provided with openings or other devices that will aid in placing the concrete so as to avoid the accumulation of hardened concrete on the forms or metal reinforcement. Where rodding or forking is impracticable, the concrete must be assisted into place by tapping, or hammering the form opposite the freshly deposited concrete. The concrete must be thoroughly worked around reinforcement, and around embedded fixtures, as well as into the corners of the forms.

Laitance.—In the use of very soft concrete, when an excess of water is used, there is a tendency for certain parts of the cement to be taken up by the surplus water and deposited on the upper surface of the concrete as a sort of light-colored slime, which is known as laitance. Its formation involves a loss of cement in the concrete and, if left in the body of the concrete, it forms a plane of weakness in the mass of concrete. Laitance is often found to an objectionable extent when very wet concrete is chuted to place, and deposited in masses of considerable vertical thickness. A column of wet concrete poured through chutes may have a cap of laitance 2 or 3 inches thick which must be removed.

Bonding to Old Work.—Joints must frequently be made with work previously placed. In massive work, subjected only to compressive stresses normal to the joints, the surface of the old concrete must be clean and should be wet before placing of the new concrete. In work where the strength of the bond of the new to the old work is of special importance, the old work should be cleaned, all laitance removed, the skin on smooth surfaces broken by scarifying, and the surface thoroughly wet. A coating of cement paste will then aid the bond with the new work. Forms should be retightened before depositing new concrete.

Joints between different days' work should be carefully located

where they will be least injurious to the strength of the structure. When feasible it is desirable to divide a structure into integral parts, each of which may be constructed without stopping the work.

Depositing under Water.—Concrete work for under-water construction is sometimes done by passing the mixed concrete through the water to the desired position. It is common to use a tremie for this purpose. This consists of a tube or closed chute, which is kept full of concrete so that the water has no chance to wash the concrete as it passes downward. The bottom of the tremie is moved about over the surface upon which the concrete is being deposited, so that the concrete does not fall through the water. Concrete for this purpose must be quite wet, in order to flow readily to place without being washed by the water through which it is passing.

Buckets which are filled with concrete, lowered through the water, and dumped by opening the bottom when in contact with the surface upon which the concrete is to be placed, are also sometimes used for under-water work. The method of enclosing the concrete in bags and placing these in contact with each other has also been used for this purpose.

The following is from the Joint Committee report for 1924:

Concrete should not be deposited under water if practicable to deposit in air. There is always uncertainty as to results obtained from placing concrete under water. Where conditions permit, the additional expense and delay of avoiding this method will be warranted. It is especially important that the aggregate be free from loam and other material which may cause laitance. Washed aggregates are preferable. Coarse aggregate consisting of washed gravel of a somewhat smaller size than that used in open-air concrete work will give best results. Concrete should never be deposited under water without experienced supervision. Many failures, especially of structures in sea water, can be traced directly to ignorance of proper methods or lack of expert supervision.

47. *General.*—The methods, equipment, and materials to be used shall be submitted to and be approved by the engineer before the work is started. Concrete shall be deposited by a method that will prevent the washing of the cement from the mixture, minimize the formation of laitance, and avoid flow of water until the concrete has fully hardened. Concrete shall be placed so as to minimize segregation of materials. Concrete shall not be placed in water having a temperature below 35°F.

48. *Proportions.*—Concrete to be deposited under water shall contain $1\frac{3}{4}$ bbl. (7 bags) or more of Portland cement per cubic yard of concrete in place.

49. *Coffer-dams.*—Coffer-dams shall be sufficiently tight to prevent flow of water through the space in which the concrete is to be deposited. Pumping will not be permitted while concrete is being deposited, nor until it has fully hardened.

50. *Depositing Continuously.*—Concrete shall be deposited continuously, keeping the top surface as nearly level as possible, until it is brought above the

water, or to the required height. The work shall be carried on with sufficient rapidity, to prevent the formation of layers.

51. *Method.*—The following method* shall be used for depositing concrete under water:

Tremie.—The tremie shall be water-tight and sufficiently large to permit a free flow of concrete. It shall be kept filled at all times during depositing. The concrete shall be discharged and spread by raising the tremie in such a manner as to maintain as nearly as practicable a uniform flow and avoid dropping the concrete through the water. If the charge is lost during depositing, the tremie shall be withdrawn and refilled.

Drop-bottom Bucket.—The bucket shall be of a type that cannot be dumped until it rests on the surface upon which the concrete is tó be deposited. The bottom doors when tripped shall open freely downward and outward. The top of the bucket shall be open. The bucket shall be completely filled, and slowly lowered to avoid back-wash. When discharged, the bucket shall be withdrawn slowly until well above the concrete.

Bags.—Bags of jute or other coarse cloth shall be filled about two-thirds ($\frac{2}{3}$) full of concrete and carefully placed by hand in a header-and-stretcher system so that the whole mass is interlocked.

52. *Laitance.*—Great care shall be exercised to disturb the concrete as little as possible when it is being deposited in order to avoid the formation of laitance. On completing a section of concrete, the laitance shall be entirely removed before work is resumed.

The tremie may be filled by one of the following methods: (1) Place the lower end in a box partly filled with concrete, so as to seal the bottom, then lower into position; (2) plug the tremie with cloth sacks or other material, which shall be forced down as the pipe is filled with concrete; (3) plug the end of the tremie with cloth sacks filled with concrete.

90. Placing Concrete in Freezing Weather.—The setting and hardening of concrete are greatly retarded at low temperatures; in cold weather much longer time is needed to gain strength, and forms must be left longer before removal. Accidents have sometimes occurred to concrete structures through premature removal of forms, on account of failure to consider the influence of temperature upon the rate of hardening. At 40° F., the time required to gain a given strength is two or three times as long as at 70° F.; below 40° F., the required time rapidly increases as the temperature is lowered.

Cement mortar or concrete made and frozen before it has time to set is uninjured by freezing and sets and hardens properly after it thaws out. If the mortar is frozen when partially set or soon after it has set and before any considerable strength has been gained, the expansion caused by freezing breaks the bond and destroys the cohesion of the mass, causing it to crumble upon thawing out.

The use of concrete in freezing weather, except in large masses,

* The engineer must strike out the method or methods not applicable to the work.

should be avoided in so far as possible. When it is necessary to place concrete at freezing temperature, or when it is likely to be frozen soon after placing, extreme care should be taken to minimize the probable effect of freezing upon the concrete. The methods employed may be intended to hasten the setting and hardening of the cement, to prevent the concrete from freezing soon after placing, or both, and for this purpose, materials may be selected that will act quickly when made into mortar. Quick-setting cements, however, are sometimes more retarded by low temperatures than others setting more slowly at normal temperatures. In selecting materials, it is more important to get those acquiring strength quickly than those setting quickly.

Heating the Materials, and mixing and placing them warm, has the effect of hastening the hardening processes, and also prevents immediate freezing. If the temperature is but little (3° or 4°) below the freezing-point, heating the materials and placing the concrete warm may be sufficient to prevent injury from freezing. Protection should also be given the concrete to delay freezing as long as possible. The materials should not be at a temperature much above 100° or 110° F. at the time of mixing. The use of hot water is injurious to the cement, and may defeat the object of heating by preventing the cement setting properly.

Having placed the concrete while warm, if the temperature is likely to be more than 3° or 4° below the freezing-point, it is necessary to have some means of keeping the work from freezing on the surface, which may sometimes be done by enclosing the work in some way and using small stoves, or steam pipes may be available for heating small enclosed spaces. In placing work in large masses, the heat of chemical action will prevent freezing in the body of the work, but exposed surfaces must be protected.

Use of Salt.—When temperaure is but little below the freezing-point, the freezing of concrete may be prevented by dissolving salt in the water used for mixing. A small addition of salt (3 to 5 per cent of the weight of the water) lowers the freezing-point of the concrete, and prevents injury from freezing at temperatures perhaps 5° or 6° below freezing. The salt also has the effect of somewhat increasing the rapidity of hardening, which is very slow at such temperatures.

Salt is sometimes used in larger proportion, 10 to 15 per cent of the weight of water, to prevent freezing at lower temperatures. This seems to retard hardening, and is considered by some engineers to be harmful to the concrete.

91. Forms for Concrete Construction.—Proper form work for concrete construction is of great importance. Wooden forms if made of well selected material, properly designed and handled, may be used several times or adapted to use on other parts of the work. Standard portable metal forms are used to some extent.

In regard to forms, the Joint Committee report for 1924 includes the following:

53. *General.*—Forms shall conform to the shape, lines, and dimensions of the concrete as called for on the plans. Lumber used in forms for exposed surfaces shall be dressed to a uniform thickness, and shall be free from loose knots or other defects. Joints in forms shall be horizontal or vertical. For unexposed surfaces and rough work undressed lumber may be used. Lumber once used in forms shall have nails withdrawn and surfaces to be in contact with concrete thoroughly cleaned before being used again.

54. *Design.*—Forms shall be substantial and sufficiently tight to prevent leakage of mortar; they shall be properly braced or tied together so as to maintain position and shape. If adequate foundation for shores cannot be secured, trussed supports shall be provided.

55. *Workmanship.*—Bolts and rods shall preferably be used for internal ties; they shall be so arranged that when the forms are removed no metal shall be within 1 inch of any surface. Wire ties will be permitted only on light and unimportant work; they shall not be used through surfaces where discoloration would be objectionable. Shores supporting successive stories shall be placed directly over those below, or so designed that the load will be transmitted directly to them. Forms shall be set to line and grade and so constructed and fastened as to produce true lines. Special care shall be used to prevent bulging.

56. *Moldings.*—Unless otherwise specified suitable moldings or bevels shall be placed in the angles of forms to round or bevel the edges of the concrete.

57. *Oiling.*—The inside of forms shall be coated with non-staining mineral oil or other approved material, or thoroughly wetted (except in freezing weather). Where oil is used, it shall be applied before the reinforcement is placed.

58. *Inspection.*—Temporary openings shall be provided at the base of column and wall forms and at other points where necessary to facilitate cleaning and inspection immediately before depositing concrete.

59. *Removal.*—Forms shall not be disturbed until the concrete has adequately hardened.* Shoring shall not be removed until the member has acquired sufficient strength to support safely its weight and the load on it. Member. subject to additional loads during construction shall be adequately shored to support both the member and construction loads in such a manner as will protect the member from damage by the loads; this shoring shall not be removed until the member has acquired sufficient strength to support safely its weight and the load on it.

92. Contraction Joints.—Cement mortar and concrete expand and contract with changes of temperature in the same manner as

* Many conditions affect the hardening of concrete, and the proper time for the removal of the forms should be determined by the engineer.

other materials. They also change in dimension with changes in moisture, expanding when wet and contracting when dry.

Temperature Changes.—The coefficient of expansion of concrete has been found by various investigators to vary from about .0000050 to .0000065 per degree F., the average result being about .0000055 per degree F., or .0000099 per degree C. If the coefficient of elasticity of the concrete is 2,000,000, this would be sufficient to produce a unit stress in the concrete of 440 lb./in.^{2}, for a change of temperature of 40° F. if the concrete be restrained from yielding.

Concrete in large masses frequently reaches a high temperature during the period of early hardening, due to the heat produced by the chemical changes taking place, temperatures of from 95° F. to 150° F. having been observed.[5] In thin walls this is largely counteracted by the radiation into the atmosphere. The influence of changes of atmospheric temperatures rapidly decreases with the distance from the surface of the concrete. Daily variations of temperature are not felt at depths of more than 2 or 3 feet, while seasonal variations may not reach more than one-third the amount of the change in the outside air at a depth of 10 feet.

Moisture Changes.—Variations in moisture conditions are of greater importance than those of temperature in causing mortar or concrete to expand and contract. These changes are of special importance during the time that the concrete is hardening. Experiments indicate that concrete kept in dry air during the period of hardening undergoes a progressive shrinkage, while that kept in water expands during the same period, but to a less extent. The results obtained by different investigators have varied considerably in the extent of the changes shown. In general, concrete exposed to dry air may be expected to contract .04 to .06 per cent of its length in six months after mixing, while if kept under water it may expand .01 to .02 per cent. The changes for cement mortar are greater than for concrete, the extent of the change being greater as the mortar is richer.

Tests indicate that concrete at any age expands if changed from dry to wet condition, and contracts if changed from wet to dry. Concrete subject to changes in moisture conditions, therefore, alternately expands and contracts with such changes, unless restrained by its position from such motion. We have no means of estimating the amount of the variations to be expected in concrete work, but

[5] Temperature Changes in Mass Concrete, by Paul and Mayhew, Transactions, American Society of Civil Engineers, Vol. LXXIX, 1915, p. 1225.

under ordinary conditions these effects must be much less than the progressive variations during hardening.

Available data are not sufficient to determine to what extent the progressive expansions or contractions taking place during hardening may be permanent.[6] Indications are that concrete which has been kept wet during the first month or more and then permitted to dry for several months, does not shrink to the same extent as that which is kept dry during the whole period. Concrete kept damp during the early period of hardening should not crack when exposed to the air to the same extent as that continuously dry.

It seems probable that under some conditions progressive changes in dimension may take place over a long period, though it must not be inferred that work in which the concrete is restrained from such changes is subjected to the stresses which would be imposed by the necessity of resisting them all at once. Concrete restrained, as in reinforced work, from yielding to the tendency to contract probably becomes adjusted to the situation so that it would not contract if the restraint were removed.

Contraction joints are commonly used to prevent the cracking of concrete by shrinkage. The compressive strength of concrete is usually sufficient to take up the stresses due to expansion without injury to the structure, but tensions due to contraction may be sufficient to crack the concrete.

Thin concrete walls usually need contraction joints 20 to 30 feet apart; in heavy walls, they may be 50 or 60 feet apart. The use of light reinforcement in the exposed surfaces between expansion joints may prevent disfiguring surface cracks.

Ordinarily these joints may be formed by building the work in sections and allowing one section to set before the adjoining one is placed. This introduces planes of weakness through the work which will yield when the wall contracts. To bond the ends together, grooves may be left in the sections first constructed and filled in placing the new work. Joints are sometimes made by inserting strips of roofing paper and placing the new concrete against these, or where water-tight work is necessary by filling a thin opening in the concrete with asphalt cement.

93. Finishing Concrete Surfaces.—The appearance of a con-

[6] See, "Expansion and Contraction of Concrete While Hardening," by A. T. Goldbeck, Proceedings, American Society for Testing Materials, Vol. XI, p. 563, also, "Volume Changes in Portland Cement Mortar and Concrete," by A. H. White, Proceedings, American Society for Testing Materials, Vol. XIV, p. 203.

crete structure depends largely upon the way in which the surfaces are finished. In general, the whole showing face between prescribed construction joints should be cast in one continuous operation, and the same brand of cement and the same kind and size of aggregate should be used throughout the whole of any showing face. The forms for such faces should be smooth and watertight, and if wood is used, the boards should be planed, tongued, and grooved, evenly matched, and tightly placed. They should be made removable by unscrewing or loosening without the necessity of hammering or prying against the face of the concrete.

When the concrete next the form has been carefully spaded in placing the concrete, the surface should be fairly smooth with no vacant spaces which require filling, and if the forms are smooth, a quite even, uniform appearance may be obtained. For certain classes of structures, such as retaining walls and bridge abutments in certain locations the appearance may be satisfactory without further treatment, although the dead color of the smooth surface skin is not particularly pleasing.

A smooth surface is sometimes obtained by plastering with cement mortar—a method not usually satisfactory, as the mortar is apt to scale off. A rough appearance is usually more suitable to the material, and the surface of the concrete itself should be used. When a smooth mortar surface is desired, it should be obtained by placing the mortar at the same time as the concrete, which may be done by using a movable form for the mortar. The form slides inside the main form and separates the mortar from the concrete, and is removed as the materials are placed so that they may be tamped together.

A pleasing appearance may often be made by scrubbing the surface with a stiff brush and water as soon as it has set sufficiently to remove the forms, which removes the marks of the forms and brings the pieces of larger aggregate into view. Scrubbing must be done before the concrete has hardened too much, usually within twenty-four hours of placing the concrete, and immediately after removing the forms, as the surface hardens rapidly after the forms are taken off. In removing forms for this purpose, care must be used to prevent breaking the corners of the concrete, as, to present a good appearance, the edges must be straight and sharp. Scrubbing involves comparatively little labor and is an inexpensive method of finishing.

After the concrete surface is hard it may be scrubbed, and the skin removed, by the use of a solution of about one part hydro-

chloric acid to five parts water, though this method is quite laborious and rather expensive.

Concrete surfaces are sometimes finished by tooling, using the axe, bush-hammer, or point. The concrete may thus be made to show a very uniformly roughened surface which is very pleasing. If neatly done, this is rather slow and expensive, although a roughly pointed effect may be produced with less work.

The appearance of the finished surface may be controlled by the choice of aggregates. If a uniform appearance is desired, small aggregates may be used on the surface. If a more rough appearance is wanted, larger and less uniform material may be employed. Pleasing color effects may often be had by care in the choice of aggregates to be used on the surface, or mortar colors may be used for the purpose. White Portland cement may also be used where special effects are desired.

A sand-blast finish may be obtained by allowing the concrete face to attain an intermediate degree of hardness and then air-blasting with hard sand until the aggregate is in uniform relief.

Breaking the continuity of a surface by introducing panels may frequently improve its appearance. These are made by nailing boards of proper shape to the inside of the forms. The surface may be broken by lines indented into the concrete by nailing strips of triangular section to the inside of the forms.

94. Floor Surfaces.—The use of monolithic floor surface finishing is gaining in favor. Specifications generally require that the finish shall be laid integral with the slab, and that it shall be at least one-quarter inch in thickness in addition to the thickness of the slab shown on the plans. The finish is usually composed of 1 part of Portland cement to $1\frac{1}{2}$ parts finely sifted limestone or granite, thoroughly steel troweled in position to insure even surface and perfect bond. After the finish is laid, the entire surface must be covered with at least two inches of wet sawdust or sand, and kept wet and protected until the surface is hard enough to withstand the usual abuse.

ART. 23. WATERTIGHT CONCRETE

95. Permeability of Concrete.—The permeability of a wall of concrete varies with the size and shape of the aggregates, the density of the mixture, and the richness of the mortar. For given aggregates, the densest and strongest mixture will usually be the least permeable, although the least porous concrete is not necessarily the least permeable when different materials are used.

Mortar composed of fine sand is more porous and less permeable than mortar of coarse sand mixed in the same proportions. Graded sand, with sufficient fine materials, shows less porosity and less permeability than either fine or coarse sand alone, but with any sand, the permeability of mortar decreases as the ratio of cement to sand increases.

The permeability of mortar decreases with age during the period of hardening, and mortar subject to the continuous filtration of water decreases in permeability. Messrs. Fuller and Thompson[7] found that the permeability of concrete decreased as the maximum size of coarse aggregate increased, and that gravel concrete was less permeable than that made with broken stone.

The use of sand cement (see Section 18) in place of Portland cement ordinarily gives a somewhat more impervious concrete, and is frequently used for the purpose. The extremely fine grinding to which the cement is subjected in preparing the sand cement is favorable to making a watertight mortar.

Waterproof Concrete.—With carefully prepared and proportioned materials and good workmanship, concrete may be made practically watertight. To secure this result, rich mortar (at least 1 to 2) should be used, and the aggregates graded to produce a dense mixture. The concrete should be thoroughly mixed to a plastic or soft, but not too wet, consistency, and placed carefully, eliminating joints if possible. When horizontal joints are unavoidable, the skin on the old surface should be broken and roughened before placing the cement paste to receive the new work. A thickness of 1 foot of well-constructed concrete wall may be expected to be practically watertight, under a head of 50 feet. No wall to hold water pressure should be less than 6 inches thick.

When a lean concrete is used for the body of the work, a thin surface of rich concrete, or of cement mortar, may be placed upon the water face; this face must be built up with the main body of the work and firmly united with it, and contraction joints must be used where cracks are likely to occur.

Tests for Permeability.—The permeability of concrete is tested by forcing water through a block of concrete under pressure, the block being so arranged that the water can escape only by passing through the concrete. In an apparatus used by Mr. Thompson[8] for this purpose the sides of the mold were made by two pieces of wrought iron bent to a half-circle and bolted together, these rest-

[7] Transactions, American Society of Civil Engineers, Vol. LIX, 1907, p. 67.

[8] Proceedings, American Society for Testing Materials, Vol. VIII, p. 506.

ing on a plank which formed the bottom of the mold until the concrete had set. The surfaces of the concrete were chipped to remove the skin, the blocks turned upside down in making the tests, and the water which passed through the block measured.

96. Integral Waterproofing.—For the purpose of increasing the watertightness of concrete, additions of other materials are sometimes made. There are a number of proprietary compounds on the market to be mixed with the concrete to make it impervious, known as integral waterproofing compounds. Some of these may be of value provided they are not used in lieu of proper proportions or good work in placing the concrete, but dependence upon making meager or improperly mixed concrete watertight by additions of waterproofing compounds is apt to end in failure.

The Joint Committee report for 1924 specifies that, "Integral compounds shall not be used for waterproofing unless specifically authorized by the Engineer."

Hydrated Lime.—The addition of hydrated lime to the cement mortar used in concrete may be useful in assisting in making the concrete watertight. As already noted (Section 37), mortar containing lime works easier, and is preferred by masons for brickwork (Section 62). A small addition of hydrated lime makes concrete flow more readily in placing and tends to prevent separation of the materials in handling, and it is sometimes used for this reason in concrete which is to be chuted to place

Hydrated lime may be used in cement mortar to replace a small per cent by weight of the cement without injury to the strength of the mortar. It is a very finely divided material, much more bulky than the cement, and therefore renders the mortar less permeable. Where the strength is sufficient, a somewhat leaner concrete may be used if hydrated lime is added. Experiments by Mr. Thompson [9] show that hydrated lime may be of considerable value in rendering concrete impervious under considerable heads. Mr. Thompson recommends the addition of hydrated lime in the following percentages of weight of dry hydrated lime to the weight of Portland cement:

For 1 part Portland cement; 2 parts sand; 4 parts stone, add 8 per cent hydrated lime.

For 1 part Portland cement; $2\frac{1}{2}$ parts sand; $4\frac{1}{2}$ parts stone, add 12 per cent hydrated lime.

For 1 part Portland cement; 3 parts sand; 5 parts stone, add 16 per cent hydrated lime.

[9] Proceedings, American Society for Testing Materials, Vol. VIII, p. 500.

Clay.—Finely divided clay in the sand used in making concrete has been found to lessen the permeability of the concrete. The clay must be free from vegetable matter and present in small proportion, not more than 5 per cent of the weight of sand. Finely pulverized rock has much the same effect. These materials are of use when the mortar is lean, although for rich mortars they may be detrimental to strength without materially affecting the permeability.

Alum and soap solution has sometimes been used to mix with the body of concrete and seems to have been fairly efficient as a waterproofing medium. In applying, it is usually best to mix powdered alum with the cement and dissolve the hard soap in the water to be used in mixing. The soap may be about 3 per cent of the weight of water, and the alum about one-half the weight of the soap.

97. Waterproof Coatings.—Various methods have been proposed and are sometimes used for the treatment of concrete surfaces to make them waterproof. For ordinary work, as already stated, the concrete may itself be made watertight and nothing is needed beyond care in the selection of materials and in preparing and placing the concrete. Under some circumstances, however, as in old work or where joints and cracks cannot be avoided, it may be necessary to provide some means of protecting the surface of concrete against penetration of water.

Layers of Waterproof Materials.—Probably the most effective method of protection is that of applying layers of waterproof paper or felt coated with asphalt or coal-tar pitch. The concrete is first coated with hot asphalt, layers of paper or felt are then placed, and each coated with the hot asphalt, the applications being made from 3-ply to 6-ply, depending upon the degree of protection needed—a method frequently employed on subways, and bridge floors with good results. Objection has been made to this method on account of it preventing the radiation of heat in subway work. Careful workmanship is necessary in placing such a protection; the layers must break joints properly, and be protected against being punctured after being placed.

Cement Grout.—Washing the surface of concrete with a grout of neat Portland cement may sometimes be of use on a surface exposed to water, serving to fill voids and cracks which may exist in the surface.

Plastering with cement mortar, or other materials mixed with cement, is sometimes adopted as a means of waterproofing. Usually such plastering needs protection against possible weather cracks,

and sometimes the plaster does not adhere. On horizontal or inclined surfaces a troweled mortar finish, similar to that commonly used on sidewalks, makes a watertight job, provided care be taken to guard against cracks, and to insure the bonding of the mortar to the concrete.

Alum and Soap.—A solution of soap and alum is often used to wash the surface of concrete; it is of the same character as the mixture employed in integral waterproofing, and is also sometimes used for mixing with cement mortar for plastering the surface.

Bituminous coatings consisting of one or more coatings of asphalt or tar painted on hot are sometimes employed, such applications being often used on the outside of the cellar walls of buildings. A number of proprietary compounds are available for use as surface washes, some of which seem quite effective, though in many cases they require renewal from time to time. Some interesting tests of a number of methods of waterproofing concrete surfaces were made by Mr. F. M. McCullough, and the results given in Bulletin No. 336 of the University of Wisconsin, on "Tests of the Permeability of Concrete."

ART. 24. DURABILITY OF CONCRETE

98. Destructive Agencies.—Well-constructed concrete, under the conditions usually met in ordinary work, is practically an indestructible material, but under special conditions, when subject to the action of agencies peculiar to particular classes of work, concrete may yield like any other material. The cracking of concrete through contraction, as explained in Section 92, may be of injury to a structure, but the body of concrete itself is not destroyed or disintegrated by cracking.

Concrete has sometimes seemed to be injured by the action of certain chemical agencies, such as oils, salts of sea water, alkalies, and acids. Destruction by electrolysis and by fire have also sometimes occured.

Effect of Oils.—Mineral oils have no ill effects upon concrete and have sometimes been used for the purpose of rendering the surface less pervious, and to prevent dust upon surfaces subject to abrasion. Some animal fats and vegetable oils seem to cause disintegration in concrete. When such oils at high temperatures come into contact with concrete, a combination of lime from the cement with acids contained in the oils produces compounds which expand when crystallizing in the pores. In manufacturing plants, where

animal or vegetable oil may come into contact with concrete, the effect of such contact should be carefully investigated.

Effect of Acids.—Water containing acids should not come into contact with concrete before it has well hardened. Hard concrete may resist the action of such solutions unless they are in rather concentrated form. The effect of any destructive agency of this character will be much greater for porous concrete into which the liquid may readily penetrate than for dense concrete.

Electrolysis.—In some instances, injury to concrete containing steel reinforcement has resulted from the leakage of electric currents through the mass. The Joint Committee on Concrete in its 1917 report makes the following statements concerning electrolysis:

Electrolysis.—The experimental data available on this subject seem to show that while reinforced concrete structures may, under certain conditions, be injured by the flow of electric current in either direction between the reinforcing material and the concrete, such injury is generally to be expected only where voltages are considerably higher than those which usually occur in concrete structures in practice. If the iron be positive, trouble may manifest itself by corrosion of the iron accompanied by cracking of the concrete, and, if the iron be negative, there may be a softening of the concrete near the surface of the iron, resulting in a destruction of the bond. The former, or anode effect, decreases much more rapidly than the voltage, and almost if not quite disappears at voltages that are most likely to be encountered in practice. The cathode effect, on the other hand, takes place even under very low voltages, and is therefore more important from a practical standpoint than that of the anode.

Structures containing salt or calcium chloride, even in very small quantities, are very much more susceptible to the effects of electric currents than normal concrete, the anode effect progressing much more rapidly in the presence of chlorine, and the cathode effect being greatly increased by the presence of an alkali metal.

There is great weight of evidence to show that normal reinforced concrete structures free from salt are in very little danger under most practical conditions, while non-reinforced concrete structures are practically immune from electrolysis troubles.

The subject of electrolysis is not touched on in the Joint Committee report for 1924.

99. Effect of Sea Water upon Concrete.—There have been numerous instances of failure of concrete subject to the action of sea water, the causes of which are not fully determined. The results of experiments seem to indicate that salts contained in sea water act upon nearly all cements to which the water has free access, producing compounds which expand, disrupting the mass of mortar, or which soften the mortar and cause disintegration. This action is probably due to sulphates in the sea water, which are decomposed in contact with the free lime of the cement, the sulphuric acid com-

bining with the lime. A considerable deposit of magnesia also commonly occurs in cement mortar when exposed to sea water,[10] indicating that the sulphate of magnesia may be the source of the trouble.

Those cements which contain the most lime are usually most affected by the action of sea water. Cements containing considerable alumina should not be used for work in sea water, siliceous cements, or those in which alumina is replaced by iron oxide being preferable. In France a siliceous hydraulic lime known as lime of teil is extensively used for such work.

The addition of finely ground puzzuolanic materials to Portland cement has been found useful in preventing the disintegrating effects of sea water. These materials probably combine with and reduce the amounts of free lime available for combination with the sea salts. As used, they also render the mortar less permeable.

Mortars of fine sand are found to be more affected by sea water than those of coarse or graded sands.

The injurious action of sea water is dependent upon the water having access to the body of the concrete, hence it is important in such work to use concrete of maximum density, or to protect the body of the concrete by a surface of dense mortar or concrete. The Joint Committee in its 1924 report makes the following reference to work in sea water:

81. *Proportions.*—Plain concrete in sea water from 2 feet below low water to 2 feet above high water, or from a plane below to a plane above wave action, shall contain a minimum of $1\frac{3}{4}$ bbl. (7 bags) of Portland cement per cubic yard in place. Other plain concrete in sea water or exposed directly along the sea coast shall contain a minimum of $1\frac{1}{2}$ bbl. (6 bags) of Portland cement per cubic yard in place. Porous or weak aggregates shall not be used.

82. *Consistency.*—The consistency shall meet the requirements of Joint Committee Report Section 30. (See p. 137.)

83. *Depositing.*—Sea water shall not be allowed to come in contact with the concrete until it has hardened for at least four (4) days. Concrete shall be placed in such a manner as to minimize the number of horizontal or inclined seams or work planes. The placing of concrete between tides shall be a continuous operaation. . . . Concrete shall be deposited in sea water only when so directed by the engineer, in which case it shall be placed in accordance with the methods described in Joint Committee Report Sections 47 to 52. (See p. 156 et seq.)

84. *Protection.*—Metal reinforcement shall be placed at least 3 inches from any plane or curved surface except at corners when it shall be at least 4 in. from adjacent surfaces. Metal chairs, supports, or ties shall not extend to the surface of the concrete. Where unusually severe conditions of abrasion are anticipated,

[10] Alexandre, Annales des Ponts et Chaussées, 1890, Vol. I, p. 408. Candlot, Ciment et Chaux Hydraulique, Paris, 1891. Feret, Annales des Ponts et Chaussées, 1891, Vol. II, p. 93.

the face of the concrete from 2 ft. below low water to 2 ft. above high water, or from a plane below to a plane above wave action, shall be protected by creosoted timber, dense vitrified shale brick, or stone of suitable quality, as designated on the plans or as required by the engineer.

100. Effect of Alkalies.—In some localities in the arid regions of the Western States difficulty has been met in the use of concrete because of the disintegrating effects of alkaline waters—effects similar to those of sea water and probably due to the same causes. The most serious disintegration is found where the concrete is alternately wet and dry, although in some cases the whole of the concrete below water has been affected.

On many irrigation projects large quantities of concrete are being used, and the problem of dealing with the alkaline salts, with which the soil is impregnated in some localities, has become a serious one. These alkaline deposits vary in character in different places, comprising salts of potassium, sodium, calcium, and magnesium. The ill effects seem to occur where sulphates are present in considerable quantities,[11] which agrees with the results of studies of the action of sea water. The same precautions may be taken in selection of materials as for work in sea water, but all cements seem to be affected to some extent by contact with these salts. The use of dense concrete, or the application of protective coatings to prevent access of the alkaline water to the interior of the mass of concrete, offers the best means of preventing disintegration.

The Joint Committee in its 1924 report recommends that concrete in alkali waters or below the ground line of alkali soils shall contain a minimum of $1\frac{3}{4}$ barrels (7 bags) of Portland cement per cubic yard in place. The Committee recommends that the consistency of such concrete shall meet the requirements of Joint Committee Report Section 30 as cited above, and that the concrete shall be placed in such a manner as to minimize the number of horizontal or inclined seams, or work planes. The Committee emphasizes the fact that special care must be used in placing concrete where it will be exposed to sulphate waters. "An impermeable concrete made with a durable aggregate is necessary. Concrete should be permitted to harden under favorable conditions before it is exposed to injurious alkalies, and wherever practicable such concrete should be made in the form of precast units." It is further suggested, "Where the foundations of important buildings or similar structures are subject to high concentrations of alkalies, under-drainage may be used as an added precaution."

[11] J. Y. Jewett, Proceedings American Society for Testing Materials, Vol. VIII, 1908, p. 484.

101. Resistance to Fire.—Experience indicates that concrete when properly used, is one of the best materials for resisting fire. The surface of concrete immediately exposed to the fire is injured and may become dehydrated, but concrete is a poor conductor of heat, and the penetration of the dehydrating effect is extremely slow. Experiments by Professor Woolson [12] show that when a mass of concrete is subjected to high heat for several hours, the temperature 1 inch below the surface is several hundred degrees below that at the surface. With the temperature of 1500° F. at the surface for two hours, the temperature at 2 inches beneath the surface was from 500° to 700° F., and at 3 inches beneath the surface about 200 to 250° F.

When concrete is used as structural material, where it is liable to be subjected to serious fire risk, a layer of concrete next the exposed surface should be considered as fireproofing and not included in the section necessary for resisting stresses.

ART. 25. STRENGTH OF PLAIN CONCRETE

102. Compressive Strength.—Concrete is used almost entirely in those positions where compressive strength is of the most importance and there are several variables which affect this desired quality. These are the quality and quantity of the cement, the quality and proportions of the fine and coarse aggregates, the quantity of water, the thoroughness of mixing, the degree of compacting, and the conditions under which hardening takes place. All of these have an influence upon the strength of the resulting concrete.

Thorough mixing and careful placing and compacting of the concrete should be obtained in all work. Carelessness always reduces its strength, and is wasteful and unnecessary.

Cement.—The expensive ingredient of concrete is the cement and its economic use consistent with the strength of concret desired must be carefully guarded.

Sizes of Aggregates.—The smaller the particles of the fine aggregate, the more cement will be required to coat the particles and to completely fill the interstices among them. Likewise, the smaller the pieces of the coarse aggregate, the more mortar will be required to fill the interstices among them. The best possible economic grading of the available materials should always be reached.

Sometimes concrete mixtures are spoken of in terms of the proportion of the cement to that of the combined fine and coarse aggregates. For example a 1 : 6 mixture may be intended to imply a

[12] Proceedings, American Society for Testing Materials, Vol. VII, 1907, p. 408.

mixture of one part of cement to two parts of sand to four parts of broken stone. A little reflection will show that this is not correct. The common run of broken stone may have approximately 50 per cent of voids and by careful manipulation about 2 cubic feet of sand may be introduced into a vessel containing 4 cubic feet of broken stone without expanding the total mass. In order to get a 1 : 2 : 4 mixture of such material, a 1 : 4 mixture of the combined aggregates should be called for. Combined mixtures are usually very uncertain as to content and the fine aggregate is likely to predominate. This means a weaker mortar and therefore a weaker concrete.

Consistency.—As already stated as little water as possible, consistent with the use to which the concrete is to be put, should be used. The time of mixing and the speed of the mixer are now commonly specified, and the slump and flow tests promise to bring about the desired results as far as the proper quantity of water is concerned.

Growth in Strength.—It is customary to use the strength at twenty-eight days in fixing the stresses to be allowed on concrete in structures. This strength would usually be attained before maximum loads could be applied. The strength of concrete under normal conditions continues to increase through a considerable period. Tests have shown that average concrete may be expected to reach about twice the twenty-eight-day strength in two or three years if kept from becoming too dry. Specimens kept dry show a considerably smaller increase, and may ultimately gain but little more than the twenty-eight-day strength.

103. Tests for Compressive Strength.—The compressive strength of concrete has commonly been tested on 6- or 12-inch cubes, and much of the available data are based upon such tests. The desirability of eliminating the corners has led to the use of cylindrical specimens, which are found much easier to make and handle satisfactorily. The use of a test piece whose height is greater than its lateral diameter is also found advantageous. Blocks of concrete under crushing loads usually yield through shearing on surfaces making angles of about 60° with the horizontal, and by using blocks whose heights are twice the diameter sufficient freedom is allowed for such action. Tests have seemed to indicate that the strength of blocks varies somewhat with the ratio of height to diameter, cubes showing 25 to 35 per cent more strength per square inch than cylinders whose heights are twice their diameters. Blocks of greater relative height show a further loss of strength, but to much less degree, those having a height five times the diameter giving an average strength about 90 per cent of those with a ratio of 2 to 1.

The Joint Committee has issued "Tentative Methods of Making Compression Tests of Concrete" (Serial Designation: C39-21 T), explaining that the purpose of publishing is to elicit criticism and stating that these methods are subject to revision before formal adoption. The Committee calls attention to the fact that these methods are intended to cover tests made in the laboratory where accurate control of quantities of materials and test conditions is possible, and that machine-mixed concrete will require certain obvious changes.

In the preparations for the tests the usual precautions must be observed. The materials must be brought to a room temperature of from 65 to 70° F. Cement must be stored preferably in covered metal cans and thoroughly mixed in order that the sample may be uniform throughout the tests. It must be passed through a No. 16 sieve and all lumps rejected. Aggregates must be in a room-dry condition, and in general the coarse aggregate should be separated on a No. 4, $\frac{3}{8}$, and $1\frac{1}{2}$ in. sieves and recombined to the average sieve analysis for each batch. Fine aggregates should also be separated into different sizes where unusual gradings are being studied.

Sampling the materials should be carefully done to insure obtaining fair samples for the tests. Cement test samples should be made up of a small quantity from each sack used in the concrete tests. To secure an average sample of the aggregates, the method of quartering should be applied. This consists in taking shovelfuls of materials from different parts of the pile, mixing them together and spreading out on the mixing surface. The resulting layer of material is then divided into quarters, two opposite quarters are shoveled away, the other two quarters are mixed again and the operation repeated until a sample of the size desired is obtained. When materials are not uniform and vary in different parts of the supply, it may be desirable to take separate samples from each part and make comparative tests.

Cement should be tested in accordance with the methods called for in the "Standard Specifications and Tests for Portland Cement" (Serial Designation: C9–21) of the A. S. T. M.

Fine aggregates (passing through a No. 4 sieve) should be subjected to sieve analysis in accordance with the "Standard Method of Test for Sieve Analysis of Aggregates for Concrete" (Serial Designation: C41–21); test for organic impurities (natural sand only), in accordance with the "Standard Method of Test for Organic Impurities in Sands for Concrete" (Serial Designation: C40–22); test for quantity of silt, clay, or dust, in accordance with the "Tentative Method of Decantation Test for Sand and Other Fine Aggregates" (Serial Designation: D136–22T); test for unit weight, in accordance with the "Standard Method of Test for Unit Weight of Aggregate for Concrete" (Serial Designation: C29–21) of the A. S. T. M.; and strength test of 1 : 3 mortar by weight at 7 and 28 days in comparison with standard sand mortar.

Coarse aggregates (retained on a No. 4 sieve) should be subjected to sieve analysis test; test for quantity of silt, clay or dust; and test for unit weight as mentioned above for fine aggregates.

Mixed aggregates as used in concrete tests should be tested for unit weight as above.

Proportioning should be based on the sieve analysis and unit weight of the mixed aggregate, the exact quantities of cement and of each size of aggregate being determined by weight. The quantity of water for each batch should be accurately measured.

The sizes of test pieces should vary with the sizes of the aggregates. The accepted form of test piece for compression is a cylinder with its diameter equal to one-half the length. The standard size is a 6- by 12-in. cylinder where the coarse aggregate does not exceed 2 in. in size. For aggregates larger than 2 in. the Joint Committee calls for the cylinders to be 8 by 16 in., and cylinders 2 by 4 in. may be used for mixtures without coarse aggregate.

Mixing concrete should be done by hand in a shallow galvanized steel pan in batches of such size that a small quantity of concrete remains after molding a single test piece. The cement and fine aggregate should first be mixed dry to an even color and the coarse aggregate added and again mixed dry. Sufficient water should then be added to produce concrete of the required workability, and the mass should be mixed thoroughly until a homogeneous appearance is obtained.

Workability or plasticity of each batch of concrete should be measured immediately after mixing by the slump test made in accordance with the "Tentative Method of Test for Consistency of Portland Cement Concrete for Pavements or for Pavement Base" (Serial Designation: D138–22T) of the A. S. T. M., or in accordance with the flow test, as follows: Place a metal form in the shape of a frustum of a cone, 6 in. in top diameter, 10 in. in bottom diameter, 5 in. deep, on the table of the flow apparatus. (See Proceedings of the A. S. T. M., Vol. XX, Part II, p. 242, for 1920.) The fresh concrete must be placed in the mold in two layers. Each layer must be puddled and the top cleaned off with a trowel. Immediately after molding, the form shall be removed by a steady upward pull, the specimen raised ½ in., and dropped 15 times in about 6 seconds by means of a suitable cam and crank. The spread of the fresh concrete due to this treatment as compared with the original bottom diameter of the cone, expressed as a percentage, is the "flow."

Forms for compression test pieces should preferably be of metal and each form should be provided with a machined metal base-plate. They should be made to fit tight so that mixing water will not escape during molding, and they should be oiled with heavy mineral oil before using.

Molding of test pieces should be done carefully, the fresh concrete being placed in layers from 3 to 4 in. in thickness and each layer puddled with twenty-five strokes with a ⅝-in. round steel bar of a length of 9 in. greater than the length of the mold, pointed at the lower end. After puddling the top layer, the surplus concrete should be cleaned off with a trowel and the mold covered with a machined metal plate or a piece of plate glass at least ¼ in. thick.

Capping the cylinders should take place from two to four hours after molding and should be effected with a thin layer of stiff neat cement paste in order that the cylinders may present a smooth end for loading.

Curing of test pieces should be effected preferably by storing in damp sand, or under damp cloths, or in a moist chamber until time of testing.

Testing.—In making the tests, spherical bearing blocks should be used and care taken to permit the adjustment of the test pieces to uniform bearing, properly centered.

104. Proportions for Given Strength.—The Joint Committee in

Advisory Appendix XVI to its 1924 report, gives a table of proportions* for concrete of given compressive strengths at twenty-eight days in which Portland cement and a wide range of sizes of fine and coarse aggregates should be mixed to obtain concrete of compressive values ranging from 1500 to 3000 lb. per sq. in. The proportions are for four different consistencies, and it is explained that the purpose of the table is two-fold: "To furnish a guide in the selection of mixtures to be used in preliminary investigations of the strength of concrete from given materials," and "To indicate proportions which may be expected to produce concrete of a given strength under average conditions where control tests are not made."

The Committee emphasizes the fact that, "If the proportions to be used in the work are selected from the table without preliminary tests of the materials, and control tests are not made during the progress of the work, the mixtures in bold-faced type shall be used."

The conditions on which the use of the table are based and the table (in four parts) follow:

1.—Concrete shall be plastic;

2.—Aggregates shall be clean and structurally sound;

3.—Aggregates shall be graded between the sizes indicated;

4.—Cement shall conform to the requirements of the "Standard Specifications and Tests for Portland Cement" (Serial Designation: C9–21) of the American Society for Testing Materials.

The plasticity of the concrete shall be determined by the slump test carried out in accordance with the "Tentative Method of Test for Consistency of Portland Cement Concrete for Pavements or for Pavement Base" (Serial Designation: D138–22T) of the American Society for Testing Materials.

Apply the following rules in determining the size assigned to a given aggregate:

1.—Not less than 15 per cent shall be retained between the sieve which is considered the maximum† size and the next smaller sieve.

2.—Not more than 15 per cent of a coarse aggregate shall be finer than the sieve considered as the minimum size.†

3.—Only the sieve sizes given in the table shall be considered in applying Rules (1) and (2).

4.—Sieve analysis shall be made in accordance with the "Tentative Method of Test for Sieve Analysis of Aggregates for Concrete" (Serial Designation: C41–24T) of the American Society for Testing Materials.

Proportions may be interpolated for concrete strengths, aggregate sizes and consistencies not covered by the table or determined by test.

* Based on the 28-day compressive strengths of 6 by 12-in. cylinders, made and stored in accordance with the "Tentative Methods of Making Compression Tests of Concrete" (Serial Designation: C39–21T) of the American Society for Testing Materials.

† For example: a graded sand with 16 per cent retained on the No. 8 sieve would fall in the 0–No. 4 size; if 14 per cent or less were retained, the sand would fall in the 0–No. 8 size. A coarse aggregate having 16 per cent coarser than 2-in. sieve would be considered as 3-in. aggregate.

TABLE XVII.—PROPORTIONS FOR 1500 LB. PER SQ. IN. CONCRETE

Proportions are expressed by volume as follows: Portland cement; fine aggregate; coarse aggregate.

Thus 1 : 2.6 : 4.6 indicates 1 part by volume of Portland cement, 2.6 parts by volume of fine aggregate, and 4.6 parts by volume of coarse aggregate.

Size of Coarse Aggregate.	Slump, in Inches.	Size of Fine Aggregate.				
		0–No. 28.	0–No. 14.	0–No. 8.	0–No. 4.	0–⅜ in.
None..........	½ to ½	1 : 2.8	1 : 3.2	1 : 3.8	1 : 4.4	1 : 5.1
	3 " 4	1 : 2.4	1 : 2.8	1 : 3.3	1 : 3.8	1 : 4.5
	6 " 7	**1 : 1.9**	**1 : 2.2**	**1 : 2.6**	**1 : 3.0**	**1 : 3.6**
	8 " 10	1 : 1.4	1 : 1.6	1 : 1.8	1 : 2.1	1 : 2.5
No. 4 to ¾ in...	½ to 1	1 : 2.6 : 4.6	1 : 2.9 : 4.3	1 : 3.4 : 4.1	1 : 3.9 : 3.6	1 : 4.6 : 3.1
	3 " 4	1 : 2.3 : 4.0	1 : 2.6 : 3.8	1 : 2.9 : 3.6	1 : 3.4 : 3.2	1 : 4.1 : 2.8
	6 " 7	**1 : 1.8 : 3.4**	**1 : 2.0 : 3.2**	**1 : 2.3 : 3.1**	**1 : 2.6 : 2.8**	**1 : 3.1 : 2.5**
	8 " 10	1 : 1.1 : 2.5	1 : 1.3 : 2.4	1 : 1.5 : 2.4	1 : 1.7 : 2.2	1 : 2.1 : 2.0
No. 4 to 1 in...	½ to 1	1 : 2.4 : 5.3	1 : 2.7 : 5.2	1 : 3.1 : 5.0	1 : 3.5 : 4.7	1 : 4.3 : 4.3
	3 " 4	1 : 2.1 : 4.7	1 : 2.4 : 4.5	1 : 2.7 : 4.4	1 : 3.1 : 4.1	1 : 3.7 : 3.7
	6 " 7	**1 : 1.6 : 3.9**	**1 : 1.8 : 3.8**	**1 : 2.1 : 3.7**	**1 : 2.4 : 3.5**	**1 : 2.9 : 3.3**
	8 " 10	1 : 1.1 : 2.9	1 : 1.2 : 2.8	1 : 1.4 : 2.8	1 : 1.6 : 2.7	1 : 1.9 : 2.5
No. 4 to 1½ in...	½ to 1	1 : 2.4 : 6.0	1 : 2.7 : 5.9	1 : 3.1 : 5.8	1 : 3.5 : 5.4	1 : 4.1 : 5.1
	3 " 4	1 : 2.0 : 5.4	1 : 2.3 : 5.3	1 : 2.7 : 5.2	1 : 3.0 : 5.0	1 : 3.5 : 4.6
	6 " 7	**1 : 1.6 : 4.4**	**1 : 1.8 : 4.3**	**1 : 2.0 : 4.3**	**1 : 2.3 : 4.1**	**1 : 2.7 : 3.9**
	8 " 10	1 : 1.0 : 3.3	1 : 1.1 : 3.2	1 : 1.3 : 3.2	1 : 1.5 : 3.1	1 : 1.8 : 2.9
No. 4 to 2 in...	½ to 1	1 : 2.2 : 6.9	1 : 2.4 : 6.8	1 : 2.8 : 6.8	1 : 3.1 : 6.6	1 : 3.7 : 6.4
	3 " 4	1 : 1.8 : 6.2	1 : 2.0 : 6.1	1 : 2.4 : 6.1	1 : 2.7 : 6.0	1 : 3.1 : 5.7
	6 " 7	**1 : 1.4 : 5.1**	**1 : 1.6 : 5.0**	**1 : 1.8 : 5.0**	**1 : 2.0 : 5.0**	**1 : 2.4 : 4.8**
	8 " 10	1 : 0.9 : 3.8	1 : 1.0 : 3.8	1 : 1.1 : 3.8	1 : 1.3 : 3.8	1 : 1.5 : 3.7
⅜ to 1 in.......	½ to 1	1 : 2.8 : 5.2	1 : 3.1 : 5.1	1 : 3.6 : 4.8	1 : 4.2 : 4.6	1 : 4.8 : 4.1
	3 " 4	1 : 2.4 : 4.5	1 : 2.6 : 4.5	1 : 3.1 : 4.3	1 : 3.6 : 4.0	1 : 4.1 : 3.6
	6 " 7	**1 : 1.9 : 3.9**	**1 : 2.1 : 3.7**	**1 : 2.4 : 3.6**	**1 : 2.8 : 3.4**	**1 : 3.2 : 3.1**
	8 " 10	1 : 1.3 : 2.8	1 : 1.4 : 2.8	1 : 1.6 : 2.7	1 : 1.9 : 2.6	1 : 2.2 : 2.4
⅜ to 1½ in......	½ to 1	1 : 2.8 : 5.8	1 : 3.1 : 5.7	1 : 3.5 : 5.5	1 : 4.1 : 5.3	1 : 4.7 : 4.9
	3 " 4	1 : 2.4 : 5.2	1 : 2.7 : 5.1	1 : 3.1 : 5.0	1 : 3.5 : 4.8	1 : 4.1 : 4.4
	6 " 7	**1 : 1.9 : 4.3**	**1 : 2.1 : 4.2**	**1 : 2.4 : 4.2**	**1 : 2.7 : 4.0**	**1 : 3.1 : 3.7**
	8 " 10	1 : 1.2 : 3.2	1 : 1.4 : 3.2	1 : 1.6 : 3.1	1 : 1.8 : 3.0	1 : 2.1 : 2.9
⅜ to 2 in.......	½ to 1	1 : 2.7 : 6.6	1 : 3.0 : 6.6	1 : 3.4 : 6.5	1 : 3.9 : 6.4	1 : 4.4 : 6.0
	3 " 4	1 : 2.3 : 5.9	1 : 2.6 : 5.9	1 : 2.9 : 5.8	1 : 3.3 : 5.6	1 : 3.7 : 5.5
	6 " 7	**1 : 1.8 : 4.9**	**1 : 2.0 : 4.8**	**1 : 2.2 : 4.8**	**1 : 2.6 : 4.8**	**1 : 3.0 : 4.5**
	8 " 10	1 : 1.2 : 3.7	1 : 1.3 : 3.7	1 : 1.5 : 3.7	1 : 1.7 : 3.6	1 : 1.9 : 3.5
¾ to 1½ in......	½ to 1	1 : 3.2 : 5.4	1 : 3.6 : 5.3	1 : 4.1 : 5.1	1 : 4.7 : 4.8	1 : 5.3 : 4.4
	3 " 4	1 : 2.8 : 4.8	1 : 3.2 : 4.8	1 : 3.6 : 4.6	1 : 4.0 : 4.4	1 : 4.6 : 4.0
	6 " 7	**1 : 2.1 : 4.0**	**1 : 2.5 : 4.0**	**1 : 2.8 : 3.9**	**1 : 3.2 : 3.7**	**1 : 3.5 : 3.4**
	8 " 10	1 : 1.5 : 3.0	1 : 1.7 : 3.0	1 : 1.9 : 2.9	1 : 2.2 : 2.8	1 : 2.5 : 2.7
¾ to 2 in.......	½ to 1	1 : 3.2 : 6.2	1 : 3.6 : 6.1	1 : 4.0 : 6.0	1 : 4.6 : 5.8	1 : 5.2 : 5.4
	3 " 4	1 : 2.8 : 5.5	1 : 3.1 : 5.5	1 : 3.5 : 5.4	1 : 3.9 : 5.2	1 : 4.5 : 4.9
	6 " 7	**1 : 2.1 : 4.5**	**1 : 2.4 : 4.6**	**1 : 2.7 : 4.5**	**1 : 3.1 : 4.4**	**1 : 3.5 : 4.1**
	8 " 10	1 : 1.4 : 3.4	1 : 1.6 : 3.4	1 : 1.8 : 3.4	1 : 2.1 : 3.4	1 : 2.4 : 3.3
¾ to 3 in.......	½ to 1	1 : 3.2 : 7.1	1 : 3.6 : 7.1	1 : 4.0 : 7.0	1 : 4.6 : 6.9	1 : 5.2 : 6.6
	3 " 4	1 : 2.7 : 6.3	1 : 3.0 : 6.3	1 : 3.4 : 6.3	1 : 4.0 : 6.2	1 : 4.5 : 5.9
	6 " 7	**1 : 2.1 : 5.1**	**1 : 2.4 : 5.2**	**1 : 2.7 : 5.2**	**1 : 3.1 : 5.1**	**1 : 3.5 : 4.9**
	8 " 10	1 : 1.4 : 3.8	1 : 1.6 : 3.9	1 : 1.8 : 3.9	1 : 2.1 : 3.9	1 : 2.4 : 3.8

Joint Committee table of proportions for concrete of a given strength at 28 days.

TABLE XVIII.—PROPORTIONS FOR 2000 LB. PER SQ. IN. CONCRETE

Proportions are expressed by volume as follows: Portland cement; fine aggregate; coarse aggregate.

Thus, 1 : 2.6 : 4.6 indicates 1 part by volume of Portland cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate.

Size of Coarse Aggregate.	Slump, in Inches.	Size of Fine Aggregate.				
		0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–⅜ in.
None	½ to 1	1 : 2.2	1 : 2.6	1 : 3.0	1 : 3.5	1 : 4.1
	3 " 4	1 : 1.9	1 : 2.2	1 : 2.6	1 : 3.0	1 : 3.5
	6 " 7	**1 : 1.5**	**1 : 1.7**	**1 : 2.0**	**1 : 2.3**	**1 : 2.7**
	8 " 10	**1 : 1.0**	**1 : 1.1**	**1 : 1.3**	1 : 1.6	1 : 1.8
No. 4 to ¾ in.	½ to 1	1 : 2.1 : 3.8	1 : 2.3 : 3.7	1 : 2.6 : 3.5	1 : 3.0 : 3.1	1 : 3.6 : 2.8
	3 " 4	1 : 1.7 : 3.3	1 : 1.9 : 3.2	1 : 2.2 : 3.1	1 : 2.6 : 2.8	1 : 3.0 : 2.4
	6 " 7	**1 : 1.3 : 2.7**	**1 : 1.4 : 2.6**	**1 : 1.7 : 2.5**	**1 : 1.9 : 2.3**	**1 : 2.3 : 2.1**
	8 " 10	1 : 0.8 : 1.9	1 : 0.9 : 1.9	1 : 1.0 : 1.8	1 : 1.2 : 1.7	1 : 1.5 : 1.6
No. 4 to 1 in.	½ to 1	1 : 1.9 : 4.5	1 : 2.2 : 4.3	1 : 2.5 : 4.2	1 : 2.8 : 3.9	1 : 3.4 : 3.6
	3 " 4	1 : 1.6 : 3.9	1 : 1.8 : 3.8	1 : 2.1 : 3.7	1 : 2.4 : 3.5	1 : 2.8 : 3.2
	6 " 7	**1 : 1.2 : 3.1**	**1 : 1.3 : 3.1**	**1 : 1.5 : 3.0**	**1 : 1.8 : 2.9**	**1 : 2.1 : 2.7**
	8 " 10	1 : 0.7 : 2.2	1 : 0.8 : 2.2	1 : 1.0 : 2.3	1 : 1.1 : 2.1	1 : 1.3 : 2.0
No. 4 to 1½ in.	½ to 1	1 : 1.9 : 5.0	1 : 2.1 : 4.9	1 : 2.4 : 4.9	1 : 2.7 : 4.6	1 : 3.2 : 4.4
	3 " 4	1 : 1.6 : 4.4	1 : 1.7 : 4.3	1 : 2.0 : 4.2	1 : 2.4 : 4.0	1 : 2.7 : 3.8
	6 " 7	**1 : 1.1 : 3.5**	**1 : 1.3 : 3.5**	**1 : 1.4 : 3.5**	**1 : 1.7 : 3.4**	**1 : 2.0 : 3.2**
	8 " 10	1 : 0.7 : 2.5	1 : 0.8 : 2.5	1 : 0.9 : 2.5	1 : 1.0 : 2.4	1 : 1.2 : 2.3
No. 4 to 2 in.	½ to 1	1 : 1.7 : 5.8	1 : 1.9 : 5.7	1 : 2.1 : 5.8	1 : 2.4 : 5.6	1 : 2.8 : 5.5
	3 " 4	1 : 1.4 : 5.0	1 : 1.5 : 5.0	1 : 1.8 : 5.0	1 : 2.0 : 4.9	1 : 2.3 : 4.7
	6 " 7	**1 : 1.0 : 4.1**	**1 : 1.1 : 4.1**	**1 : 1.2 : 4.1**	**1 : 1.4 : 4.1**	**1 : 1.7 : 3.9**
	8 " 10	1 : 0.6 : 2.9	1 : 0.7 : 2.9	1 : 0.7 : 3.0	1 : 0.8 : 2.9	1 : 1.0 : 2.9
⅜ to 1 in.	½ to 1	1 : 2.2 : 4.4	1 : 2.5 : 4.2	1 : 2.8 : 4.1	1 : 3.3 : 3.8	1 : 3.8 : 3.4
	3 " 4	1 : 1.9 : 3.8	1 : 2.1 : 3.7	1 : 2.4 : 3.6	1 : 2.8 : 3.4	1 : 3.2 : 3.1
	6 " 7	**1 : 1.4 : 3.1**	**1 : 1.5 : 3.0**	**1 : 1.8 : 3.0**	**1 : 2.1 : 2.8**	**1 : 2.4 : 2.5**
	8 " 10	1 : 0.9 : 2.2	1 : 1.0 : 2.2	1 : 1.1 : 2.2	1 : 1.3 : 2.0	1 : 1.5 : 1.9
⅜ to 1½ in.	½ to 1	1 : 2.2 : 4.9	1 : 2.5 : 4.8	1 : 2.8 : 4.7	1 : 3.2 : 4.6	1 : 3.7 : 4.2
	3 " 4	1 : 1.9 : 4.3	1 : 2.1 : 4.2	1 : 2.4 : 4.1	1 : 2.7 : 4.0	1 : 3.1 : 3.7
	6 " 7	**1 : 1.4 : 3.5**	**1 : 1.5 : 3.4**	**1 : 1.7 : 3.4**	**1 : 2.0 : 3.3**	**1 : 2.3 : 3.1**
	8 " 10	1 : 0.9 : 2.5	1 : 1.0 : 2.5	1 : 1.1 : 2.4	1 : 1.3 : 2.4	1 : 1.5 : 2.3
⅜ to 2 in.	½ to 1	1 : 2.1 : 5.6	1 : 2.3 : 5.5	1 : 2.6 : 5.5	1 : 3.0 : 5.4	1 : 3.5 : 5.1
	3 " 4	1 : 1.7 : 4.8	1 : 2.0 : 4.8	1 : 2.2 : 4.8	1 : 2.5 : 4.7	1 : 2.9 : 4.4
	6 " 7	**1 : 1.3 : 4.0**	**1 : 1.4 : 3.9**	**1 : 1.6 : 3.9**	**1 : 1.8 : 3.9**	**1 : 2.1 : 3.8**
	8 " 10	1 : 0.8 : 2.9	1 : 0.9 : 2.9	1 : 1.0 : 2.9	1 : 1.2 : 2.9	1 : 1.3 : 2.8
¾ to 1½ in.	½ to 1	1 : 2.6 : 4.5	1 : 2.9 : 4.5	1 : 3.3 : 4.4	1 : 3.8 : 4.2	1 : 4.3 : 3.9
	3 " 4	1 : 2.2 : 3.9	1 : 2.5 : 3.9	1 : 2.8 : 3.8	1 : 3.2 : 3.6	1 : 3.6 : 3.3
	6 " 7	**1 : 1.6 : 3.2**	**1 : 1.8 : 3.2**	**1 : 2.1 : 3.1**	**1 : 2.4 : 3.0**	**1 : 2.7 : 2.8**
	8 " 10	1 : 1.0 : 2.3	1 : 1.2 : 2.3	1 : 1.4 : 2.2	1 : 1.6 : 2.2	1 : 1.8 : 2.1
¾ to 2 in.	½ to 1	1 : 2.5 : 5.2	1 : 2.8 : 5.2	1 : 3.2 : 5.1	1 : 3.6 : 5.0	1 : 4.1 : 4.7
	3 " 4	1 : 2.1 : 4.5	1 : 2.4 : 4.5	1 : 2.7 : 4.4	1 : 3.1 : 4.3	1 : 3.5 : 4.0
	6 " 7	**1 : 1.6 : 3.7**	**1 : 1.8 : 3.7**	**1 : 2.0 : 3.7**	**1 : 2.3 : 3.6**	**1 : 2.6 : 3.5**
	8 " 10	1 : 1.0 : 2.6	1 : 1.1 : 2.6	1 : 1.3 : 2.6	1 : 1.5 : 2.7	1 : 1.7 : 2.6
¾ to 3 in.	½ to 1	1 : 2.5 : 6.0	1 : 2.9 : 5.9	1 : 3.2 : 5.9	1 : 3.6 : 5.8	1 : 4.1 : 5.6
	3 " 4	1 : 2.1 : 5.1	1 : 2.4 : 5.2	1 : 2.7 : 5.2	1 : 3.1 : 5.1	1 : 3.5 : 4.9
	6 " 7	**1 : 1.5 : 4.1**	**1 : 1.7 : 4.2**	**1 : 2.0 : 4.2**	**1 : 2.3 : 4.2**	**1 : 2.5 : 4.0**
	8 " 10	1 : 1.0 : 2.9	1 : 1.1 : 3.0	1 : 1.3 : 3.0	1 : 1.5 : 3.0	1 : 1.7 : 3.0

Joint Committee table of proportions for concrete of a given strength at 28 days.

TABLE XIX.—PROPORTIONS FOR 2500 LB. PER SQ. IN. CONCRETE

Proportions are expressed by volume as follows: Portland cement; fine aggregate; coarse aggregate.

Thus, 1 : 2.6 : 4.6 indicates 1 part by volume of Portland cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate.

Size of Coarse Aggregate.	Slump, in Inches.	Size of Fine Aggregate.				
		0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–⅜ in.
None..	½ to 1	1 : 1.8	1 : 2.1	1 : 2.4	1 : 2.9	1 : 3.3
	3 " 4	1 : 1.5	1 : 1.8	1 : 2.1	1 : 2.4	1 : 2.8
	6 " 7	**1 : 1.1**	**1 : 1.3**	**1 : 1.6**	**1 : 1.8**	**1 : 2.1**
	8 " 10	1 : 0.7	1 : 0.8	1 : 0.9	1 : 1.1	1 : 1.3
No. 4 to ¾ in...	½ to 1	1 : 1.6 : 3.2	1 : 1.8 : 3.1	1 : 2.1 : 3.0	1 : 2.4 : 2.7	1 : 2.9 : 2 4
	3 " 4	1 : 1.3 : 2.8	1 : 1.5 : 2.7	1 : 1.7 : 2.6	1 : 2.0 : 2.4	1 : 2.4 : 2.2
	6 " 7	**1 : 1.0 : 2.2**	**1 : 1.1 : 2.2**	**1 : 1.1 : 2.2**	**1 : 1.5 : 2.0**	**1 : 1.8 : 1.8**
	8 " 10	1 : 0.5 : 1.4	1 : 0.6 : 1.4	1 : 0.7 : 1.4	1 : 0.8 : 1.4	1 : 1.0 : 1.3
No. 4 to 1 in...	½ to 1	1 : 1.5 : 3.7	1 : 1.7 : 3.7	1 : 2.0 : 3.5	1 : 2.2 : 3.4	1 : 2.7 : 3.1
	3 " 4	1 : 1.2 : 3.3	1 : 1.4 : 3.2	1 : 1.6 : 3.1	1 : 1.9 : 3.0	1 : 2.2 : 2.7
	6 " 7	**1 : 0.9 : 2.6**	**1 : 1.0 : 2.5**	**1 : 1.1 : 2.5**	**1 : 1.3 : 2.4**	**1 : 1.6 : 2.3**
	8 " 10	1 : 0.5 : 1.7	1 : 0.6 : 1.7	1 : 0.6 : 1.7	1 : 0.7 : 1.6	1 : 0.9 : 1.5
No. 4 to 1½ in..	½ to 1	1 : 1.4 : 4.2	1 : 1.6 : 4.1	1 : 1.9 : 4.1	1 : 2.2 : 4.0	1 : 2.5 : 3.8
	3 " 4	1 : 1.2 : 3.7	1 : 1.3 : 3.6	1 : 1.5 : 3.6	1 : 1.8 : 3.5	1 : 2.1 : 3.3
	6 " 7	**1 : 0.9 : 2.9**	**1 : 0.9 : 2.8**	**1 : 1.1 : 2.8**	**1 : 1.3 : 2.8**	**1 : 1.5 : 2.6**
	8 " 10	1 : 0.5 : 1.9	1 : 0.5 : 1.9	1 : 0.6 : 1.9	1 : 0.7 : 1.8	1 : 0.8 : 1.8
No. 4 to 2 in...	½ to 1	1 : 1.3 : 4.9	1 : 1.4 : 4.8	1 : 1.6 : 4.9	1 : 1.9 : 4.8	1 : 2.2 : 4.7
	3 " 4	1 : 1.1 : 4.3	1 : 1.2 : 4.2	1 : 1.3 : 4.3	1 : 1.6 : 4.2	1 : 1.8 : 4.1
	6 " 7	**1 : 0.7 : 3.3**	**1 : 0.8 : 3.3**	**1 : 0.9 : 3.4**	**1 : 1.1 : 3.3**	**1 : 1.2 : 3.3**
	8 " 10	1 : 0.4 : 2.2	1 : 2.4 : 2.2	1 : 0.5 : 2.2	1 : 0.6 : 2.2	1 : 0.6 : 2.2
⅜ to 1 in.......	½ to 1	1 : 1.8 : 3.7	1 : 2.0 : 3.6	1 : 2.3 : 3.5	1 : 2.6 : 3.3	1 : 3.0 : 2.9
	3 " 4	1 : 1.4 : 3.2	1 : 1.6 : 3.1	1 : 1.9 : 2.9	1 : 2.2 : 2.9	1 : 2.5 : 2.6
	6 " 7	**1 : 1.0 : 2.5**	**1 : 1.2 : 2.5**	**1 : 1.3 : 2.4**	**1 : 1.6 : 2.3**	**1 : 1.8 : 2.2**
	8 " 10	1 : 0.6 : 1.6	1 : 0.7 : 1.6	1 : 0.8 : 1.6	1 : 0.9 : 1.6	1 : 1.0 : 1.5
⅜ to 1½ in......	½ to 1	1 : 1.7 : 4.1	1 : 1.9 : 4.1	1 : 2.2 : 4.0	1 : 2.5 : 3.9	1 : 2.9 : 3.6
	3 " 4	1 : 1.5 : 3.6	1 : 1.6 : 3.6	1 : 1.8 : 3.5	1 : 2.1 : 3.4	1 : 2.3 : 3.2
	6 " 7	**1 : 1.0 : 2.9**	**1 : 1.2 : 2.8**	**1 : 1.3 : 2.8**	**1 : 1.5 : 2.7**	**1 : 1.8 : 2.6**
	8 " 10	1 : 0.6 : 1.9	1 : 0.6 : 1.9	1 : 0.8 : 1.8	1 : 0.9 : 1.8	1 : 1.0 : 1.8
⅜ to 2 in.......	½ to 1	1 : 1.7 : 4.7	1 : 1.8 : 4.7	1 : 2.1 : 4.7	1 : 2.4 : 4.6	1 : 2.7 : 4.4
	3 " 4	1 : 1.4 : 4.1	1 : 1.5 : 4.1	1 : 1.7 : 4.1	1 : 2.0 : 4.0	1 : 2.3 : 3.9
	6 " 7	**1 : 1.0 : 3.2**	**1 : 1.1 : 3.2**	**1 : 1.2 : 3.2**	**1 : 1.4 : 3.2**	**1 : 1.6 : 3.1**
	8 " 10	1 : 0.5 : 2.1	1 : 0.6 : 2.1	1 : 0.7 : 2.2	1 : 0.8 : 2.2	1 : 0.9 : 2.1
¾ to 1½ in......	½ to 1	1 : 2.0 : 3.8	1 : 2.3 : 3.8	1 : 2.6 : 3.7	1 : 3.0 : 3.6	1 : 3.4 : 3.3
	3 " 4	1 : 1.7 : 3.3	1 : 2.0 : 3.3	1 : 2.2 : 3.2	1 : 2.5 : 3.2	1 : 2.9 : 2.9
	6 " 7	**1 : 1.2 : 2.6**	**1 : 1.4 : 2.6**	**1 : 1.6 : 2.6**	**1 : 1.9 : 2.5**	**1 : 2.1 : 2.3**
	8 " 10	1 : 0.7 : 1.7	1 : 0.8 : 1.7	1 : 0.9 : 1.7	1 : 1.1 : 1.7	1 : 1.2 : 1.6
¾ to 2 in.......	½ to 1	1 : 2.0 : 4.4	1 : 2.2 : 4.4	1 : 2.5 : 4.3	1 : 2.9 : 4.3	1 : 3.3 : 4.1
	3 " 4	1 : 1.7 : 3.8	1 : 1.9 : 3 8	1 : 2.1 : 3.8	1 : 2.5 : 3.7	1 : 2.8 : 3.6
	6 " 7	**1 : 1.2 : 3.0**	**1 : 1.4 : 3 0**	**1 : 1.5 : 3.0**	**1 : 1.8 : 3.0**	**1 : 2.0 : 2.8**
	8 " 10	1 : 0.7 : 2.0	1 : 0.8 : 2.0	1 : 0.9 : 2.0	1 : 1.0 : 2.0	1 : 1.2 : 2.0
¾ to 3 in.......	½ to 1	1 : 2.0 : 5.0	1 : 2.2 : 5.0	1 : 2.5 : 5.0	1 : 2.7 : 5.0	1 : 3.2 : 4.7
	3 " 4	1 : 1.7 : 4.3	1 : 1.9 : 4.3	1 : 2.1 : 4.3	1 : 2.4 : 4.3	1 : 2.7 : 4.1
	6 " 7	**1 : 1.2 : 3.3**	**1 : 1.4 : 3.4**	**1 : 1.5 : 3.4**	**1 : 1.8 : 3.4**	**1 : 2.0 : 3.3**
	8 " 10	1 : 0.7 : 2.2	1 : 0.8 : 2.2	1 : 0.9 : 2.2	1 : 1.0 : 2.3	1 : 1.2 : 2.3

Joint Committee table of proportions for concrete of a given strength at 28 days.

TABLE XX.—PROPORTIONS FOR 3000 LB. PER SQ. IN. CONCRETE

Proportions are expressed by volume as follows: Portland cement; fine aggregate; coarse aggregate.

Thus, 1 : 2.6 : 4.6 indicates 1 part by volume of Portland cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate.

Size of Coarse Aggregate.	Slump, in Inches.	Size of Fine Aggregate.				
		0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–⅜ in.
None..........	½ to 1	1 : 1.5	1 : 1.7	1 : 2.0	1 : 2.3	1 : 2.7
	3 " 4	1 : 1.2	1 : 1.4	1 : 1.7	1 : 1.9	1 : 2.3
	6 " 7	**1 : 0.9**	**1 : 1.0**	**1 : 1.2**	**1 : 1.4**	**1 : 1.6**
	8 " 10	1 : 0.5	1 : 0.6	1 : 0.7	1 : 0.8	1 : 0.9
No. 4 to ¾ in...	½ to 1	1 : 1.3 : 2.7	1 : 1.5 : 2.6	1 : 1.7 : 2.5	1 : 1.9 : 2.4	1 : 2.3 : 2.1
	3 " 4	1 : 1.0 : 2.3	1 : 1.2 : 2.2	1 : 1.4 : 2.2	1 : 1.6 : 2.0	1 : 1.9 : 1.8
	6 " 7	**1 : 0.7 : 1.7**	**1 : 0.8 : 1.7**	**1 : 0.9 : 1.7**	**1 : 1.1 : 1.6**	**1 : 1.3 : 1.4**
	8 " 10	1 : 0.3 : 1.0	1 : 0.4 : 1.0	1 : 0.5 : 1.0	1 : 0.5 : 1.0	1 : 0.6 : 0.9
No. 4 to 1 in...	½ to 1	1 : 1.2 : 3.1	1 : 1.3 : 3.1	1 : 1.5 : 3.0	1 : 1.8 : 2.9	1 : 2.1 : 2.7
	3 " 4	1 : 0.9 : 2.7	1 : 1.1 : 2.6	1 : 1.2 : 2.6	1 : 1.4 : 2.5	1 : 1.7 : 2.3
	6 " 7	**1 : 0.6 : 2.0**	**1 : 0.7 : 2.0**	**1 : 0.8 : 2.0**	**1 : 0.9 : 1.9**	**1 : 1.1 : 1.8**
	8 " 10	1 : 0.3 : 1.2	1 : 0.3 : 1.2	1 : 0.4 : 1.2	1 : 0.5 : 1.2	1 : 0.6 : 1.2
No. 4 to 1½ in..	½ to 1	1 : 1.1 : 3.6	1 : 1.2 : 3.5	1 : 1.5 : 3.5	1 : 1.7 : 3.4	1 : 2.0 : 3.2
	3 " 4	1 : 0.9 : 3.0	1 : 1.0 : 2.9	1 : 1.2 : 2.9	1 : 1.4 : 2.9	1 : 1.6 : 2.7
	6 " 7	**1 : 0.6 : 2.2**	**1 : 0.7 : 2.2**	**1 : 0.8 : 2.2**	**1 : 0.9 : 2.2**	**1 : 1.1 : 2.1**
	8 " 10	1 : 0.3 : 1.4	1 : 0.3 : 1.3	1 : 0.4 : 1.4	1 : 0.5 : 1.4	1 : 0.5 : 1.3
No. 4 to 2 in...	½ to 1	1 : 1.0 : 4.1	1 : 1.1 : 4.1	1 : 1.2 : 4.1	1 : 1.4 : 4.1	1 : 1.6 : 4.0
	3 " 4	1 : 0.8 : 3.4	1 : 0.9 : 3.4	1 : 1.0 : 3.5	1 : 1.1 : 3.4	1 : 1.3 : 3.4
	6 " 7	**1 : 0.5 : 2.6**	**1 : 0.6 : 2.6**	**1 : 0.6 : 2.7**	**1 : 0.7 : 2.6**	**1 : 0.9 : 2.6**
	8 " 10	1 : 0.2 : 1.6	1 : 0.3 : 1.6	1 : 0.3 : 1.7	1 : 0.4 : 1.7	1 : 0.4 : 1.7
⅜ to 1 in.......	½ to 1	1 : 1.4 : 3.1	1 : 1.5 : 3.0	1 : 1.8 : 2.9	1 : 2.1 : 2.8	1 : 2.4 : 2.6
	3 " 4	1 : 1.1 : 2.6	1 : 1.3 : 2.6	1 : 1.5 : 2.5	1 : 1.7 : 2.4	1 : 2.0 : 2.2
	6 " 7	**1 : 0.8 : 2.0**	**1 : 0.8 : 2.0**	**1 : 1.0 : 1.9**	**1 : 1.1 : 1.9**	**1 : 1.3 : 1.8**
	8 " 10	1 : 0.4 : 1.2	1 : 0.4 : 1.2	1 : 0.5 : 1.2	1 : 0.6 : 1.2	1 : 0.7 : 1.1
⅜ to 1½ in......	½ to 1	1 : 1.4 : 3.5	1 : 1.5 : 3.4	1 : 1.7 : 3.4	1 : 2.0 : 3.3	1 : 2.3 : 3.1
	3 " 4	1 : 1.1 : 3.0	1 : 1.2 : 2.9	1 : 1.4 : 2.9	1 : 1.6 : 2.8	1 : 1.9 : 2.6
	6 " 7	**1 : 0.6 : 2.2**	**1 : 0.8 : 2.2**	**1 : 1.0 : 2.2**	**1 : 1.1 : 2.1**	**1 : 1.3 : 2.0**
	8 " 10	1 : 0.4 : 1.4	1 : 0.4 : 1.4	1 : 0.5 : 1.4	1 : 0.6 : 1.3	1 : 0.7 : 1.3
⅜ to 2 in.......	½ to 1	1 : 1.3 : 4.0	1 : 1.4 : 4.0	1 : 1.6 : 4.0	1 : 1.9 : 3.9	1 : 2.1 : 3.8
	3 " 4	1 : 1.0 : 3.4	1 : 1.2 : 3.4	1 : 1.3 : 3.3	1 : 1.5 : 3.3	1 : 1.7 : 3.2
	6 " 7	**1 : 0.7 : 2.6**	**1 : 0.8 : 2.5**	**1 : 0.9 : 2.6**	**1 : 1.0 : 2.6**	**1 : 1.1 : 2.5**
	8 " 10	1 : 0.4 : 1.6	1 : 0.4 : 1.6	1 : 0.5 : 1.6	1 : 0.5 : 1.6	1 : 0.6 : 1.6
¾ to 1½ in......	½ to 1	1 : 1.6 : 3.2	1 : 1.8 : 3.2	1 : 2.1 : 3.2	1 : 2.4 : 3.1	1 : 2.7 : 2.9
	3 " 4	1 : 1.3 : 2.7	1 : 1.5 : 2.7	1 : 1.7 : 2.7	1 : 2.0 : 2.6	1 : 2.3 : 2.5
	6 " 7	**1 : 0.9 : 2.0**	**1 : 1.0 : 2.1**	**1 : 1.2 : 2.0**	**1 : 1.4 : 2.0**	**1 : 1.5 : 1.8**
	8 " 10	1 : 0.5 : 1.2	1 : 0.5 : 1.3	1 : 0.6 : 1.3	1 : 0.7 : 1.3	1 : 0.8 : 1.2
¾ to 2 in.......	½ to 1	1 : 1.6 : 3.7	1 : 1.8 : 3.7	1 : 2.0 : 3.7	1 : 2.4 : 3.6	1 : 2.6 : 3.5
	3 " 4	1 : 1.3 : 3.1	1 : 1.5 : 3.1	1 : 1.6 : 3.1	1 : 1.9 : 3.1	1 : 2.2 : 3.0
	6 " 7	**1 : 0.9 : 2.4**	**1 : 1.1 : 2.4**	**1 : 1.1 : 2.4**	**1 : 1.3 : 2.4**	**1 : 1.5 : 2.3**
	8 " 10	1 : 0.5 : 1.5	1 : 0.5 : 1.5	1 : 0.6 : 1.5	1 : 0.7 : 1.5	1 : 0.8 : 1.5
¾ to 3 in.......	½ to 1	1 : 1.6 : 4.2	1 : 1.8 : 4.2	1 : 2.0 : 4.2	1 : 2.3 : 4.1	1 : 2.6 : 4.0
	3 " 4	1 : 1.3 : 3.5	1 : 1.5 : 3.6	1 : 1.6 : 3.6	1 : 1.9 : 3.6	1 : 2.1 : 3.5
	6 " 7	**1 : 0.9 : 2.6**	**1 : 1.0 : 2.6**	**1 : 1.1 : 2.6**	**1 : 1.3 : 2.6**	**1 : 1.4 : 2.6**
	8 " 10	1 : 0.5 : 1.6	1 : 0.5 : 1.6	1 : 0.6 : 1.7	1 : 0.7 : 1.7	1 : 0.8 : 1.7

Joint Committee table of proportions for concrete of a given strength at 28 days.

105. Field Tests Sponsored by Joint Committee.—With the object of determining whether the recommendations of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete were practical, a series of field tests, sponsored by the Joint Committee, was made under the auspices of a Joint Committee of Contractors (on which was represented the Associated General Contractors of America).

One series of tests was made during the summer of 1923 on concrete used in the construction of Building No. 10 of the Victor Talking Machine Co., at Camden N. J., and another on the concrete being used in the construction of the piers for the Newark Bay Bridge of the Central Railroad of New Jersey. The tests were conducted through the coöperation of the owners and contractors directly concerned.

The report in full is published in the Proceedings of the Am. Soc. C. E. for January, 1925.

The report states that the aggregates were of washed sand and gravel at both places; that the plant at Camden was not designed to provide great accuracy in measuring the aggregates, which were measured loose in a metal charging-hopper; that at Newark Bay the aggregates were measured in adjustable Blaw-Knox " batchers " designed to give uniform quantities; that Ransome mixers of 1 cubic yard capacity were used on both jobs.

The report states further that at Camden nominal mixes of 1 : 2 : 4 and 1 : 2 : 2 were used, having a consistency represented by a slump of about 8 inches; that at Newark Bay most of the concrete was approximately 1 : 2.4 : 3.6 and of a consistency represented by a slump of 3 to 4 inches, and some of the concrete was a 1 : 2 : 3 mix of about 3 to 4 inch slump.

It is only possible here to mention some of the outstanding points of the twenty-two principal results of the tests:

1.—The grand averages of the 28-day strengths of the concrete in the two construction projects were as follows:

Camden....	1 : 2 : 4 concrete........	2190 lb. per sq. in.
	1 : 1 : 1 concrete........	3830 lb. per sq. in.
Newark Bay	1 : 2.4 : 3.6 concrete	3150 lb. per sq. in.
	1 : 2 : 3 concrete........	3800 lb. per sq. in.

7.—The results of the slump and flow tests varied considerably. The slump test seemed to be more reliable at slumps of 6 to 8 in. than for lower slumps. The slump test and flow test seemed to be of about equal value for measuring the workability of the concrete.

8.— . . . The indication is that the slump test should be considered as a measurement of workability and not a criterion for strength unless much more

accurate methods of measuring materials are employed than those now commonly used.

12.—With the slump and quantity of cement maintained constant, the strength increased with an increase in fineness modulus up to the point beyond which the concrete was no longer workable due to the presence of too little fine aggregate.

13.—In general, the strengths at 28 days and 3 months of moulded cylinders cured in damp sand were about equal to or slightly greater than the strengths of specimens cut from test slabs and columns which had stood in the building with no treatment other than that to which the building itself was subjected. The moulded cylinders were made from the same batches as the cores.

14.—The strengths of field made specimens were about the same as those made in the laboratory for the same mixtures and slumps, and similar conditions of curing.

15.—The Camden tests, in which fairly wet concrete was used, showed no consistent variation in strength of concrete with variations in time of mixing for periods of from 15 sec. to 5 min. At Newark Bay, where a dryer concrete was used, the strength at 28 days was increased on an average of about 170 lb. per sq. in. for each addtional 1 min. of mixing, up to 6 min.

16.—Concrete which had stood in the forms for periods up to 1 hour before moulding into test specimens showed strengths about 30 per cent greater than that of specimens from the same batch made at the time the concrete was placed in the form. Similar results were found for specimens taken from concrete which had stood in a water-tight box protected from evaporation losses.

18.—Cylinders of 1 : 2 : 4 concrete cured in air at Camden during June and July showed strengths at 28 days about 80 per cent of the strength of cylinders from the same batch cured in damp sand.

20.—An accurate water-measuring device is necessary to maintain a uniform workability, but such a device must be capable of ready adjustment to differences in quantities of water due to the variation in the moisture content of the aggregates.

21.—Greater accuracy of measuring quantities of materials on the work than is now generally secured, is the feature which seems to be most worthy of future effort.

22.—In so far as general results may be predicted from these tests, they indicate that it is possible to meet requirements based on the Tables of Proportions as contained in the report of the Joint Committee or on average strengths shown by preliminary laboratory tests of specimens using materials from the job under consideration. This assumes that a tolerance which permits about 10 per cent of the strengths to fall below 80 per cent of the average strength is satisfactory.

106. Tensile and Transverse Strength.—There are very few data available concerning the tensile strength of concrete, as it is not used where it is subject to direct tension, and this strength is of comparatively little interest. The tensile strength is called into play in unreinforced beams, but the action is quite different from that of a direct pull. The results of tests indicate that the tensile strength of concrete commonly varies from about one-fifteenth to one-twelfth of the compressive strength.

The transverse strength is dependent upon the tensile resistance

of the material, and plain concrete is therefore a weak material for use in beams. On account of this weakness, concrete is seldom used for beams without reinforcement, and in the computation of reinforced beams, the resistance of the concrete on the tension side of the beam is neglected.

The few data available indicate that the modulus of rupture for plain concrete beams varies from about one-eighth to one-fifth of the compressive unit strength, or that it is approximately twice the strength in direct tension. These values are based upon the application of the common theory of flexure, and the usual formulas for homogeneous materials. The difference between the modulus of rupture and tensile strength may be partly accounted for by the fact that the modulus of elasticity is not constant and the neutral axis does not remain at the gravity axis, but changes in position, approaching the compression side of the beam as the load increases, so that the actual tension does not reach the computed modulus of rupture.

ART. 26. COST OF CONCRETE WORK

107. Cost of Concrete Work.—In a country of the size of the United States, it is difficult to give anything other than the roughest approximations of the cost of concrete.

In the issue of January 8, 1925, the Engineering News-Record tabulates the spring prices of materials and labor from 1913 to 1924 inclusive.

The average price of Portland Cement per barrel, *net without bags*, f.o.b. Chicago in 1913 was $1.19. A peak price of $2.20 was reached in 1923.

The average cost of Crushed Stone per cubic yard, ¾-inch, f.o.b. New York City in 1913 was $0.90. A peak price of $2.10 was reached in 1920.

The average price of Gravel per cubic yard, ¾-inch, f.o.b. New York City in 1913 was $0.85. A peak price of $2.58 was reached in 1920.

The average price of Sand per cubic yard, f.o.b. New York City in 1913 was $0.50. A peak price of $1.42 was reached in 1920.

The average price of Common Labor per hour for the United States in 1913 was $0.17. A peak price of $0.57 was reached in 1920.

The price of Portland Cement per barrel without bags, in February, 1925, f.o.b. Denver was $2.84, and f.o.b. Dallas was $2.05.

The price of Crushed Stone per cubic yard at that time f.o.b. Kansas City was $1.50, and f.o.b. Chicago and Seattle was $3.00.

The price of Gravel and Sand per cubic yard f.o.b. Baltimore was \$1.40, and f.o.b. Chicago was \$3.00.

The price of Common Labor per hour in February, 1925, was lowest in Atlanta at from 25 to 30 cents, and highest in Chicago at from 75 to $82\frac{1}{2}$ cents.

108. Cost of Materials.—As shown above, the cost of cement varies with the demand, and for different localities, with the cost of transportation. Prices for cement delivered should always be obtained in making estimates for work. The cost of wagon transportation in delivery of cement to the work varies with the condition of the roads and the means of transport available. A team may haul 10 or 12 barrels of cement on a fair country road, and travel about 20 miles per day; on hard-surface roads the loads may be increased and auto transportation over good roads is less expensive.

The approximate quantities of materials required may be taken from the tables in Section 84. The following figures represent rough approximations of the costs of work. The unsettled prices existing make it impracticable to give data of much, if any value.

Sand.—For ordinary work in urban communities, sand may usually be purchased delivered, and estimates should be based upon the local prices, which vary in different localities according to the nearness of the source of supply and the way in which the sand is delivered.

When sand is purchased by the ton, the weight per cubic yard must be ascertained to figure the cost accurately. Ordinary screened sand, with about 45 per cent voids, weighs from 2400 to 2500 pounds per cubic yard.

For sand taken from a pit, the costs include digging the sand, loading it into wagons, and hauling to the work. In digging and loading, 1 to $1\frac{1}{2}$ hours labor is ordinarily required per cubic yard and at 56 cents per hour the cost may be from 70 to 98 cents per cubic yard. On well-organized work, where hauling is continuous, hauling may cost 12 to 18 cents per 1000 feet of distance, while on smaller and less well-arranged work, the cost may reach 20 to 25 cents, with team and driver at \$9 per day, but decreases as the length of haul increases. Hand screening usually costs somewhat less than loading, perhaps from 25 to 40 cents per cubic yard.

Stone and Gravel.—Broken stone or gravel, like sand, may usually be purchased for ordinary work by the cubic yard or by the ton. The weight per cubic yard depends upon the specific gravity of the stone and the percentage of voids. For ordinary crusher-run stone with chips removed and about 45 per cent voids, granite or hard

limestone weighs about 2500 pounds per cubic yard, trap about 2700 pounds; stone containing more fine material, in which the voids are 40 per cent, weighs about 10 per cent more than these figures.

When gravel is to be obtained from a pit, the cost of digging and loading, with labor at 60 cents an hour, commonly varies from about 55 to 80 cents per cubic yard, according to the quantity required and the arrangements for loading, while hauling is about the same as for sand. The cost of screening gravel by hand may vary from 50 to 90 cents per cubic yard, 75 cents being an ordinary average.

Average Costs.—Under average conditions, with a moderate haul the price of sand delivered on work varies from about $1.50 to $2.00 per cubic yard; similarly screened gravel, $1.80 to $2.25, and broken stone, $2.10 to $2.60. When gravel or broken stone is delivered on cars, about 40 cents per cubic yard must be allowed for unloading.

109. Cost of Labor.—The labor required in concrete work includes handling the materials to the mixing platform, mixing the concrete and handling the concrete to place. The cost varies with the organization of the work and the experience of the men. The ability of the foreman to systematize and control the work is one of the most important items. The author on one occasion had two almost identical pieces of work in progress at the same time, and found after the first few days that one job was costing about 40 per cent more than the other, due almost entirely to differences in the management of the foremen.

When materials are conveniently arranged and the concrete, after mixing, may be shoveled into place from the mixing platform, the cost of mixing and placing concrete is commonly from $2.50 to $3.50 per cubic yard, with labor at 56 cents per hour. Wheeling the concrete to place costs about 42 cents per cubic yard for the first 50 feet and 14 cents for each additional 50 feet. For work of moderate size the cost of machine mixing does not differ very materially from hand mixing, when the overhead charges are included.

In placing mass concrete in large work, the labor costs may be materially reduced where machinery is used for mixing and handling the concrete. Costs from $1.00 to $1.25 per cubic yard are not uncommon.

In reinforced concrete structural work, the placing of concrete is more expensive than ordinary work mentioned above, and the labor coast of mixing and placing concrete may be from $3 to $4 per cubic yard—figures which will vary with the difficulty of spading

and compacting in the forms, and with the means of distribution over the work.

In work for which considerable machinery is required, a charge for use of plant and supplies must be included, which will vary with the size of the job and the extent of the plant required. For building operations where a hoisting plant is necessary, it will be from $1 to $2 per cubic yard.

110. Total Costs.—The costs for concrete in place include the costs for materials, labor, and plant. In the construction of a 1 : 3 : 6 concrete foundation for street pavement, an average cost is approximately as follows:

Cement, 1.02 barrels at $2.75	$2.80
Sand, 0.46 cubic yard at $1.75	0.80
Stone, 0.92 cubic yard at $2.40	2.22
Labor, per cubic yard	3.08
Total cost per cubic yard	$8.90

For the construction of a concrete building, an average cost for 1 : 2 : 4 concrete of crusher run stone (45 per cent voids) may be as follows:

Cement, 1.47 barrels at $2.75	$4.04
Sand, 0.43 cubic yard at $1.75	0.76
Stone, 0.87 cubic yard at $2.40	2.09
Labor, per cubic yard	4.20
Plant, per cubic yard	2.24
Total cost per cubic yard	$11.33

Allowance should be made for waste of materials, which always occurs to some extent—in some cases 5 per cent of the amount of materials necessary to form the required concrete is needed to cover this loss. Care in handling the materials and close supervision of the mixing may reduce the loss to a negligible quantity, where the work is concentrated in large units.

The above figures do not include the cost of forms or of steel for reinforcement. Forms must be specially designed for each structure and the cost varies widely. In massive construction or in foundation work, the forms may be a comparatively small item, while for heavy walls requiring support on the sides, they may cost $.75 per cubic yard or less. On structural work, including beams

and columns, the cost of forms is frequently greater than that of the concrete. In such cases, only a careful design for the forms, and estimate of the materials and labor required for their erection, can give accurate data as to cost. In building work, the cost of forms is sometimes roughly estimated as about 12 to 15 cents per square foot of surface of concrete, where the use of the materials may be repeated, and when the forms can be used but once the cost is much greater.

Valuable information concerning the cost of concrete construction may be found in " Concrete Costs " by Taylor and Thompson, and in " Concrete Construction " by Gillette and Hill.

" Concrete, Plain and Reinforced," by Taylor, Thompson, and Smulski (1925) gives a very complete discussion of the materials, proportions, methods of mixing and placing, and properties of plain concrete.

" Researches in Concrete " by W. K. Hatt, Bulletin of Purdue University, Vol. IX, No. 11, November, 1925, is a comprehensive review of selected concrete researches, with extensive bibliography on cement and concrete.

CHAPTER VI

REINFORCED CONCRETE

ART. 27. GENERAL PRINCIPLES

111. Object of Reinforcement.—Concrete and steel are frequently combined in structural work in two distinct types of construction:

1. *Structural Members of Steel Encased in Concrete.*—In this type of construction, the steel member is designed to carry the loads, and the concrete is used for protection of the steel against the weather or fire, or sometimes to give lateral stiffness to the member.

2. *Reinforced concrete,* in which the load-carrying member is made of concrete, the steel being used to strengthen the concrete by taking stresses that the concrete is unfitted to resist.

Structures of the first type, in which the concrete is used to give stiffness to the structure, are often classed as reinforced concrete, although reliance is placed upon the steel alone for carrying the loads. These are not, however, designed in accordance with the theories of reinforced concrete.

The advantages to be gained by combining steel and concrete are due to the fact that concrete is extremely weak and uneconomical when subjected to tension, but has much greater strength and is a convenient and economical material for resistance to compressions, while steel must be made of special forms satisfactorily to carry compression, but may be used for resisting tension in the form of ordinary bars.

In structural forms, such as beams, in which both tensile and compressive stresses are developed, the combination of the two materials offers an economical means of construction when the conditions are favorable, and the use of this type of construction has been rapidly extending during the past few years.

In the use of concrete and steel in combination, the following properties of the materials are important:

1. When steel bars are embedded in concrete, the concrete adheres to the steel and develops a considerable bond strength, which may be relied upon to make the two materials act together.

2. Concrete acts as a protection to the steel against rust. In a number of instances in removing concrete structures, it has been found that the steel, after being embedded for several years, was in good condition and free from rust. To form an efficient protection the concrete must be mixed rather wet, so that the steel is completely covered with a coating of mortar.

3. The coefficients of expansion for the two materials are so nearly the same that no stresses need be considered as resulting from differences of expansions or contractions due to changes in temperature.

4. Changes in dimension occur in unreinforced concrete during hardening (see Section 92) and with variations in moisture conditions. When these changes are restrained by reinforcement, the concrete seems to adjust itself to the situation, adopting permanently the form in which it is held, without being placed under appreciable stress.

112. Bond Strength.—The stresses carried by the steel in a reinforced concrete structural member must usually be transmitted to the steel through the bond between the steel and concrete. Tests and experience show that plain steel bars embedded in concrete develop considerable bond strength which may be relied upon to hold the bars permanently in place in resisting stresses which tend to separate them from the concrete. Experiments upon the adhesion of plain bars to concrete show that the bond strength is approximately proportional to the area of surface contact, and varies with the quality of the concrete, being nearly proportional to the strength in compression.

When tests are made by pulling a bar of steel out of a block of concrete in which its end has been embedded, the compression of the concrete may influence the results, and the bond strength shown be greater than would be developed in a beam where both steel and concrete are under tension. When the length of the bar embedded in the concrete is considerable, the bar may begin to slip at the surface of the block before the resistance of the more deeply embedded part is fully brought into play, and the bond resistance per unit of surface area be less than for shorter lengths. In tests at the University of Wisconsin no difference in unit bond strength was found between 6-inch and 12-inch embedments.

Various tests have shown ultimate bond strengths for ordinary concrete from about 200 to 700 pounds per square inch of surface area of bar. In general, for concrete as commonly employed in

structural work the unit bond resistance for plain bars may be from 200 to 300 pounds per square inch.

Twisted and deformed bars are made in a number of forms for the purpose of increasing the bond strength, and are extensively used in reinforced concrete work; their raised projections or uneven surfaces give a mechanical bond and carry considerable more load before finally yielding than plain bars. although initial slip may occur under about the same stresses.

113. Reinforcing Steel.—The Joint Committee on Concrete makes the following recommendations [1] concerning steel for reinforcement bars:

21.—*Quality.*—Metal reinforcement shall meet the requirements of the "Standard Specifications for Billet Steel Concrete Reinforcement Bars" (Serial Designation: A15–14) of the specified grade, or of the "Standard Specifications for Rail-Steel Concrete Reinforcement Bars" (Serial Designation: A16–14) of the A. S. T. M. as required by the Engineer, except that the machining of deformed bars before testing shall be eliminated.

22.—*Standard Sizes of Bars.*—Reinforcement bars shall conform to the areas and equivalent sizes (as recommended by the Joint Conference of Representatives of Manufacturers, Distributers, and Users of Concrete Reinforcement Bars held Sept. 29, 1924) shown in the following table:

TABLE XXI.—STANDARD REINFORCEMENT BARS

Size of Bar, in Inches.	Area, in Square Inch.		Size of Bar, in Inches.	Area, in Square Inch.	
	Round Bar.	Square Bar.		Round Bar.	Square Bar.
$\frac{1}{4}$	0.049	....	$\frac{7}{8}$	0.601	
$\frac{3}{8}$	0.110	....	1	0.785	1.000
$\frac{1}{2}$	0.196	0.250	$1\frac{1}{8}$	....	1.265
$\frac{5}{8}$	0.306	....	$1\frac{1}{4}$	....	1.562
$\frac{3}{4}$	0.441				

23.—*Deformed Bars.*—Deformed bars, in order to meet approval, must develop a bond of at least 25 per cent greater than that of a plain bar of equivalent cross-sectional area. Areas of deformed bars shall be determined by the minimum cross-section thereof.

24.—*Structural Shapes.*—Structural shapes used in reinforcement shall conform to the requirements of "Standard Specifications for Structural Steel Bridges" (Serial Designation: A7–24) or to requirements of "Standard Specifications for Structural Steel Buildings" (Serial Designation: A9–24) of the A. S. T. M. as required by the Engineer.

[1] Proceedings, American Society of Civil Engineers, Oct., 1924.

25. *Wire*.—Wire for concrete reinforcement shall conform to the requirements of "Tentative Specifications for Cold-drawn Steel Wire for Concrete Reinforcement" (Serial Designation: A82–21T) of the A. S. T. M.

26.—*Cast Iron*.—Cast iron used in composite columns shall conform to the requirements of the "Standard Specifications for Cast-iron Pipe and Special Castings" (Serial Designation: A44–04) of the A. S. T. M.

The Specifications of the American Society for Testing Materials are given in their Book of Standards or may be obtained in reprints from the Secretary of the Society.

On important work, it is common to purchase steel subject to these specifications, and to submit steel to careful inspection at the mills.

Engineers differ as to the advisability of using "hard-grade" steel for reinforcement. As a concrete beam usually gives way when the yield point of the steel is reached, through the cracking and crushing of the concrete, the yield point may be considered as the ultimate strength for concrete work, and some engineers prefer to use hard-grade steel on account of its high yield point. Medium steel is, however, usually preferred as less expensive and less likely to be brittle. When hard-grade steel is used, either high carbon or cold deformed material, it should be carefully tested, as it is more variable in quality than medium steel, but when meeting the specifications is a superior material.

For ordinary reinforced concrete work, mild steel as commonly found upon the market is usually employed. It is desirable to subject this to the cold bending test, which is the most important test for reinforcing steel, and upon failure the material should always be rejected.

114. Ratio of Moduli of Elasticity.—The modulus of elasticity of a material is the ratio of unit stress to the corresponding unit deformation, within the elastic limit of the material.

When two materials with different moduli of elasticity, like steel and concrete, are combined in a structural member so that they must act together, as in a column, they will each be extended or compressed to the same amount, and the unit stress carried by each material will be proportional to the modulus of elasticity of the material.

When a beam is loaded so as to cause it to bend, it is lengthened on the convex and shortened on the concave side. Tests of reinforced concrete beams show that any plane section of the beam before bending remains approximately plane when bent, and that the amount of extension or shortening is proportional to the distance

from its neutral surface. In such beams the stresses upon steel and concrete at the same distance from the neutral surface are proportional to the moduli of elasticity of the materials.

In the discussion of stresses in any structural member of steel and concrete subject to deformation, it is therefore necessary to know the ratio of the moduli of elasticity of the two materials in order to determine the amount of stress carried by each.

The modulus of elasticity of steel is practically the same for the different grades and is independent of the ultimate strength or yield point. An average value is about 30,000,000 lb./in.^{2}, and this value is usually employed in reinforced concrete computations.

The modulus of elasticity of concrete is not a constant, but varies with the stress, becoming less as the stress becomes greater. For small stresses, within the limits of allowable working stress, however, the variation is very small, and the modulus of elasticity may be taken as constant without appreciable error. The formulas in common use are based upon the assumption of a constant modulus of elasticity, and variation of stress in beam design proportional to distance from the neutral axis.

Tests for the determination of the moduli of elasticity of concretes vary considerably in results, and indicate that the modulus depends upon the quality of the concrete, being approximately proportional to the compressive strength. The modulus also varies with the age of the concrete, increasing with age more rapidly than does the strength of the concrete.

The Joint Committee makes the following recommendation [2] concerning the modulus of elasticity:

> The modulus of elasticity of concrete in compression is constant within the limits of working stresses and the distribution of compressive stress in beams is rectilinear.
>
> The moduli of elasticity of concrete in computations for the position of the neutral axis, for the resisting moment of beams, and for compression in columns are as follows:
>
> 1.—One-fifteenth ($\frac{1}{15}$) that of steel, when the compressive strength of the concrete at 28 days exceeds 1500 and does not exceed 2200 lb. per sq. in.
> 2.—One-twelfth ($\frac{1}{12}$) that of steel, when the compressive strength of the concrete at 28 days exceeds 2200 and does not exceed 2900 lb. per sq. in.
> 3.—One-tenth ($\frac{1}{10}$) that of steel, when the compressive strength of concrete at 28 days is greater than 2900 lb. per sq. in.

115. Reinforced Concrete in Tension.—When reinforced concrete is subject to tensile stresses, the two materials act together,

[2] Proceedings, American Society of Civil Engineers, Oct., 1924.

each carrying unit stresses in proportion to its modulus of elasticity, so long as the stresses do not exceed the strength of the concrete. When, however, the steel is stressed to a fair working load, the stress upon the concrete will have passed its breaking strength, and it can no longer be considered as carrying stress—a condition which usually exists in reinforced concrete beams when carrying normal working loads. The steel in such beams is designed to carry all the tensions, the concrete on the tension side merely holding the steel in place.

In the earlier studies of reinforced beams, it was supposed that the concrete when reinforced became capable of carrying greater tensions than plain concrete, and beam formulas were proposed in which it was assumed that the concrete carried part of the tension. Later investigations, however, showed that this was erroneous and these formulas are no longer used in design.

Observations upon beams under tests have shown that minute cracks, invisible to the naked eye, frequently exist in the concrete surface on the tension side while the beam is carrying only a safe load—a discovery made in testing damp beams with the tension side uppermost at the University of Wisconsin. Dark, wet lines appeared upon the surface at about the time that the ultimate strength of the concrete was reached, and these later developed into fine cracks. Experience with this type of construction indicates that, when the materials are properly used, no injury results from this overstressing of the concrete, and that the steel is fully protected by the concrete.

ART. 28. RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

116. Flexure Formulas.—The common or straight-line formulas for reinforced concrete beams are based upon the ordinary theory of flexure, and involve the following assumptions:

(1) A section of the beam that is plane before bending remains plane when bent.

(2) The modulus of elasticity of concrete is constant within the limits of safe unit stresses.

(3) The concrete resists compression only, all tensions being carried by the steel.

(4) Initial stresses due to expansion or contraction of the concrete are negligible.

These assumptions greatly simplify the computations and are found experimentally to be sufficiently accurate within the limits

of stresses used in ordinary beam design; they are not applicable to ultimate loads and can be used only for working loads and working stresses.

The following notation will be used:

A_s = effective cross-sectional area of metal reinforcement in tension in beams or compression in columns;
A_v = total area of web reinforcement in tension within a distance of s, or the total area of all bars bent up in any one plane;
b = width of rectangular beam or width of flange of T-beam;
b' = width of stem of T-beam;
C = total compressive stress in concrete;
C' = total compressive stress in reinforcement;
d = depth from compression surface of beam or slab to center of longitudinal tension reinforcement;
d' = depth from compression surface of beam or slab to center of compression reinforcement;
d_1 = total depth of beam or slab;
E_c = modulus of elasticity of concrete in compression;
E_s = modulus of elasticity of steel in tension = 30,000,000 lb. per sq. in.;
f_c = compressive unit stress in extreme fiber of concrete;
f_c' = ultimate compressive unit strength of concrete at age of 28 days, based on standard tests of A. S. T. M.
f_s = tensile unit stress in longitudinal reinforcement;
f_s' = compressive unit stress in longitudinal reinforcement;
f_v = tensile unit stress in web reinforcement;
j = ratio of lever arm of resisting couple to depth, d;
$jd = d - z$ = arm of resisting couple;
k = ratio of depth of neutral axis to depth, d;
l = span length of beam or slab from center to center of supports;
M = bending moment or moment of resistance in general;
$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete;
Σ_o = sum of perimeters of bars in one set;
p = ratio of effective area of tension reinforcement to effective area of concrete in beams $= \frac{A_s}{bd}$;
p' = ratio of effective compression reinforcement to effective area of concrete in beams;
s = spacing of web members, measured at the plane of the lower reinforcement and in the direction of the longitudinal axis of the beam;

t = thickness of flange of T-beam;
T = total tensile stress in longitudinal reinforcement;
u = bond stress per unit of area of surface of bar;
v = shearing unit stress;
V = total shear;
$V_d = V - 40bjd$ = shear used in designing stirrups and bent up bars to resist diagonal tension;
w = uniformly distributed load per unit of length of beam or slab;
W = total dead and live load uniformly distributed over length of beam or slab;
z = depth from compression surface of beam or slab to resultant of compressive stresses.

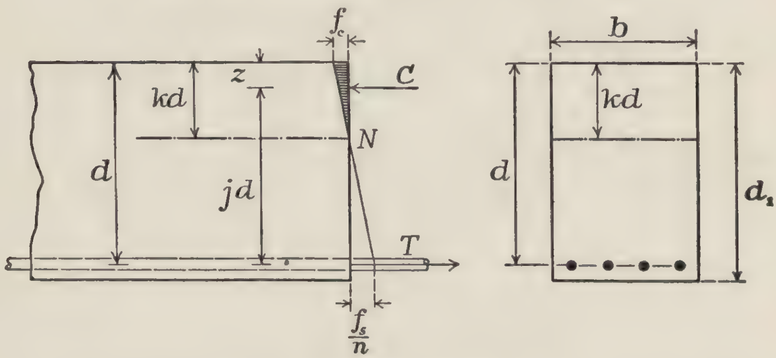


FIG. 52.—Reinforced Concrete Beam.

Assuming that a plane section before bending remains plane after bending, from similar triangles (see Fig. 52),

$$\frac{kd}{d-kd}=\frac{f_c}{f_s/n},$$

or

$$\frac{k}{1-k}=\frac{nf_c}{f_s},$$

from which

$$k=\frac{nf_c-knf_c}{f_s},$$

or

$$k=\frac{nf_c}{f_s+nf_c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The total compression on the concrete is, $C=\frac{1}{2}f_ckbd$.

The total tension on the steel is, $T=A_sf_s=f_spbd$. These are equal for equilibrium; equating and reducing,

$$f_sp=\tfrac{1}{2}f_ck,$$

or

$$\frac{f_s}{f_c}=\frac{k}{2p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Combining (1) and (2) we have

$$p=\frac{k^2}{2n(1-k)}, \qquad (3)$$

and solving for k,

$$k=\sqrt{2pn+(pn)^2}-pn. \qquad (4)$$

The centroid of compressive stresses is at a distance $kd/3$ from the compressive face of the beam, and

$$jd=d-kd/3,$$

or

$$j=1-\frac{k}{3}. \qquad (5)$$

From the foregoing it is readily seen that the ratio of the unit stresses on the steel and concrete, and the values of k, j and p are interdependent. If the unit stresses and value of n be assumed, k and the required percentage of steel may be found from Formulas (1) and (2). If the percentage of steel be known and the ratio n assumed, the values of k and the ratio f_s/f_c may be found from (4) and (2).

The resisting moment of the beam is due to the couple formed by the tensions and compressions and is equal to either of them into the arm of the couple:

$$M=Tjd=A_sf_sjd=f_spjbd^2, \qquad (6)$$

or

$$M=Cjd=\tfrac{1}{2}f_ckjbd^2, \qquad (7)$$

and

$$bd^2=\frac{M}{f_spj}=\frac{2M}{f_ckj}. \qquad (8)$$

Formulas (6) and (7) give a means of determining the moment of resistance of a beam of known dimensions and safe unit stresses, while from (8) the necessary dimensions may be found to resist any given bending moment with assumed unit stresses in steel and concrete.

Examples.—The problems arising in the use of these formulas are of two kinds—the design of beams to carry certain loads; the investigation of existing beams to determine the loads they may safely carry, or the unit stresses resulting from given loads. The following examples illustrate the use of the formulas for these purposes:

1. A reinforced concrete beam is to carry a bending moment of 152,000 in.-lb. The safe unit stresses upon concrete and steel are

650 and 16,000 lb./in.^{2} respectively, and $n=15$. Find dimensions for the beam and area of steel required.

Solution.—Formula 1 (p. 194) gives,

$$k=\frac{15\times 650}{16000+15\times 650}=.379,$$

from Formula 5 (p. 195),

$$j=1-\frac{.379}{3}=.874,$$

and Formula 3 (p. 195),

$$p=\frac{(.379)^2}{2\times 15(1-.379)}=.0077.$$

Formula 8 (p. 195) now gives,

$$bd^2=\frac{152000}{16000\times .0077\times .874}=1410.$$

A value may now be assumed for either b or d, or a relation may be fixed between them. Assuming $b=8$ in., $d^2=1410/8=176$ in.^{2}, and from Table XXV (p. 202), $d=13\frac{1}{4}$ in. If the concrete extends $1\frac{3}{4}$ in. below the center of the steel, the total depth $d_1=13\frac{1}{4}+1\frac{3}{4}=$ 15 inches.

$$A_s=.0077\times 8\times 13.25=.82 \text{ in.}^2$$

2. A concrete beam is 9 inches wide and 16 inches deep, and is reinforced with four $\frac{3}{4}$-inch round steel bars, with centers 2 inches above the lower surface of the beam. The safe unit stresses for the concrete and steel are 700 and 14,000 lb./in.^{2} respectively. $n=15$. What is the safe bending moment for the beam?

Solution.—The area of steel is, from Table XXVI (p. 203), $A_s=.4418\times 4=1.767$ in.^{2}, and $p=\frac{A_s}{bd}=\frac{1.767}{9\times 14}=.014$.

Using (4) (p. 195),

$$k=\sqrt{2\times 15\times .014+(15\times .014)^2}-15\times .014=.47.$$

(2) (p. 194), gives

$$\frac{f_s}{f_c}=\frac{.47}{2\times .014}=16.8,$$

or

$$f_c=\frac{14000}{16.8}=833 \text{ lb./in.}^2$$

This shows that if a stress of 14,000 lb./in.^{2} be brought upon the steel, the stress upon the concrete will be greater than 700 lb./in.^{2}

Hence the safe moment is that which causes a stress of 700 lb./in.^{2} in the concrete, or applying (7) (p. 195),

$$M=\frac{700}{2}.47\times.84\times9\times\overline{14}^2=243750 \text{ in.-lb.}$$

3. If the beam in the preceding example is subjected to a bending moment of 225,000 in.-lb., what are the maximum stresses upon the steel and concrete?

Solution.—As in the preceding case, we find $A_s=1.767$ in.^{2}, $k=.47$, $j=0.84$ and $f_s/f_c=16.8$.

From (6) (p. 195),

$$f_s=\frac{M}{A_sjd}=\frac{225000}{1.767\times.84\times14}=10810 \text{ lb./in.}^2$$

$$f_c=10{,}810/16.8=644 \text{ lb./in.}^2$$

117. Tables.—The labor of computation in the use of the above formulas may be considerably lessened by the tabulation of values of some of the terms involved.

In Formula (8) (p. 195), the denominators f_spj and $\frac{1}{2}f_ckj$ are constant for any particular values of f_s and f_c, and may be represented by a single term,

$$R=f_spj=\tfrac{1}{2}f_ckj.$$

Substituting this in (8),

$$M=Rbd^2,$$

or

$$bd^2=M/R. \qquad (9)$$

In Table XXII (p. 199), values of j, k, p, and R are given for several values of f_s and f_c when $n=15$.

Table XXIII (p. 200), gives values of the same quantities when $n=12$.

In Table XXIV (p. 201), values of f_s/f_c, k, j, and $\frac{1}{2}jk$ are given for various values of p, with $n=12$ and $n=15$.

Table XXV (p. 202), gives the values of d to the nearest quarter-inch from 3 to $62\frac{3}{4}$ in., inclusive, for various values of d^2.

Table XXVI (p. 203), gives the perimeters, weights, and areas of square and round steel bars from $\frac{1}{4}$ to $1\frac{1}{2}$ in. in diameter, the sizes furnished by the manufacturers of reinforcing steel being printed in bold-face type.

Table XXVIII (p. 205), gives the bending moment in 1000 in.-lb. that will safely be resisted by twelve inch-width slabs of depths varying from 3 to 30 in., together with areas of steel required, for values of $f_s=16{,}000$ and $f_c=650$ lb. per sq. in., and $n=15$.

Table XXIX (p. 206), gives the bending moment in 1000 in.-lb. that will safely be resisted by twelve inch width slabs of *cinder concrete* of depths varying from 3 to 12 in., together with areas of steel required, for values of $f_s = 16{,}000$ and $f_c = 350$ lb. per sq. in. and $n = 30$.

Tables XXX, XXXI, XXXII, and XXXIII (pp. 207–211), give the bending moment in 1000 in.-lb. that will safely be resisted by various rectangular beams from 4×5 in. to 30×60 in. for different values of f_s and f_c, the value of n in all cases being $= 15$.

Tables XXVIII to XXXIII will be found useful in reviewing designs, and similar ones may easily and quickly be prepared consistent with the practice of the designer.

Examples.—The use of the tables will be illustrated by solving a few problems.

4. A reinforced concrete beam is to resist a bending moment of 312,000 in.-lb. The safe unit stresses upon the concrete and steel are 650 and 16,000 lb./in.^{2}, respectively. The value of $n = 15$. Find dimensions for the beam and the area of steel required.

Solution.—From Table XXII (p. 199), for $f_s = 16{,}000$ and $f_c = 650$, it is found that $R = 108$ and $p = .0077$. Substituting this value of R in (9), $bd^2 = M/R$ or $d^2 = M/bR$. Assuming b as 10 in., $d^2 = 312{,}000/(10 \times 108) = 288.9$ in.^{2}, and from Table XXV (p. 202), $d = 17$ in.

If the center of the steel is taken as $1\frac{1}{2}$ in. from the lower surface of the concrete, the total depth of the beam, $d_1 = 17 + 1\frac{1}{2} = 18\frac{1}{2}$ in.

The area of steel required, $A_s = pbd = .0077 \times 10 \times 17 = 1.31$ in.^{2}

Example 5.—A concrete beam is 10 in. wide and 18 in. deep and is reinforced with four $\frac{3}{4}$-in. round steel bars with centers 2 in. above the lower surface of the beam. The safe unit stresses for concrete and steel are 700 and 16,000 lb./in.^{2} respectively. $n = 15$. What is the safe bending moment for the beam?

Solution.—From Table XXVI (p. 203), four $\frac{3}{4}$-in. round bars have area of 1.77 in.^{2} and $p = 1.77/160 = .0111$. From Table XXIV (p. 201), for $p = .0111$, and $n = 15$, we find $f_s/f_c = 19.6$ and $jk/2 = .185$.

When $f_s = 16{,}000$, $f_c = 16{,}000/19.6 = 816$ lb./in.^{2} This is greater than the allowable stress on concrete, and the safe bending moment is that which causes a stress of 700 lb./in.^{2} on the concrete, or by Formula (7), $700 \times .185 \times 10 \times 16 \times 16 = 331{,}520$ in.-lb.

Example 6.—A concrete beam 9 in. wide and 15 in. deep is reinforced with five $\frac{1}{2}$-in. round bars of steel, with centers 2 in. above lower face of beam. The beam carries a bending moment of 185,000 in.-lb. If $n = 12$, find the unit stresses on the steel and concrete.

TABLE XXII.—RECTANGULAR BEAMS

$n=15$

f_s		Values of f_c, Lbs./In.²								
		500	550	600	650	700	750	800	850	900
14,000	k	.349	.370	.391	.410	.429	.446	.462	.477	.491
	j	.884	.877	.870	.863	.857	.851	.846	.841	.836
	p	.0062	.0073	.0084	.0095	.0107	.0120	.0133	.0146	.0158
	R	77.	89.	102.	115.	128.	143.	157.	171.	185.
15,000	k	.333	.354	.375	.394	.412	.428	.444	.459	.474
	j	.889	.882	.875	.869	.863	.857	.852	.847	.842
	p	.0055	.0065	.0075	.0086	.0097	.0107	.0118	.0130	.0142
	R	74.	86.	99.	112.	125.	138.	151.	166.	180.
16,000	k	.319	.339	.360	.379	.397	.414	.429	.444	.457
	j	.894	.887	.880	.874	.868	.862	.857	.852	.848
	p	.0050	.0058	.0068	.0077	.0087	.0097	.0107	.0118	.0128
	R	72.	83.	95.	108.	121.	134.	147.	161.	174.
18,000	k	.294	.314	.333	.351	.369	.385	.400	.415	.429
	j	.902	.895	.889	.883	.877	.872	.867	.862	.857
	p	.0041	.0048	.0055	.0063	.0072	.0080	.0089	.0098	.0107
	R	66.	77.	88.	100.	113.	126.	139.	152.	165.
20,000	k	.273	.292	.310	.327	.344	.360	.375	.389	.403
	j	.909	.903	.897	.891	.885	.880	.875	.870	.866
	p	.0034	.0040	.0046	.0053	.0060	.0068	.0075	.0083	.0091
	R	62.	72.	83.	94.	106.	119.	132.	145.	157.

TABLE XXIII.—RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

$n = 12$

f_s		f_c—LB./IN.²						
		700	750	800	850	900	950	1000
15,000	k	.359	.375	.390	.405	.419	.432	.444
	j	.880	.875	.870	.865	.860	.856	.852
	p	.0084	.0094	.0104	.0115	.0125	.0136	.0147
	R	111.	123.	136.	149.	162.	175.	189.
16,000	k	.344	.360	.375	.389	.403	.416	.429
	j	.885	.880	.875	.870	.866	.861	.857
	p	.0075	.0084	.0094	.0104	.0114	.0123	.0134
	R	107.	119.	131.	144.	157.	170.	184.
18,000	k	.318	.333	.348	.362	.375	.388	.409
	j	.894	.889	.884	.879	.875	.871	.867
	p	.0062	.0070	.0078	.0086	.0094	.0103	.0111
	R	100.	111.	123.	135.	148.	161.	174.
20,000	k	.296	.310	.324	.338	.351	.363	.375
	j	.901	.897	.892	.887	.883	.879	.875
	p	.0052	.0058	.0065	.0072	.0079	.0086	.0094
	R	93.	104.	116.	128.	140.	152.	164.

From Table XXVI (p. 203), five $\frac{1}{2}$-in. round bars have an area of .98 in.², and $p = A_s/bd = .98/117 = .0084$. From Table XXIV, (p. 201), for $p = .0084$ and $n = 12$, we have $jk/2 = .158$ and $f_s/f_c = 21.5$. By Formula (7),

$$f_c = \frac{M}{bd^2jk/2} = \frac{185000}{9 \times 13 \times 13 \times .158} = 770 \text{ lb./in.}^2$$

$$f_s = 21.5 f_c = 21.5 \times 770 = 16550 \text{ lb./in.}^2$$

TABLE XXIV.—RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

p	$n=12$					$n=15$				
	k	j	$jk/2$	f_s/f_c	R/f_s	k	j	$jk/2$	f_s/f_c	R/f_s
.0015	.173	.942	.081	57.7	.0014	.191	.936	.089	63.7	.0014
.002	.196	.935	.092	49.0	.0019	.217	.928	.101	54.3	.0019
.0025	.217	.928	.100	43.4	.0023	.239	.920	.110	47.8	.0023
.003	.235	.922	.108	39.3	.0027	.258	.914	.118	43.0	.0027
.0035	.251	.916	.115	35.8	.0032	.276	.908	.125	39.4	.0032
.004	.266	.911	.121	33.3	.0037	.292	.903	.132	36.3	.0036
.0045	.279	.907	.126	31.0	.0041	.307	.898	.137	33.9	.0040
.005	.291	.903	.131	29.1	.0045	.320	.893	.142	31.6	.0045
.0055	.303	.899	.136	27.6	.0049	.332	.889	.147	30.2	.0049
.006	.314	.895	.141	26.7	.0054	.344	.885	.152	28.7	.0053
.0065	.325	.892	.145	24.9	.0058	.355	.882	.156	27.5	.0057
.007	.334	.889	.149	23.9	.0062	.365	.878	.160	26.1	.0061
.0075	.344	.885	.153	23.0	.0066	.375	.875	.164	25.0	.0066
.008	.353	.882	.156	22.1	.0071	.384	.872	.167	24.0	.0070
.0085	.361	.879	.159	21.3	.0075	.393	.869	.171	23.1	.0074
.009	.369	.877	.162	20.5	.0079	.402	.866	.174	22.3	.0078
.0095	.377	.874	.165	19.8	.0083	.410	.863	.177	21.6	.0082
.0100	.384	.872	.167	19.2	.0087	.418	.861	.180	21.0	.0086
.011	.398	.867	.175	18.1	.0095	.433	.856	.185	19.7	.0094
.012	.412	.863	.178	17.2	.0103	.446	.851	.190	18.6	.0102
.013	.425	.859	.182	16.4	.0111	.458	.847	.194	17.7	.0110
.014	.436	.855	.186	15.6	.0120	.470	.843	.198	16.9	.0118
.015	.447	.851	.190	14.9	.0128	.482	.839	.202	16.1	.0126
.016	.457	.848	.194	14.3	.0136	.493	.836	.206	15.4	.0134
.017	.467	.845	.197	13.7	.0144	.503	.832	.210	14.8	.0142
.018	.476	.842	.200	13.2	.0152	.513	.829	.213	14.3	.0149
.019	.485	.839	.203	12.8	.0159	.522	.826	.216	13.8	.0157
.020	.493	.836	.206	12.4	.0167	.531	.823	.219	13.3	.0165

TABLE XXV.—VALUES OF d TO THE NEAREST QUARTER-INCH FOR VARIOUS VALUES OF d^2

d^2	d	d^2	d	d^2	d	d^2	d	d^2	d	d^2	d
9	3.00	169	13.00	529	23.00	1089	33.00	1849	43.00	2809	53.00
10	3.25	175	13.25	540	23.25	1105	33.25	1870	43.25	2835	53.25
12	3.50	182	13.50	552	23.50	1122	33.50	1892	43.50	2862	53.50
14	3.75	189	13.75	564	23.75	1139	33.75	1914	43.75	2889	53.75
16	4.00	196	14.00	576	24.00	1156	34.00	1936	44.00	2916	54.00
18	4.25	203	14.25	588	24.25	1173	34.25	1958	44.25	2943	54.25
20	4.50	210	14.50	600	24.50	1190	34.50	1980	44.50	2970	54.50
22	4.75	217	14.75	612	24.75	1207	34.75	2002	44.75	2997	54.75
25	5.00	225	15.00	625	25.00	1225	35.00	2025	45.00	3025	55.00
27	5.25	232	15.25	637	25.25	1242	35.25	2047	45.25	3052	55.25
30	5.50	240	15.50	650	25.50	1260	35.50	2070	45.50	3080	55.50
33	5.75	248	15.75	663	25.75	1278	35.75	2093	45.75	3108	55.75
36	6.00	256	16.00	676	26.00	1296	36.00	2116	46.00	3136	56.00
39	6.25	264	16.25	689	26.25	1314	36.25	2139	46.25	3164	56.25
42	6.50	272	16.50	702	26.50	1332	36.50	2162	46.50	3192	56.50
45	6.75	280	16.75	715	26.75	1350	36.75	2185	46.75	3220	56.75
49	7.00	289	17.00	729	27.00	1369	37.00	2209	47.00	3249	57.00
52	7.25	297	17.25	742	27.25	1387	37.25	2232	47.25	3277	57.25
56	7.50	306	17.50	756	27.50	1406	37.50	2256	47.50	3306	57.50
60	7.75	315	17.75	770	27.75	1425	37.75	2280	47.75	3335	57.75
64	8.00	324	18.00	784	28.00	1444	38.00	2304	48.00	3364	58.00
68	8.25	333	18.25	798	28.25	1463	38.25	2328	48.25	3393	58.25
72	8.50	342	18.50	812	28.50	1482	38.50	2352	48.50	3422	58.50
76	8.75	351	18.75	826	28.75	1501	38.75	2376	48.75	3451	58.75
81	9.00	361	19.00	841	29.00	1521	39.00	2401	49.00	3481	59.00
85	9.25	370	19.25	855	29.25	1540	39.25	2425	49.25	3510	59.25
90	9.50	380	19.50	870	29.50	1560	39.50	2450	49.50	3540	59.50
95	9.75	390	19.75	885	29.75	1580	39.75	2475	49.75	3570	59.75
100	10.00	400	20.00	900	30.00	1600	40.00	2500	50.00	3600	60.00
105	10.25	410	20.25	915	30.25	1620	40.25	2525	50.25	3630	60.25
110	10.50	420	20.50	930	30.50	1640	40.50	2550	50.50	3660	60.50
115	10.75	430	20.75	945	30.75	1660	40.75	2575	50.75	3690	60.75
121	11.00	441	21.00	961	31.00	1681	41.00	2601	51.00	3721	61.00
126	11.25	451	21.25	976	31.25	1701	41.25	2626	51.25	3751	61.25
132	11.50	462	21.50	992	31.50	1722	41.50	2652	51.50	3782	61.50
138	11.75	473	21.75	1008	31.75	1743	41.75	2678	51.75	3813	61.75
144	12.00	484	22.00	1024	32.00	1764	42.00	2704	52.00	3844	62.00
150	12.25	495	22.25	1040	32.25	1785	42.25	2730	52.25	3875	62.25
156	12.50	506	22.50	1056	32.50	1806	42.50	2756	52.50	3906	62.50
162	12.75	517	22.75	1072	32.75	1827	42.75	2782	52.75	3937	62.75

TABLE XXVI.—STEEL BARS. AREAS AND WEIGHTS

Diam. in Inches.	Weight per Ft. Pounds.	Perimeter Inches.	Areas of Different Numbers of Bars—Square Inches.								
			1	2	3	4	5	6	7	8	9
Square Bars.											
1/4	0.212	1.00	0.0625	0.12	0.19	0.25	0.31	0.38	0.44	0.50	0.56
5/16	0.333	1.25	0.0977	0.20	0.29	0.39	0.49	0.59	0.68	0.78	0.88
3/8	0.478	1.50	0.1406	0.28	0.42	0.56	0.70	0.84	0.98	1.12	1.27
7/16	0.671	1.75	0.1914	0.38	0.57	0.77	0.96	1.15	1.34	1.53	1.72
1/2	**0.850**	**2.00**	**0.2500**	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**
9/16	1.076	2.25	0.3164	0.63	0.95	1.27	1.58	1.90	2.21	2.53	2.85
5/8	1.328	2.50	0.3906	0.78	1.17	1.56	1.95	2.34	2.73	3.12	3.52
11/16	1.608	2.75	0.4727	0.94	1.42	1.89	2.36	2.84	3.31	3.78	4.25
3/4	1.913	3.00	0.5625	1.12	1.69	2.25	2.81	3.37	3.94	4.50	5.06
13/16	2.245	3.25	0.6602	1.32	1.98	2.64	3.30	3.96	4.62	5.28	5.94
7/8	2.603	3.50	0.7656	1.53	2.30	3.06	3.83	4.59	5.36	6.12	6.89
15/16	2.988	3.75	0.8789	1.76	2.64	3.52	4.39	5.27	6.15	7.03	7.91
1	**3.400**	**4.00**	**1.0000**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**	**7.00**	**8.00**	**9.00**
1/8	**4.303**	**4.50**	**1.2656**	**2.53**	**3.79**	**5.06**	**6.33**	**7.59**	**8.86**	**10.12**	**11.39**
1/4	**5.312**	**5.00**	**1.5625**	**3.12**	**4.69**	**6.25**	**7.81**	**9.37**	**10.94**	**12.50**	**14.06**
3/8	6.428	5.50	1.8806	3.78	5.67	7.56	9.45	11.34	13.23	15.12	17.02
1/2	7.650	6.00	2.2500	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25
Round Bars.											
1/4	**0.167**	**0.785**	**0.0491**	**0.10**	**0.15**	**0.20**	**0.25**	**0.29**	**0.34**	**0.39**	**0.44**
5/16	0.261	0.982	0.0767	0.15	0.23	0.31	0.38	0.46	0.54	0.61	0.69
3/8	**0.375**	**1.178**	**0.1104**	**0.22**	**0.33**	**0.44**	**0.55**	**0.66**	**0.77**	**0.88**	**0.99**
7/16	0.511	1.374	0.1503	0.30	0.45	0.60	0.75	0.90	1.05	1.20	1.35
1/2	**0.667**	**1.571**	**0.1963**	**0.39**	**0.59**	**0.79**	**0.98**	**1.18**	**1.37**	**1.57**	**1.77**
9/16	0.845	1.767	0.2485	0.50	0.75	0.99	1.24	1.49	1.74	1.99	2.24
5/8	**1.043**	**1.964**	**0.3068**	**0.61**	**0.92**	**1.23**	**1.53**	**1.84**	**2.15**	**2.45**	**2.76**
11/16	1.262	2.160	0.3712	0.74	1.11	1.48	1.86	2.23	2.60	2.97	3.34
3/4	**1.502**	**2.356**	**0.4418**	**0.88**	**1.33**	**1.77**	**2.21**	**2.65**	**3.09**	**3.53**	**3.98**
13/16	1.763	2.553	0.5185	1.04	1.55	2.07	2.59	3.11	3.63	4.15	4.67
7/8	**2.044**	**2.749**	**0.6013**	**1.20**	**1.80**	**2.40**	**3.01**	**3.61**	**4.21**	**4.81**	**5.41**
15/16	2.347	2.945	0.6903	1.38	2.07	2.76	3.45	4.14	4.83	5.52	6.21
1	**2.570**	**3.142**	**0.7854**	**1.57**	**2.36**	**3.14**	**3.93**	**4.71**	**5.50**	**6.28**	**7.07**
1/8	3.379	3.534	0.9940	1.99	2.98	3.98	4.97	5.97	6.96	7.95	8.95
1/4	4.173	3.927	1.2272	2.45	3.68	4.91	6.14	7.36	8.59	9.82	11.04
3/8	5.049	4.320	1.4849	2.97	4.45	5.94	7.42	8.91	10.39	11.88	13.36
1/2	6.008	4.712	1.7671	3.53	5.30	7.07	8.84	10.60	12.37	14.14	15.90

Note—Bars shown in bold-face type are the standard sizes adopted by the Joint Conference of Representatives of Manufacturers, Distributers, and Users of Concrete Reinforcement Bars.

TABLE XXVII.—STEEL BARS. SPACING IN SLABS

Sectional area per foot of slab—square inches.

Diam. in Inches.	Weight per Ft. Pounds.	Area Sq. In.	Distances Apart of Bars—Inches.									
			2	2½	3	3½	4	4½	5	6	7	8
Square Bars												
$\frac{1}{4}$	0.212	0.0625	0.37	0.30	0.25	0.21	0.19	0.17	0.15	0.12	0.11	0.09
$\frac{5}{16}$	0.333	0.0977	0.59	0.47	0.39	0.33	0.29	0.26	0.23	0.20	0.17	0.15
$\frac{3}{8}$	0.478	0.1406	0.84	0.67	0.56	0.48	0.42	0.37	0.34	0.28	0.24	0.21
$\frac{7}{16}$	0.651	0.1914	1.15	0.92	0.77	0.66	0.57	0.51	0.46	0.38	0.33	0.29
$\frac{1}{2}$	**0.850**	**0.2500**	**1.50**	**1.20**	**1.00**	**0.86**	**0.75**	**0.67**	**0.60**	**0.50**	**0.43**	**0.37**
$\frac{9}{16}$	1.076	0.3164	1.90	1.52	1.37	1.08	0.95	0.84	0.76	0.63	0.54	0.47
$\frac{5}{8}$	1.328	0.3906	2.34	1.87	1.56	1.34	1.17	1.04	0.94	0.78	0.67	0.59
$\frac{11}{16}$	1.608	0.4727	2.84	2.27	1.89	1.62	1.42	1.26	1.13	0.95	0.81	0.71
$\frac{3}{4}$	1.913	0.5625	3.37	2.70	2.25	1.93	1.69	1.50	1.35	1.12	0.96	0.84
$\frac{7}{8}$	2.603	0.7656	....	3.67	3.06	2.62	2.30	2.04	1.84	1.53	1.31	1.15
1	**3.400**	**1.0000**	....	....	**4.00**	**3.43**	**3.00**	**2.67**	**2.40**	**2.00**	**1.71**	**1.50**
1$\frac{1}{8}$	**4.303**	**1.2656**	....	....	....	**4.34**	**3.80**	**3.37**	**3.04**	**2.53**	**2.17**	**1.89**
1$\frac{1}{4}$	**5.312**	**1.5625**	....	....	....	....	**4.69**	**4.17**	**3.75**	**3.12**	**2.68**	**2.34**
Round Bars												
$\frac{1}{4}$	**0.167**	**0.0491**	**0.29**	**0.25**	**0.20**	**0.17**	**0.15**	**0.13**	**0.12**	**0.10**	**0.08**	**0.07**
$\frac{5}{16}$	0.261	0.0767	0.46	0.36	0.31	0.26	0.23	0.20	0.18	0.15	0.13	0.12
$\frac{3}{8}$	**0.375**	**0.1104**	**0.66**	**0.53**	**0.44**	**0.38**	**0.33**	**0.29**	**0.26**	**0.22**	**0.19**	**0.17**
$\frac{7}{16}$	0.511	0.1503	0.90	0.72	0.60	0.51	0.45	0.40	0.36	0.30	0.26	0.23
$\frac{1}{2}$	**0.667**	**0.1963**	**1.18**	**0.94**	**0.78**	**0.67**	**0.59**	**0.52**	**0.47**	**0.39**	**0.34**	**0.29**
$\frac{9}{16}$	0.845	0.2385	1.49	1.19	0.99	0.85	0.75	0.66	0.60	0.50	0.43	0.37
$\frac{5}{8}$	**1.043**	**0.3068**	**1.84**	**1.47**	**1.23**	**1.05**	**0.92**	**0.82**	**0.74**	**0.61**	**0.53**	**0.46**
$\frac{11}{16}$	1.262	0.3912	2.23	1.78	1.48	1.27	1.11	0.99	0.89	0.74	0.64	0.56
$\frac{3}{4}$	**1.502**	**0.4418**	....	**2.12**	**1.77**	**1.51**	**1.32**	**1.18**	**1.06**	**0.88**	**0.76**	**0.66**
$\frac{7}{8}$	**2.044**	**0.6013**	....	....	**2.40**	**2.06**	**1.80**	**1.60**	**1.44**	**1.20**	**1.03**	**0.90**
1	**2.670**	**0.7854**	....	....	**3.14**	**2.69**	**2.36**	**2.09**	**1.88**	**1.57**	**1.35**	**1.18**
1$\frac{1}{8}$	3.379	0.9940	....	....	....	3.41	2.98	2.65	2.39	1.99	1.70	1.49
1$\frac{1}{4}$	4.173	1.2272	....	....	....	....	3.68	3.27	2.95	2.45	2.10	1.84

NOTE.—Standard bars are shown in bold-face type.

TABLE XXVIII.—TWELVE INCH WIDTH SLABS WITH TENSION REINFORCEMENT

$f_s = 16{,}000$. $f_c = 650$. $R = 108$. $p = .0077$. $n = 15$

$\frac{M}{1000}$ In.-Lb.	d Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	d Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	d Inches.	A_s Sq. In.
11.6	3.00	0.28	156.8	11.00	1.02	467.8	19.00	1.76
13.7	3.25	0.30	164.0	11.25	1.04	480.2	19.25	1.78
15.8	3.50	0.33	171.3	11.50	1.06	492.8	19.50	1.81
18.2	3.75	0.35	178.9	11.75	1.09	505.5	19.75	1.83
20.7	4.00	0.37	186.6	12.00	1.11	518.4	20.00	1.85
23.4	4.25	0.40	194.4	12.25	1.13	531.4	20.25	1.88
26.2	4.50	0.42	202.5	12.50	1.16	544.4	20.50	1.90
29.3	4.75	0.44	210.6	12.75	1.18	558.0	20.75	1.92
32.4	5.00	0.47	219.0	13.00	1.20	571.5	21.00	1.95
35.8	5.25	0.49	227.5	13.25	1.23	585.1	21.25	1.97
39.2	5.50	0.51	236.2	13.50	1.25	599.0	21.50	1.99
42.8	5.75	0.53	245.0	13.75	1.27	613.0	21.75	2.01
46.6	6.00	0.56	254.0	14.00	1.30	627.2	22.00	2.04
50.6	6.25	0.58	263.1	14.25	1.32	641.6	22.25	2.06
54.7	6.50	0.60	272.4	14.50	1.34	656.1	22.50	2.08
59.1	6.75	0.63	281.9	14.75	1.37	670.7	22.75	2.11
63.5	7.00	0.65	291.6	15.00	1.39	685.5	23.00	2.13
68.2	7.25	0.67	301.3	15.25	1.41	700.5	23.25	2.15
72.9	7.50	0.70	311.3	15.50	1.44	715.7	23.50	2.18
77.8	7.75	0.72	321.4	15.75	1.46	731.0	23.75	2.20
82.9	8.00	0.74	331.7	16.00	1.48	746.5	24.00	2.22
88.2	8.25	0.77	342.1	16.25	1.50	777.9	24.50	2.27
93.6	8.50	0.79	352.8	16.50	1.53	810.0	25.00	2.32
99.3	8.75	0.81	363.6	16.75	1.55	842.4	25.50	2.36
104.9	9.00	0.84	374.5	17.00	1.57	876.2	26.00	2.41
110.9	9.25	0.86	385.6	17.25	1.60	910.2	26.50	2.45
116.9	9.50	0.88	396.9	17.50	1.62	944.8	27.00	2.50
123.2	9.75	0.90	408.3	17.75	1.64	980.1	27.50	2.55
129.6	10.00	0.93	419.9	18.00	1.67	1016.0	28.00	2.59
136.1	10.25	0.95	431.6	18.25	1.69	1052.0	28.50	2.64
142.8	10.50	0.97	443.5	18.50	1.71	1090.0	29.00	2.69
149.8	10.75	0.99	455.6	18.75	1.74	1166.0	30.00	2.78

TABLE XXIX.—TWELVE INCH WIDTH CINDER CONCRETE SLABS WITH TENSION REINFORCEMENT

$f_s = 16{,}000$. $f_c = 350$. $R = 60$. $p = .00433$. $n = 30$

$\frac{M}{1000}$ In.-Lb.	d Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	d Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	d Inches.	A_s Sq. In.
M Kip In.	d Inches.	A_s Sq. In.	M Kip In.	d Inches.	A_s Sq. In.	M Kip In.	d Inches.	A_s Sq. In.
6.4	3.00	0.16	25.9	6.00	0.31	58.3	9.00	0.47
7.6	3.25	0.17	28.1	6.25	0.33	61.7	9.25	0.48
8.7	3.50	0.18	30.4	6.50	0.34	65.0	9.50	0.50
10.1	3.75	0.20	32.8	6.75	0.35	68.4	9.75	0.51
11.5	4.00	0.21	35.2	7.00	0.37	72.0	10.00	0.52
13.0	4.25	0.22	37.9	7.25	0.38	75.6	10.25	0.53
14.5	4.50	0.24	40.5	7.50	0.39	79.3	10.50	0.55
16.2	4.75	0.25	43.2	7.75	0.40	83.2	10.75	0.56
18.0	5.00	0.26	46.0	8.00	0.42	87.1	11.00	0.57
19.8	5.25	0.27	49.0	8.25	0.43	91.1	11.25	0.58
21.7	5.50	0.29	52.0	8.50	0.44	95.2	11.50	0.60
23.7	5.75	0.30	55.1	8.75	0.46	103.7	12.00	0.62

Example 7.—A reinforced concrete slab of indefinite width has a span of 16 feet c. to c. of bearings, and is to carry a load of 200 lb./ft.^{2} in addition to its own weight. The allowed unit stresses on the steel and concrete are 16,000 and 650 lb./in.^{2} respectively, and the value of n is 15. Find the depth of slab and the size and spacing of the reinforcing bars.

Assume a strip 1 foot wide with a total depth, d_1, of 12 inches, weighing 150 lb./ft.^{2} Then,

$$M = \frac{wl^2}{8} = \frac{350 \times 16 \times 16 \times 12}{8} = 134{,}000 \text{ in.-lb.}$$

Table XXVIII (p. 205) gives a value of $d = 10\frac{1}{4}$ in., and $A_s = 0.95$ in.^{2} Make the distance from the center of the steel to the lower surface of the slab $1\frac{3}{4}$ inches, and the value of d_1 will be 12 inches as assumed. Table XXVII (p. 204) shows that $\frac{3}{4}$-inch round bars spaced 5 inches on centers may be used.

TABLE XXX.—RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

$f_s = 16{,}000.\quad f_c = 650.\quad R = 108.\quad p = .0077.\quad n = 15$

$\frac{M}{1000}$ In.-Lb.	$b \times d$ Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	$b \times d$ Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	$b \times d$ Inches.	A_s Sq. In.
10.8	4×5	0.16	685.2	12×23	2.13	1,693	20×28	4.32
15.5	4×6	0.19	746.5	12×24	2.22	1,944	20×30	4.63
21.1	4×7	0.22				2,211	20×32	4.93
27.6	4×8	0.25	506.8	13×19	1.90	2,496	20×34	5.24
			561.6	13×20	2.00	2,800	20×36	5.55
26.4	5×7	0.27	619.2	13×21	2.10	3,119	20×38	5.86
34.5	5×8	0.31	679.5	13×22	2.20	3,556	20×40	6.17
43.7	5×9	0.35	742.6	13×23	2.30			
54.0	5×10	0.39	808.6	13×24	2.40	2,041	21×30	4.86
			949.0	13×26	2.61	2,322	21×32	5.18
41.4	6×8	0.37				2,621	21×34	5.50
52.5	6×9	0.42	604.8	14×20	2.16	2,939	21×36	5.83
64.8	6×10	0.46	666.8	14×21	2.27	3,275	21×38	6.15
78.4	6×11	0.51	731.8	14×22	2.37	3,629	21×40	6.47
93.3	6×12	0.56	800.0	14×23	2.48	4,000	21×42	6.80
			870.9	14×24	2.59			
75.6	7×10	0.54	1022.0	14×26	2.81	2,433	22×32	5.43
91.5	7×11	0.59	1185.0	14×28	3.02	2,747	22×34	5.77
108.8	7×12	0.65				3,078	22×36	6.11
127.7	7×13	0.70	714.4	15×21	2.43	3,430	22×38	6.44
148.1	7×14	0.75	783.6	15×22	2.55	3,802	22×40	6.78
			857.0	15×23	2.66	4,191	22×42	7.12
104.5	8×11	0.68	933.1	15×24	2.78	4,600	22×44	7.46
124.4	8×12	0.74	1095.0	15×26	3.01			
146.0	8×13	0.80	1270.0	15×28	3.24	2,996	24×34	6.29
169.3	8×14	0.86	1458.0	15×30	3.46	3,359	24×36	6.66
194.4	8×15	0.93				3,743	24×38	7.03
221.2	8×16	0.99	836.4	16×22	2.71	4,147	24×40	7.40
			914.1	16×23	2.84	4,572	24×42	7.77
164.2	9×13	0.90	995.3	16×24	2.96	5,018	24×44	8.14
190.5	9×14	0.97	1168.0	16×26	3.21	5,485	24×46	8.51
218.7	9×15	1.04	1354.0	16×28	3.45	5,972	24×48	8.88
248.8	9×16	1.11	1555.0	16×30	3.70			
280.9	9×17	1.18	1769.0	16×32	3.95	4,055	26×38	7.62
314.9	9×18	1.25				4,493	26×40	8.02
			971.2	17×23	3.02	4,952	26×42	8.41
211.7	10×14	1.08	1057.0	17×24	3.15	5,436	26×44	8.81
243.0	10×15	1.16	1241.0	17×26	3.41	5,941	26×46	9.21
276.4	10×16	1.24	1439.0	17×28	3.67	6,470	26×48	9.61
312.1	10×17	1.31	1652.0	17×30	3.93	7,020	26×50	10.01
349.9	10×18	1.39	1880.0	17×32	4.20	7,593	26×52	10.41
389.9	10×19	1.47	2122.0	17×34	4.46			
432.0	10×20	1.54				4,832	28×40	8.63
			1119.0	18×24	3.33	5,334	28×42	9.06
301.1	11×16	1.36	1314.0	18×26	3.61	5,853	28×44	9.49
343.3	11×17	1.44	1524.0	18×28	3.89	6,397	28×46	9.92
384.9	11×18	1.53	1750.0	18×30	4.16	6,967	28×48	10.35
428.8	11×19	1.61	1990.0	18×32	4.44	7,560	28×50	10.78
475.2	11×20	1.70	2247.0	18×34	4.72	8,176	28×52	11.21
523.9	11×21	1.78	2519.0	18×36	5.00	8,817	28×54	11.65
575.0	11×22	1.87				9,482	28×56	12.08
			1387.0	19×26	3.81			
374.5	12×17	1.57	1608.0	19×28	4.10	6,272	30×44	10.17
420.0	12×18	1.67	1847.0	19×30	4.40	6,855	30×46	10.63
467.8	12×19	1.76	2000.0	19×32	4.69	7,464	30×48	11.09
518.4	12×20	1.85	2362.0	19×34	4.98	8,760	30×52	12.02
571.5	12×21	1.95	2659.0	19×36	5.27	10,160	30×56	12.94
627.2	12×22	2.04	2963.0	19×38	5.57	11,664	30×60	13.86

118. Shearing Stresses.—The distribution of shearing stresses in the section of a reinforced concrete beam differs from that in a homogeneous beam. The concrete between the neutral axis and the steel is not supposed to carry tension and consequently the unit shear is constant over this area. Figure 53 represents a portion of a reinforced beam, the length s being very short, so that the shear V may be the same upon its two ends. Let C_1 and C_2 represent the compression in the concrete on the two sides, and T_1 and T_2 the corresponding tensions in the steel. The difference of tensions $T_1—T_2$ must be communicated to the concrete and carried as horizontal shear to the compression side of the beam.

The intensity of the horizontal shear at any point is equal to the

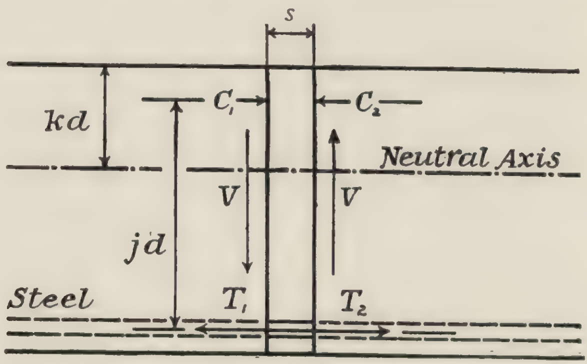


FIG. 53.—Shearing Stresses.

intensity of the vertical shear at the same point, as in any beam. If v is the shearing stress upon unit area and b the width of the beam, the total shear upon any horizontal section below the neutral axis is

$$vbs = T_1 - T_2.$$

For equilibrium of the forces acting upon the portion of the beam of length s, as shown in Fig. 53, $T_1 - T_2 = C_1 - C_2$, and the two couples are also equal, or $Vs = (T_1 - T_2)jd$. Equating these values of $T_1 - T_2$, and reducing, we find,

$$v = \frac{V}{bjd}. \qquad \qquad (10)$$

TABLE XXXI.—RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

$f_s=18{,}000$. $f_c=650$. $R=100$. $p=.0063$. $n=15$

$\frac{M}{1000}$ In.-Lb.	$b\times d$ Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	$b\times d$ Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	$b\times d$ Inches.	A_s Sq. In.
10.0	4×5	0.13	634.8	12×23	1.74	1,568	20×28	3.53
14.4	4×6	0.15	691.2	12×24	1.82	1,800	20×30	3.78
19.6	4×7	0.18				2,048	20×32	4.03
25.6	4×8	0.20	469.3	13×19	1.56	2,312	20×34	4.28
			520.0	13×20	1.64	2,592	20×36	4.54
24.5	5×7	0.22	573.3	13×21	1.72	2,888	20×38	4.78
32.0	5×8	0.25	629.2	13×22	1.80	3,200	20×40	5.04
40.5	5×9	0.28	687.7	13×23	1.88			
50.0	5×10	0.32	748.8	13×24	1.97	1,890	21×30	3.97
			878.8	13×26	2.13	2,150	21×32	4.23
38.4	6×8	0.30				2,427	21×34	4.50
48.6	6×9	0.34	560.0	14×20	1.76	2,721	21×36	4.76
60.0	6×10	0.38	617.4	14×21	1.85	3,032	21×38	5.03
72.6	6×11	0.42	677.6	14×22	1.94	3,360	21×40	5.29
86.4	6×12	0.45	740.6	14×23	2.03	3,704	21×42	5.56
			806.4	14×24	2.12			
70.0	7×10	0.44	946.4	14×26	2.29	2,252	22×32	4.44
84.7	7×11	0.49	1097.0	14×28	2.47	2,543	22×34	4.71
100.8	7×12	0.53				2,851	22×36	4.99
118.3	7×13	0.57	661.5	15×21	1.98	3,176	22×38	5.27
137.2	7×14	0.62	726.0	15×22	2.08	3,520	22×40	5.54
			793.5	15×23	2.17	3,880	22×42	5.82
96.8	8×11	0.55	864.0	15×24	2.27	4,259	22×44	6.10
115.2	8×12	0.61	1014.0	15×26	2.46			
135.2	8×13	0.66	1176.0	15×28	2.65	2,774	24×34	5.13
156.8	8×14	0.71	1350.0	15×30	2.84	3,110	24×36	5.44
180.0	8×15	0.76				3,465	24×38	5.74
204.8	8×16	0.81	774.4	16×22	2.22	3,840	24×40	6.04
			846.4	16×23	2.32	4,233	24×42	6.34
152.1	9×13	0.74	921.6	16×24	2.42	4,646	24×44	6.64
176.4	9×14	0.79	1081.0	16×26	2.62	5,078	24×46	6.95
202.5	9×15	0.85	1254.0	16×28	2.82	5,229	24×48	7.25
230.4	9×16	0.91	1440.0	16×30	3.02			
260.1	9×17	0.96	1638.0	16×32	3.23	3,754	26×38	6.23
291.6	9×18	1.02				4,160	26×40	6.56
			899.3	17×23	2.46	4,586	26×42	6.88
196.0	10×14	0.88	979.2	17×24	2.57	5,033	26×44	7.21
225.0	10×15	0.95	1149.0	17×26	2.78	5,501	26×46	7.54
256.0	10×16	1.01	1332.0	17×28	3.00	5,990	26×48	7.87
289.0	10×17	1.07	1530.0	17×30	3.21	6,500	26×50	8.19
324.0	10×18	1.13	1740.0	17×32	3.42	7,030	26×52	8.52
361.0	10×19	1.20	1965.0	17×34	3.64			
400.0	10×20	1.26				4,480	28×40	7.06
			1036.0	18×24	2.72	4,939	28×42	7.41
281.6	11×16	1.11	1216.0	18×26	2.95	5,420	28×44	7.76
317.9	11×17	1.18	1411.0	18×28	3.18	5,924	28×46	8.12
356.4	11×18	1.25	1620.0	18×30	3.40	6,451	28×48	8.47
397.1	11×19	1.32	1843.0	18×32	3.63	7,000	28×50	8.82
440.0	11×20	1.39	2080.0	18×34	3.86	7,571	28×52	9.18
485.1	11×21	1.46	2332.0	18×36	4.08	8,164	28×54	9.53
532.4	11×22	1.52				8,780	28×56	9.88
			1284.0	19×26	3.11			
346.8	12×17	1.29	1489.0	19×28	3.35	5,808	30×44	8.32
388.8	12×18	1.36	1710.0	19×30	3.59	6,348	30×46	8.70
433.2	12×19	1.44	1945.0	19×32	3.83	6,912	30×48	9.07
480.0	12×20	1.51	2196.0	19×34	4.07	8,112	30×52	9.83
529.2	12×21	1.59	2462.0	19×36	4.31	9,408	30×56	10.59
580.8	12×22	1.66	2743.0	19×38	4.55	10,800	30×60	11.34

TABLE XXXII.—RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

$f_s=16{,}000$. $f_c=800$. $R=147$. $p=.0107$. $n=15$

$\frac{M}{1000}$ In.-Lb.	$b\times d$ Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	$b\times d$ Inches.	A_s Sq. In.	$\frac{M}{1000}$ In.-Lb.	$b\times d$ Inches.	A_s Sq. In.
14.7	4×5	0.22	933.1	12×23	2.96	2,305	20×28	5.99
21.1	4×6	0.26	1016.0	12×24	3.08	2,646	20×30	6.42
28.8	4×7	0.30				3,011	20×32	6.85
37.6	4×8	0.35	690.0	13×19	2.65	3,399	20×34	7.28
			764.6	13×20	2.78	3,811	20×36	7.71
36.0	5×7	0.38	842.8	13×21	2.92	4,246	20×38	8.14
47.0	5×8	0.43	925.0	13×22	3.06	4,705	20×40	8.56
59.5	5×9	0.48	1011.0	13×23	3.20			
73.5	5×10	0.54	1100.0	13×24	3.34	2,778	21×30	6.74
			1292.0	13×26	3.62	3,160	21×32	7.19
56.4	6×8	0.52				3,568	21×34	7.64
71.4	6×9	0.58	823.3	14×20	2.92	4,000	21×36	8.09
88.2	6×10	0.64	907.7	14×21	3.06	4,458	21×38	8.54
106.7	6×11	0.71	996.2	14×22	3.21	4,940	21×40	8.99
127.0	6×12	0.78	1088.0	14×23	3.35	5,446	21×42	9.44
			1185.0	14×24	3.50			
102.9	7×10	0.75	1391.0	14×26	3.78	3,310	22×32	7.54
124.5	7×11	0.83	1613.0	14×28	4.08	3,738	22×34	8.00
148.1	7×12	0.90				4,191	22×36	8.48
173.9	7×13	0.98	972.4	15×21	3.37	4,669	22×38	8.95
201.7	7×14	1.05	1067.0	15×22	3.53	5,175	22×40	9.42
			1165.0	15×23	3.69	5,704	22×42	9.89
142.3	8×11	0.95	1270.0	15×24	3.85	6,262	22×44	10.36
169.3	8×12	1.03	1490.0	15×26	4.18			
198.7	8×13	1.12	1729.0	15×28	4.50	4,078	24×34	8.73
230.5	8×14	1.20	1985.0	15×30	4.82	4,572	24×36	9.25
264.6	8×15	1.29				5,094	24×38	9.76
301.1	8×16	1.37	1138.0	16×22	3.77	5,648	24×40	10.28
			1244.0	16×23	3.94	6,224	24×42	10.79
223.6	9×13	1.25	1354.0	16×24	4.11	6,830	24×44	11.30
259.2	9×14	1.35	1589.0	16×26	4.28	7,466	24×46	11.82
297.7	9×15	1.44	1844.0	16×28	4.45	7,688	24×48	12.33
338.7	9×16	1.54	2117.0	16×30	4.62			
382.4	9×17	1.64	2408.0	16×32	4.80	5,520	26×38	10.57
428.7	9×18	1.74				6,116	26×40	11.13
			1322.0	17×23	5.46	6,741	26×42	11.69
288.1	10×14	1.50	1438.0	17×24	5.64	7,400	26×44	12.24
330.8	10×15	1.61	1689.0	17×26	5.82	8,086	26×46	12.80
376.3	10×16	1.72	1958.0	17×28	6.00	8,806	26×48	13.36
424.8	10×17	1.82	2249.0	17×30	6.19	9.555	26×50	13.91
476.3	10×18	1.93	2558.0	17×32	6.37	10,330	26×52	14.47
530.7	10×19	2.04	2889.0	17×34	6.55			
588.0	10×20	2.14				6,585	28×40	11.95
			1523.0	18×24	4.63	7,260	28×42	12.59
414.0	11×16	1.89	1786.0	18×26	5.01	7,967	28×44	13.19
467.3	11×17	2.00	2074.0	18×28	5.40	8,709	28×46	13.78
524.0	11×18	2.12	2382.0	18×30	5.78	9,482	28×48	14.38
583.8	11×19	2.24	2710.0	18×32	6.17	10,290	28×50	14.98
646.9	11×20	2 36	3058.0	18×34	6.55	11,130	28×52	15.58
713.1	11×21	2.47	3428.0	18×36	6.94	12,000	28×54	16.18
782.6	11×22	2.59				12,900	28×56	16.78
			1887.0	19×26	5.29			
509.9	12×17	2.19	2189.0	19×28	5.70	7,479	30×44	14.13
571.9	12×18	2.31	2514.0	19×30	6.10	9,328	30×46	14.77
636.9	12×19	2.44	2860.0	19×32	6.51	10,160	30×48	15.41
705.7	12×20	2.57	3228.0	19×34	6.92	11,920	30×52	16.70
778.0	12×21	2.70	3620.0	19×36	7.32	13,830	30×56	17.98
853.2	12×22	2.83	4033.0	19×38	7.73	15,870	30×60	19.26

TABLE XXXIII.—RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

$f_s = 18{,}000.$ $f_c = 800.$ $R = 139.$ $p = .0089.$ $n = 15$

$\frac{M}{1000}$ In.-Lb.	$b \times d$ Inches.	A_s Sq. In.
13.9	4×5	0.19
20.0	4×6	0.22
27.1	4×7	0.26
35.5	4×8	0.29
33.9	5×7	0.31
44.4	5×8	0.35
56.2	5×9	0.40
69.5	5×10	0.44
53.3	6×8	0.43
67.6	6×9	0.48
83.4	6×10	0.53
100.8	6×11	0.59
120.1	6×12	0.64
97.3	7×10	0.62
117.7	7×11	0.69
140.0	7×12	0.75
164.3	7×13	0.81
190.6	7×14	0.87
134.5	8×11	0.78
160.1	8×12	0.85
187.9	8×13	0.93
217.9	8×14	1.00
250.2	8×15	1.07
284.7	8×16	1.14
211.3	9×13	1.04
245.2	9×14	1.12
281.5	9×15	1.20
320.2	9×16	1.28
361.6	9×17	1.36
405.3	9×18	1.44
272.5	10×14	1.25
312.7	10×15	1.34
355.7	10×16	1.42
401.7	10×17	1.51
450.4	10×18	1.60
501.8	10×19	1.69
556.0	10×20	1.78
391.4	11×16	1.57
441.8	11×17	1.66
495.4	11×18	1.76
551.6	11×19	1.86
611.6	11×20	1.96
674.2	11×21	2.06
740.0	11×22	2.15
482.0	12×17	1.72
540.6	12×18	1.82
602.2	12×19	1.93
667.2	12×20	2.03
735.5	12×21	2.14
807.4	12×22	2.24

$\frac{M}{1000}$ In.-Lb.	$b \times d$ Inches.	A_s Sq. In.
882.0	12×23	2.35
961.0	12×24	2.46
652.3	13×19	2.20
722.9	13×20	2.31
797.0	13×21	2.43
874.5	13×22	2.55
955.9	13×23	2.66
1040.0	13×24	2.78
1221.0	13×26	3.01
778.4	14×20	2.48
858.9	14×21	2.60
942.6	14×22	2.72
1029.0	14×23	2.83
1121.0	14×24	2.95
1316.0	14×26	3.24
1526.0	14×28	3.49
919.5	15×21	2.80
1009.0	15×22	2.94
1103.0	15×23	3.07
1201.0	15×24	3.20
1409.0	15×26	3.47
1635.0	15×28	3.74
1876.0	15×30	4.00
1076.0	16×22	3.13
1176.0	16×23	3.28
1281.0	16×24	3.42
1503.0	16×26	3.70
1743.0	16×28	3.99
2001.0	16×30	4.27
2277.0	16×32	4.56
1250.0	17×23	3.49
1361.0	17×24	3.64
1597.0	17×26	3.93
1852.0	17×28	4.24
2126.0	17×30	4.54
2415.0	17×32	4.84
2732.0	17×34	5.14
1441.0	18×24	3.84
1691.0	18×26	4.17
1961.0	18×28	4.49
2251.0	18×30	4.81
2562.0	18×32	5.13
2892.0	18×34	5.45
3242.0	18×36	5.77
1785.0	19×26	4.40
2071.0	19×28	4.73
2377.0	19×30	5.07
2704.0	19×32	5.41
3053.0	19×34	5.75
3423.0	19×36	6.09
3814.0	19×38	6.43

$\frac{M}{1000}$ In.-Lb.	$b \times d$ Inches.	A_s Sq. In.
2,180	20×28	5.98
2,502	20×30	5.34
2,846	20×32	5.70
3,214	20×34	6.05
3,603	20×36	6.41
4,015	20×38	6.76
4,448	20×40	7.12
2,627	21×30	5.61
2,989	21×32	5.98
3,375	21×34	6.35
3,784	21×36	6.73
4,216	21×38	7.10
4,671	21×40	7.48
5,149	21×42	7.85
2,502	22×30	5.87
3,131	22×32	6.27
3,535	22×34	6.66
3,963	22×36	7.05
4,416	22×38	7.44
4,893	22×40	7.83
5,400	22×42	8.22
5,920	22×44	8.62
3,853	24×34	7.26
4,324	24×36	7.69
4,818	24×38	8.12
5,338	24×40	8.54
5,886	24×42	8.97
6,460	24×44	9.40
7,059	24×46	9.83
7,687	24×48	10.25
5,220	26×38	8.79
5,783	26×40	9.26
6,374	26×42	9.72
6,998	26×44	10.18
7,646	26×46	10.64
8,328	26×48	11.11
9,035	26×50	11.57
9,774	26×52	12.03
6,228	28×40	9.97
6,868	28×42	10.47
7,534	28×44	10.95
8,235	28×46	11.47
8,968	28×48	11.96
9,731	28×50	12.46
10,520	28×52	12.96
11,350	28×54	13.46
12,200	28×56	13.96
8,074	30×44	11.75
8,824	30×46	12.29
9,609	30×48	12.82
11,270	30×52	13.89
13,070	30×56	14.96
15,010	30×60	16.02

In designing reinforced concrete beams, it is usual to adopt a limiting value for v and make the section of the beam large enough so that the safe value of v as given by the above formula shall not be exceeded. The Joint Committee on Concrete recommends that the maximum value of v shall not exceed 6 per cent of the ultimate compressive strength. This gives $v = 120$ lb./in.^{2} as a safe value for ordinary concrete as commonly used in structural work (with crushing strength of about 2000 lb./in.^{2} at thirty days).

Rectangular beams, reinforced for tension only, are usually sufficiently strong to resist direct shearing stresses when properly designed for flexural stresses. The areas of such beams are not affected by providing for shear, although they may sometimes need reinforcement against diagonal tension.

119. Diagonal Tension.—The intensity of the horizontal shear at any point in a beam is equal to the intensity of the vertical shear at the same point. In Fig. 54 let $ABCD$ be an extremely small prism

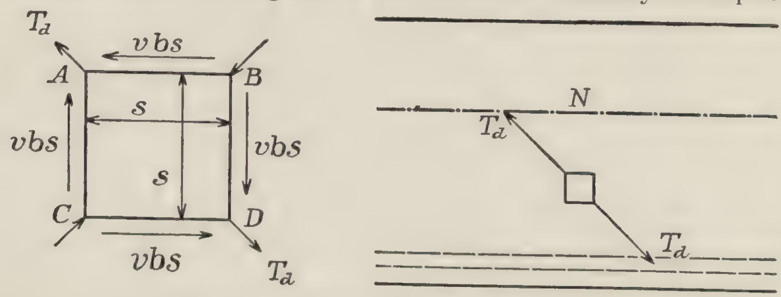


Fig. 54.—Diagonal Tension due to Shear.

in a reinforced beam, the vertical and horizontal dimensions of which, parallel to the side of the beam, are represented by s, and the thickness normal to the side by b. If v represent the unit shearing stress, the shear acting upon each of the four sides of the prism is vsb. If the two forces meeting at A and the two meeting at C be combined into resultants, T_d, there will result two equal and opposite forces producing tension in a diagonal direction upon the prism. The value of this tension is $T_d = \frac{2vbs}{\sqrt{2}}$, and it is distributed over an area, $bs/\cos 45°$. The unit tension due to shear is

$$\frac{T_d}{bs\sqrt{2}} = \frac{2vbs}{\sqrt{2}} \times \frac{1}{bs\sqrt{2}} = v.$$

The unit diagonal tension due to shear is therefore equal to the unit shear and acts at an angle of 45° with the axis of the beam.

As is readily seen from Fig. 54, diagonal compression equal to the diagonal tension exists in a direction at right angles to the tension. The diagonal compression is unimportant and need not be considered in beam design, as these stresses are always small in comparison with the compressive resistance of the concrete. It is, however, commonly necessary to reinforce concrete beams to prevent diagonal tension cracks, as failures of beams frequently occur from this cause.

In a homogeneous beam, the maximum tension at any point on the tension side of the neutral axis is the resultant obtained by combining the diagonal tension due to shear with the horizontal tension due to moment at the same point. In a reinforced concrete beam, the steel is supposed to carry all of the horizontal tension, and the concrete none. Some horizontal tension will necessarily be carried by the concrete, but, if sufficient horizontal reinforcement be used, the reinforcement for diagonal tension need provide only for tensions due to shear.

Figure 55 shows the form of failure likely to occur from diagonal tension, where horizontal reinforcement only is used. The diagonal

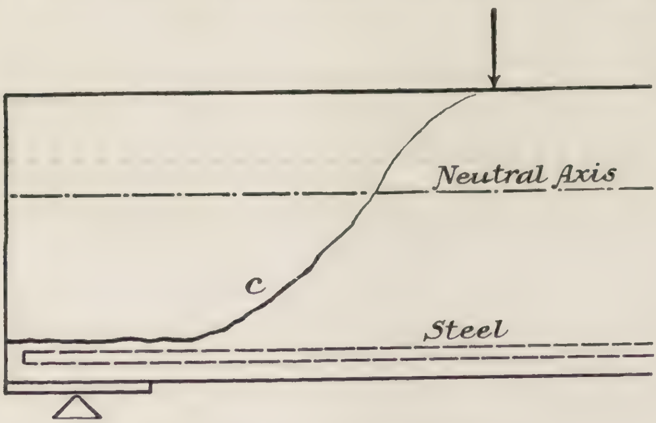


Fig. 55.—Diagonal Tension Failure.

tension at c becomes greater than the tensile strength of the concrete and the concrete cracks. A horizontal crack above the steel then follows, which separates the concrete from the steel and causes failure.

The safe resistance of concrete, without reinforcement, to diagonal tension is stated by the Joint Committee to be about one-third of the safe resistance to direct shear, or about 2 per cent of the ultimate compressive strength. For ordinary concrete, breaking at

2000 lb./in.^{2} when twenty-eight days old, reinforcement against diagonal tension is necessary when v is greater than 40 lb./in.^{2}

120. Web Reinforcement.—For the design of web reinforcement the Joint Committee in its report of October, 1924, makes the following recommendations:

122. *Variation of Shear in Beams with Uniform Load.*—For purpose of design of beams carrying uniform loads, not less than one-fourth ($\frac{1}{4}$) of the total shearing resistance required at either end of the span shall be provided at the section where the computed shearing stress is zero; from that section to the ends of the span the required shearing resistance shall be assumed to vary uniformly.

123. *Width of Beams in Shear Computations.*—The shearing unit stress shall be computed on the minimum width of rectangular beams and on the minimum thickness of the web in beams of I- or T-section.

124. *Shear in Beam and Tile Construction.*—The width of the effective section for shear as governing diagonal tension shall be assumed as the thickness of the concrete web plus one-half ($\frac{1}{2}$) the thickness of the vertical of the concrete or clay tile in contact with the beam.

125. *Types and Spacing of Web Reinforcement.*—Web reinforcement may consist of:

(*a*) Vertical stirrups or web reinforcing bars.

(*b*) Inclined stirrups or web reinforcing bars forming an angle of 30° or more with the longitudinal bars.

(*c*) Longitudinal bars bent up at an angle of 15° or more with the direction of the longitudinal bars.

Stirrups or bent-up bars which are not anchored at both ends, according to the provisions of Section 141, shall not be considered effective as web reinforcement. When the shearing stress is not greater than $0.06f'_c$, the distance, s, measured in the direction of the axis of the beam between two successive stirrups, or between two successive points of bending up of bars, or from the point of bending up of a bar to the edge of the support, shall not be greater than

$$s = \frac{45d}{\alpha + 10}, \quad \ldots\ldots\ldots\ldots \quad (30)^*$$

in which the angle, α, is in degrees.

When the shearing stress is greater than $0.06f_c'$, the distance, s, shall not be greater than two-thirds ($\frac{2}{3}$) of the values given by Formula (30).

127. *Beams without Special Anchorage of Longitudinal Reinforcement.*—The shearing unit stress computed by Formula (29)

$$\left(v = \frac{V}{bjd}\right)$$

in beams in which the longitudinal reinforcement is without special anchorage shall not exceed the values given by Formulas (31) and (32) and in no case shall it exceed $0.06f'_c$.

When α is between 45° and 90°,

$$v = 0.02f_c' + \frac{f_v A_v}{bs \sin \alpha} \quad \ldots\ldots\ldots\ldots \quad (31)^*$$

* Formulae marked with an asterisk are Joint Committee formula numbers.

When is less than 45°,

$$v = 0.02f_c' + \frac{f_v A_v}{bs}(\sin\alpha + \cos\alpha). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)^*$$

(In which f_v = tensile unit stress in web reinforcement.)

128. *Beams with Special Anchorage of Longitudinal Reinforcement.*—The shearing unit stress computed by Formula (29) in beams in which longitudinal reinforcement is anchored by means of hooked ends or otherwise, as specified by Section 140, shall not exceed the value given by Formulas (31) and (32), when $0.03f_c'$ is substituted for $0.02f_c'$ in those formulas; in no case shall the shearing unit stress exceed $0.12f_c'$.

129. *Beams with Bars Bent up at a Single Point.*—Where the web reinforcement consists of bars bent up at a single point, the point of bending shall be a distance, s, from the edge of the support, not greater than that given in Section 125 and the value of the quantity,

$$\frac{f_v' A_v}{bs}(\sin\alpha + \cos\alpha),$$

used in the design shall not exceed 75 lb. per sq. in.

130.—*Combined Web Reinforcement.*—When two or more types of web reinforcement are used in conjunction, the total shearing resistance of the beam shall be assumed as the sum of the shearing resistances computed for the various types separately. In such computations the shearing resistance of the concrete [the term, $0.02f'_c$ or $0.03f'_c$ in Formulas (31) and (32)] shall be included only once. In no case shall the maximum shearing stresses be greater than the limiting values given in Sections 127 and 128.

132. *Shear and Diagonal Tension in Footings.*—The shearing stress shall be taken as not less than that computed by Formula (29) *

$$\left(v = \frac{V}{bjd}\right).$$

The stress on the critical section shall not exceed $0.02f_c'$ for footings with straight reinforcement bars, nor $0.03f_c'$ for footings in which the reinforcement bars are anchored at both ends by adequate hooks or otherwise as specified in Section 140.

140. *Special Anchorage Requirements.*—Where increased shearing stresses are used as provided in Sections 128 and 132, or increased bond stresses as provided in Section 137 are necessary, special anchorage of all reinforcement in addition to that required in Section 139 shall be provided as follows:

(*a*) In continuous and restrained beams, anchorage beyond points of inflection of one-third ($\frac{1}{3}$) the area of the negative reinforcement and beyond the face of the support of one-third ($\frac{1}{3}$) the area of the positive reinforcement, shall be provided to develop one-third ($\frac{1}{3}$) of the maximum working stress in tension, with bond stresses not greater than those specified in Section 136.

141. *Anchorage of Web Reinforcement.*—Web bars shall be anchored at both ends by:

(*a*) Providing continuity with the longitudinal reinforcement; or
(*b*) Bending around the longitudinal bar; or
(*c*) A semicircular hook which has a radius not less than four (4) times the diameter of the web bar.

* Joint Committee formula numbers.

Stirrup anchorage shall be so provided in the compression and the tension regions of a beam as to permit the development of safe working tensile stress in the stirrup at a point $0.3d$ from either face. (Generally, a properly anchored stirrup the diameter of which does not exceed one-fiftieth ($\frac{1}{50}$) of the depth of the beam will meet these requirements.)

The end anchorage of a web member not in bearing on the longitudinal reinforcement shall be such as to engage an amount of concrete sufficient to prevent the bar from pulling out. In all cases the stirrups shall be carried as close to the upper and lower surfaces as fireproofing requirements will permit.

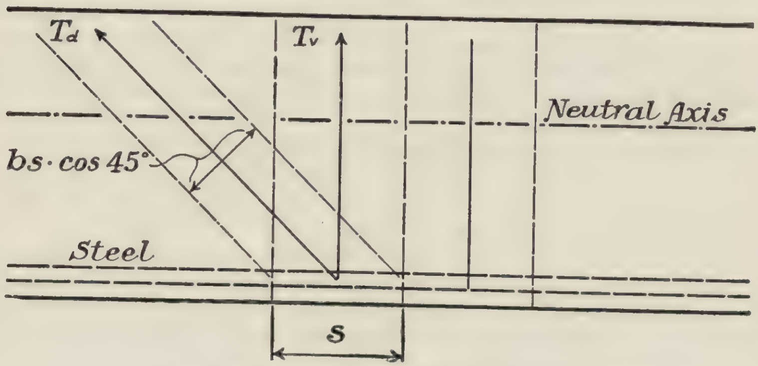


FIG. 56.—Vertical Stirrup Reinforcement.

Vertical Stirrups.—Figure 56 shows a beam reinforced for diagonal tension by the use of vertical stirrups.

Let s = length of beam to be reinforced by one stirrup;
V_d = Total vertical shear in section, s;
v = unit shearing stress;
T_d = Total diagonal tension in distance s;
T_v = Total vertical tension in stirrup.

The average unit shear is, $v=\frac{V_d}{bjd}$. This is also the unit diagonal tension due to shear, acting at an angle of 45° with the horizontal, and from formula, $T_d=\frac{2vbs}{\sqrt{2}}$, on p. 212, the total tension is,

$$T_d=2vbs \cos 45° = 2\frac{V_d s \cos 45°}{jd}. \qquad (11)$$

The vertical component of this carried by the stirrup is

$$T_v=2vbs \cos^2 45° = vbs = \frac{V_d s}{jd}. \qquad (12)$$

The total area of stirrup required to carry this stress is

$$A_v = \frac{vbs}{f_v} = \frac{V_d s}{f_v jd} \quad \text{and} \quad s = \frac{A_v f_v}{vb} = \frac{A_v f_v jd}{V_d}. \quad . \quad . \quad . \quad (13)$$

In designing stirrup reinforcement, the spaces s may be assumed and the required area A_v of stirrups computed, or the spacing may be determined for stirrups of given area. The value of v to be used should be the average value for the space s.

In order to avoid danger of cracks between stirrups, the spaces s should not exceed forty-five hundredths of the effective depth of beam, $0.45d$. The shear is a maximum at the support, and the first space should be measured from the middle of the bearing on the support.

Diagonal tension reinforcement is needed only in the portion of the beam in which the shear exceeds the allowable unit shear for plain

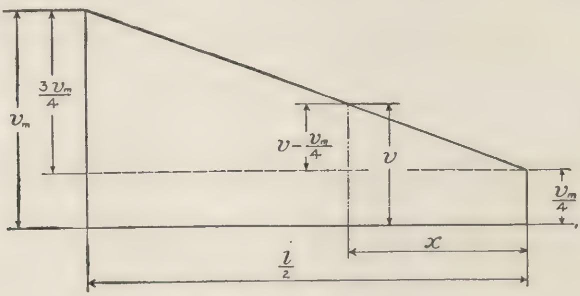


Fig. 57.—Distribution of Shear.

concrete (where v is greater than 2 per cent of the ultimate compressive strength of the concrete).

In a uniformly loaded beam, the shear is zero at the middle of the beam. The Joint Committee recommends for beams with uniform loads, that not less than one-fourth ($\frac{1}{4}$) of the total shearing resistance required at either end of the span shall be provided at the section where the computed shearing stress is zero, and that from that section to the ends of the span, the required shearing resistance shall be assumed to vary uniformly.

Reinforcement for diagonal tension is needed from the point where v equals the allowable shear for unreinforced concrete to the end of the beam. For ordinary concrete, the allowable unit shear is 40 lb./in.^{2}

Figure 57 shows that if v_m is the maximum unit shear at the support and l is the span of the beam, the distance from the middle of the beam to the section where the unit stress, v, is 40 lb./in.^{2} is,

$$x=\frac{(160-v_m)l}{6v_m}. \qquad (14)$$

Diagonal Reinforcement.—Figure 58 represents a beam reinforced for diagonal tension by bars inclined at 45° with the horizontal. As before, if the unit shear is more than 2 per cent of the ultimate compressive strength of the concrete, the beam needs reinforcement against diagonal tension.

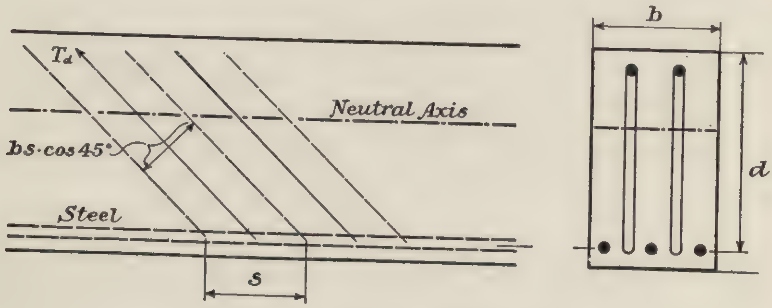


FIG. 58.—Diagonal Reinforcement.

Let s be the length of beam for which the steel is to carry the diagonal tension, and T_d the total tension in the steel.

The unit diagonal tension on the concrete is $v=\frac{V_d}{bjd}$, and $bs \cos 45°$ is the section normal to the tension over which this unit diagonal tension is distributed. The total tension to be carried by the diagonal steel where it crosses the neutral plane is

$$T_d=2vbs \cos 45°=\frac{2V_ds \cos 45°}{jd}=\frac{V_ds\sqrt{2}}{jd}. \qquad (15)$$

The area of steel required is

$$A_d=\frac{2vbs \cos 45°}{f_v}=\frac{2V_ds}{f_vjd\sqrt{2}}=\frac{V_ds\sqrt{2}}{f_vjd},$$

and

$$s=\frac{A_df_v}{vb\sqrt{2}}. \qquad (16)$$

The limits within which reinforcement is necessary, and its proper spacing, may be determined in the same manner as for vertical stir-

rups. Where diagonal reinforcement is used, the spacing should not exceed $s=\frac{45d}{\alpha+10}$ (in which α is the angle in degrees) as required by the Joint Committee.

Bending up Horizontal Steel.—Diagonal reinforcement is commonly provided by bending up a portion of the horizontal steel near the supports where it is not needed for horizontal tension. In a simple beam uniformly loaded, the moment diagram is a parabola (see Fig. 59) and the diminution of the moment from the middle toward the ends is proportional to the square of the distance from the middle of the beam. Thus if M is the moment at the middle, M_x,

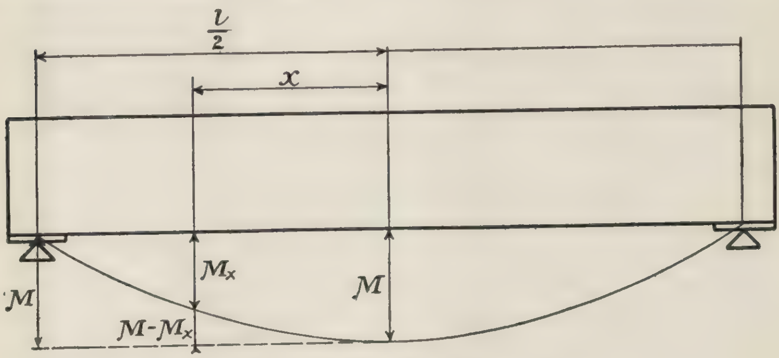


Fig. 59.—Distribution of Moment.

the moment at a point distant x from the middle, and $l/2$, the distance from the middle of the beam to the support,

$$\frac{M-M_x}{M}=\frac{x^2}{(l/2)^2},$$

or

$$M-M_x=\frac{Mx^2}{(l/2)^2}. \qquad (17)$$

The area of horizontal steel needed at any point varies directly with the moment at the point. If A is the area required at the middle and A_x the area needed at any point distant x from the middle,

$$A-A_x=\frac{Ax^2}{(l/2)^2}, \qquad (18)$$

and

$$x=\frac{l}{2}\sqrt{\frac{A-A_x}{A}}. \qquad (19)$$

$A-A_x$ is the area of steel that it is allowable to turn up at distance x from the middle of the beam.

121. Shear and Vertical Stirrup Diagrams.—In some cases the placing of stirrups to resist diagonal tension is inconvenient or the fastening of them in position is expensive. In some instances, therefore, it may be more economical to increase the effective depth, d,

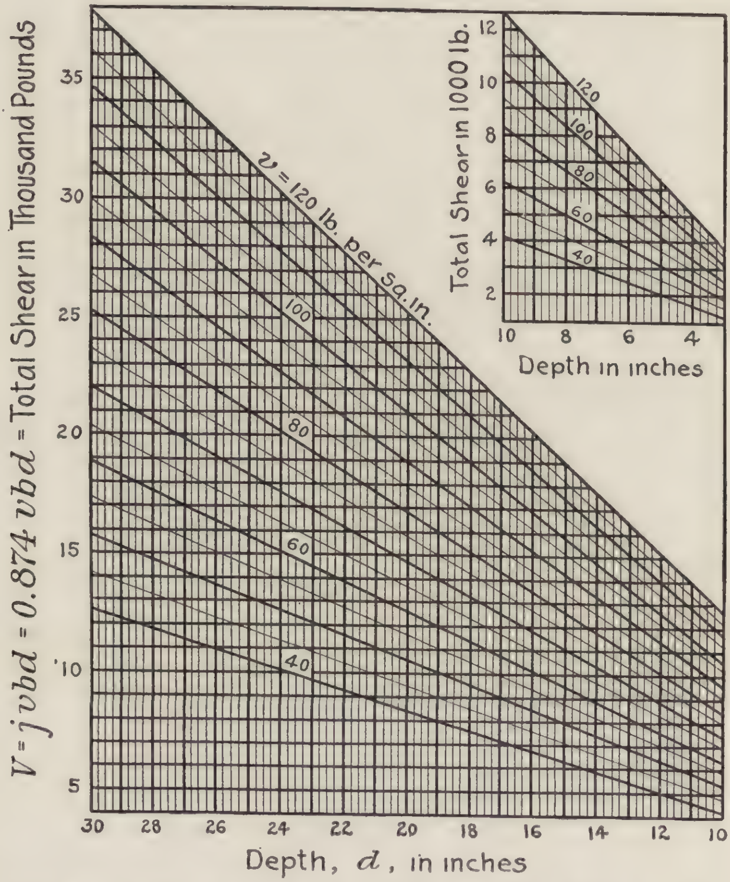


Diagram IX.—Shear for Slabs and Beams of 12 Inches Width for Depths Varying from 3 to 30 Inches.

thus reducing the unit shearing stress, v, and obviating the need for diagonal tension reinforcement.

Diagrams IX to XIII, for shear and vertical stirrups, are based on 2000 lb. concrete, and values of $f_s = f_v = 16000$ lb. and $n = 15$.

Diagram IX gives values of v for slabs and beams having a width of 12 inches and depths varying from 3 to 30 inches. This diagram will be found useful in retaining wall design.

Diagram X gives values of v for rectangular beams with cross-sectional areas varying from 20 to 100 in.^{2}, and Diagram XI gives values of v for similar beams with cross-sectional areas from 100 to 1000 in.^{2}

Diagram XII gives the size and spacing of vertical U-stirrups

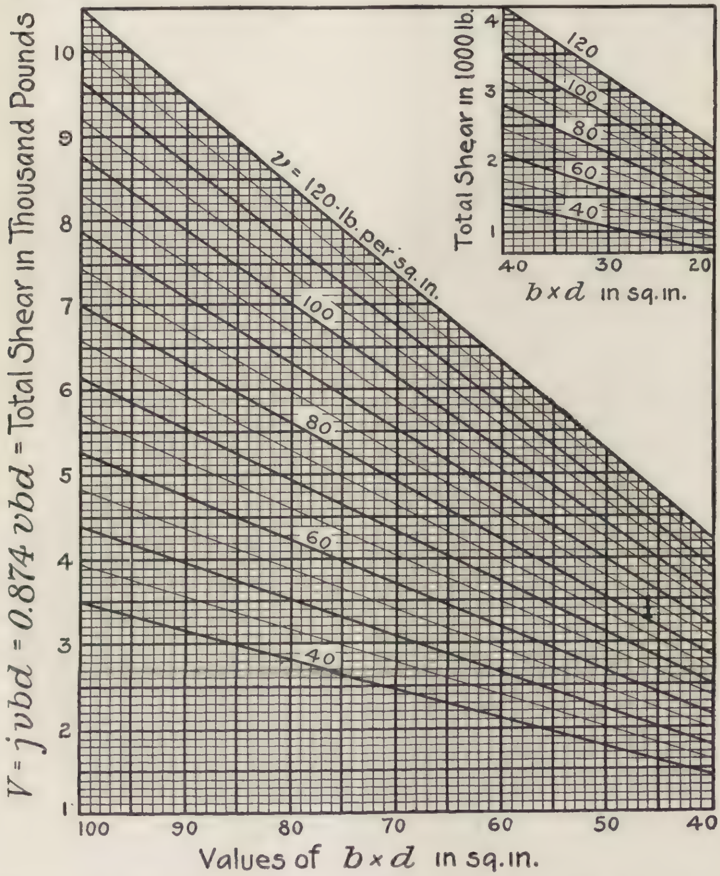


DIAGRAM X.—Shear for Beams with Cross-sectional Areas from 20 to 100 Square Inches.

for values of V_d up to 70,000 pounds, and Diagram XIII gives the size and spacing of vertical double U-stirrups for values of V_d up to 130,000 pounds.

122. Bond Resistance and Lateral Spacing.—The stress carried by the steel in a reinforced concrete beam is transmitted to the steel through the bond existing between the concrete and steel.

Horizontal Tension Bars.—The amount of stress that may be transmitted to the horizontal steel at any cross-section of the beam is equal to the horizontal shear at the section.

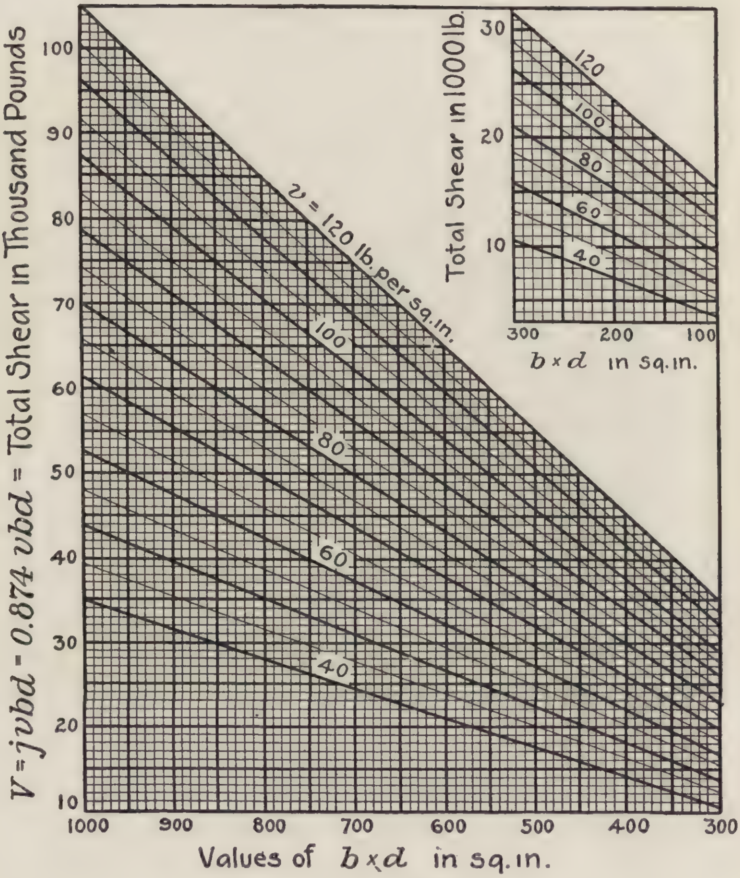


DIAGRAM XI.—Shear for Beams with Cross-sectional Areas from 100 to 1000 Square Inches.

Let $v = \frac{V}{bjd}$ = the unit horizontal shear in the concrete at any section;

u = the unit bond stress between the steel and concrete;

Σo = sum of perimeters of bars in one set;

b = width of beam.

The total horizontal shear for unit length of beam, $bv = \frac{V}{jd}$; then

$$u = \frac{bv}{\Sigma o} = \frac{V}{\Sigma ojd}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and

$$\Sigma o = \frac{bv}{u} = \frac{V}{ujd}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

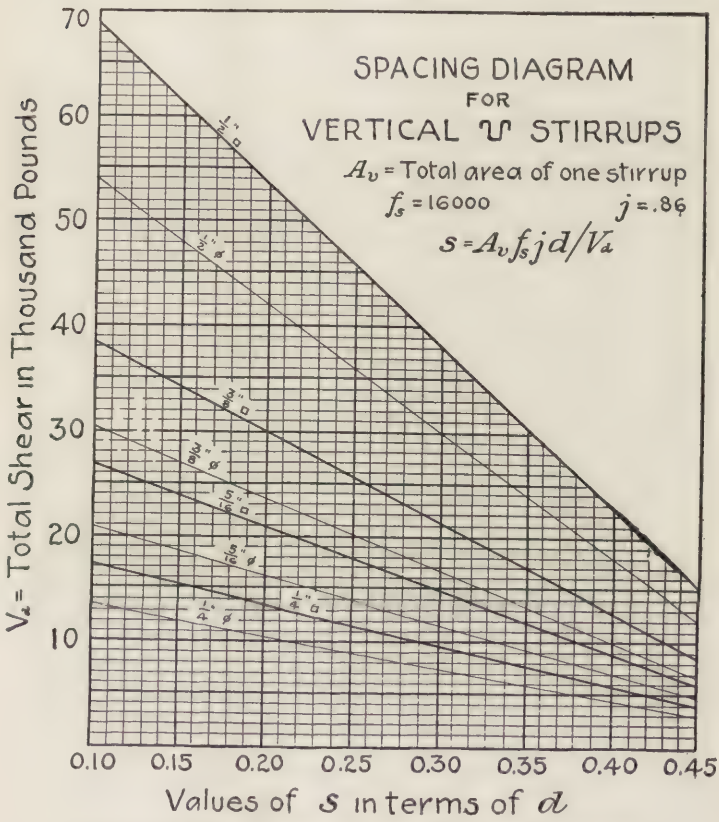


Diagram XII.—Size and Spacing of Vertical U-Stirrups.

If u does not exceed the safe unit bond stress between the steel and concrete at the section of maximum shear, the horizontal shear may be communicated to the steel without danger of the bars slipping. The Joint Committee recommends that the safe bond stress between concrete and plain reinforcing bars be limited to 4 per cent of the compressive strength of the concrete, and for good deformed bars not to exceed 5 per cent. For ordinary concrete (compression

2000 lb./in.²) this would give a value for plain bars, $u = 80$ lb./in.², and for the best deformed bars, $u = 100$ lb./in.²

The Joint Committee recommends, " For simple beams or freely supported ends of continuous beams, the critical section for bond shall be assumed to be at the face of the support. For continuous or restrained members, the critical section for bond for the positive

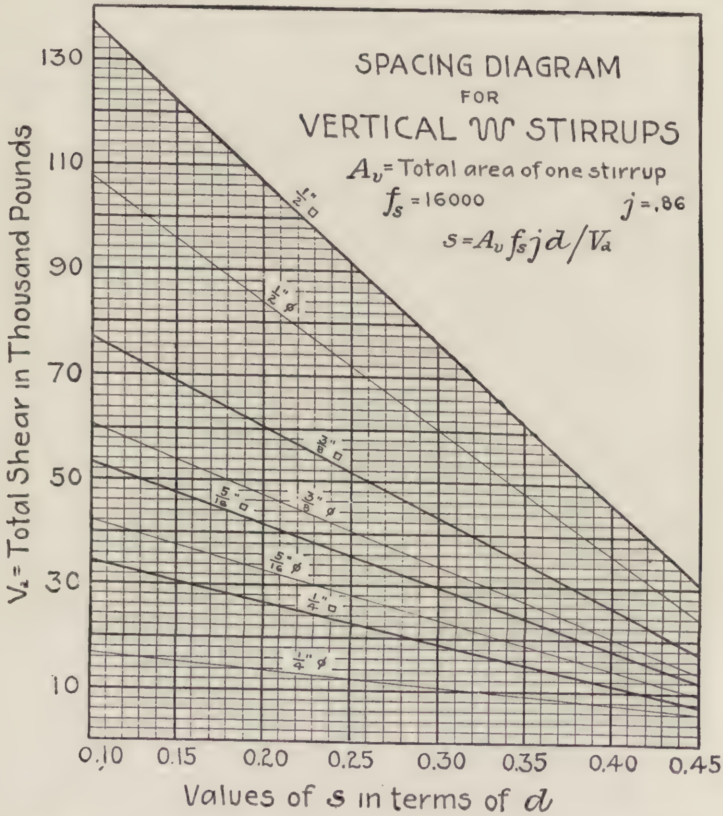


Diagram XIII.—Size and Spacing of Vertical Double U-Stirrups.

reinforcement shall be assumed to be at the point of inflection; that for the negative reinforcement shall be assumed to be at the face of the support, and at the point of inflection."

" Bent up longitudinal bars, which, at the critical section, are within a distance, $d = .3$, from horizontal reinforcement under consideration, may be included with the straight bars in computing $= \Sigma o$.

" The permissible bond stress for . . . . members in which

reinforcement is placed in more than one direction shall not exceed 75 per cent of the values prescribed for ordinary anchorage."

"In continuous, restrained or cantilever beams, anchorage of the tensile negative reinforcement beyond the face of the support shall provide for the full maximum tension with bond stresses not greater than those prescribed for ordinary anchorage. Such anchorage shall provide a length of bar not less than the depth of the beam. In the case of end supports which have a width less than three-fourths ($\frac{3}{4}$) of the depth of the beam, the bars shall be bent down toward the support a distance not less than the effective depth of the beam. The portion of the bar so bent down shall be as near to the end of the beam as the protective covering permits. In continuous or restrained beams, negative reinforcement shall be carried to or beyond the point of inflection. Not less than one-fourth ($\frac{1}{4}$) of the area of the positive reinforcement shall extend into the support to provide an embedment of ten (10) or more bar diameters.

"In simple beams, or freely supported spans of continuous beams, at least one-fourth ($\frac{1}{4}$) of the area of the tensile reinforcement shall extend along the tension side of the beam and beyond the face of the support to provide an embedment of ten (10) or more bar diameters.

In selecting sizes of bars for horizontal tension steel, care should be taken that the bars are not too large to give sufficient surface area to provide properly for bond stress. Thus, suppose a beam, in which $b=6$ inches, $d=10$ inches, and $j=0.86$, requires for tension steel, $A_s=0.60$ in.^{2} If the maximum value of shear $V=3200$ pounds and allowable unit bond stress $u=80$ lb./in.^{2}, the required surface area of steel per inch of length,

$$\Sigma o = \frac{V}{ujd} = \frac{3200}{80 \times .85 \times 10} = 4.7 \text{ in.}^2$$

Referring to Table XXVI (p. 203), we find:

For two $\frac{5}{8}$-in. round bars, $A_s=0.61$ in.^{2} and $\Sigma o = 2\times 1.96 = 3.92$ in.
three $\frac{1}{2}$-in. round bars, $A_s=0.59$ in.^{2} and $\Sigma o = 3\times 1.57 = 4.71$ in.
four $\frac{7}{16}$-in. round bars, $A_s=0.60$ in.^{2} and $\Sigma o = 4\times 1.37 = 5.48$ in.
six $\frac{5}{16}$-in. square bars, $A_s=0.59$ in.^{2} and $\Sigma o = 6\times 1.25 = 7.50$ in.

The $\frac{5}{8}$-inch bars are too large for the bond stress; the $\frac{1}{2}$-inch bars are just sufficient and would probably be selected.

Length of Bar to Prevent Slipping.—The stress carried by any reinforcing bar must be transmitted to the concrete between the point at which the stress exists and the end of the bar, which must be

accomplished either by having a sufficient length of bar to develop bond stress equal to the maximum tension or by anchoring the bar by other means.

Let f_s = tensile stress, in pounds per square inch, in the bar;
i = diameter of bar in inches;
u = allowable bond stress per square inch
l_b = length required for bond.

For round bar, the total stress $=\frac{\pi i^2 f_s}{4}=\pi i l_b u.$

For square bars, the total stress $=i^2 f_s = 4 i l_b u$

Then for either round or square bars,

$$l_b=\frac{f_s i}{4u}. \qquad \qquad (22)$$

If $f_s=16{,}000$ lb./in.^{2} and $u=80$ lb./in.^{2}, $l_b=50i$, or for safety, the length between the point where the stress of 16,000 lb./in.^{2} exists and the end of the bar must be 50 diameters.

Table XXXV (p. 227) gives lengths of embedment for different values of f_s for Standard reinforcing bars.

Anchoring Bar by Bending.—When it is not feasible to secure the length of bar necessary for bond, the end of the bar may be anchored in the concrete by bending to a semicircle. Experiments indicate that, in general, the full strength of a bar in tension may be developed by bending the end to a semicircle, the diameter of which is four times the diameter of the bar. Short right-angled bends are found to be much less effective than curves through 180°.

In the case of restrained beams, or cantilevers, when maximum tension occurs nears the support, careful attention must be given to the anchorage of the bars. Bars used for diagonal tension reinforcement, either vertical stirrups or inclined bars, have maximum tension at the neutral axis, and must have a sufficient embedment on the compression side of the neutral axis to resist the maximum tension in the steel.

Lateral Spacing of Steel.—The horizontal tension rods in a reinforced concrete beam must be so spaced as to leave a sufficient area of concrete between them to carry the shear communicated to the concrete by the portion of the bars below the minimum section of concrete. This would require that for circular bars the horizontal section between rods be capable of carrying a shearing stress equal to the bond stress on the lower half of the bars. If s_c be the clear spacing between the bars and i the diameter of the bar, for the round bar

$$s_c v=\frac{\pi i u}{2} \quad \text{or} \quad s_c=\frac{\pi i u}{2v}$$

TABLE XXXIV.—VALUES OF Σ_o FOR DIFFERENT NUMBERS OF STANDARD REINFORCING BARS

Square Bars.		Values of Σ_0 for Different Numbers of Bars—Sq. In.								
Diam. in Inches.	Perimeter, Inches.	1	2	3	4	5	6	7	8	9
$\frac{1}{2}$	2.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00
1	4.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00
$1\frac{1}{8}$	4.50	4.50	9.00	13.50	18.00	22.50	27.00	31.50	36.00	40.50
$1\frac{1}{4}$	5.00	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00
Round Bars.										
$\frac{1}{4}$	0.785	0.78	1.57	2.35	3.14	3.92	4.71	5.49	6.28	7.06
$\frac{3}{8}$	1.178	1.17	2.35	3.53	4.71	5.89	7.06	8.24	9.42	10.60
$\frac{1}{2}$	1.570	1.57	3.14	4.71	6.28	7.85	9.42	10.99	12.56	14.13
$\frac{5}{8}$	1.963	1.96	3.92	5.89	7.85	9.81	11.78	13.74	15.70	17.66
$\frac{3}{4}$	2.356	2.35	4.71	7.06	9.42	11.78	14.13	16.49	18.84	21.20
$\frac{7}{8}$	2.748	2.74	5.49	8.24	10.99	13.74	16.49	19.24	21.99	24.74
1	3.141	3.14	6.28	9.52	12.56	15.70	18.84	21.99	25.13	28.27

TABLE XXXV.—LENGTHS OF EMBEDMENT FOR BOND FOR DIFFERENT VALUES OF f_s FOR STANDARD REINFORCING BARS

Size Inch.	Lengths of Embedment—Inches.					
	Unit Bond, 80 lb./in.²			Unit Bond, 100 lb./in.²		
	$f_s = 16{,}000$	$f_s = 18{,}000$	$f_s = 20{,}000$	$f_s = 16{,}000$	$f_s = 18{,}000$	$f_s = 20{,}000$
$\frac{1}{4}$	13	14	16	10	11	13
$\frac{3}{8}$	19	21	23	15	17	19
$\frac{1}{2}$	25	28	31	20	23	25
$\frac{5}{8}$	31	35	39	25	28	31
$\frac{3}{4}$	37	42	47	30	34	38
$\frac{7}{8}$	44	49	54	35	39	44
1	50	56	62	40	45	50
$1\frac{1}{8}$	56	63	70	45	51	56
$1\frac{1}{4}$	63	70	78	50	56	63

For the values of unit stress recommended by the Joint Committee ($v=6$ per cent and $u=4$ per cent of the ultimate compressive strength), $v=\frac{3}{2}u$, and for round bars, $s_c=\frac{\pi iu}{2v}=1.05i$.

For square bars with sides vertical, $s_c v=3iu$ or $s_c=2i$, and for square bars with diagonals vertical, $s_c=\frac{4}{3}i$.

For deformed bars these values would be increased in the ratio of 5 to 4.

The Joint Committee recommends [3] that:

> The lateral spacing of parallel bars should not be less than three diameters from center to center, nor should the distance from the side of the beam to the center of the nearest bar be less than two diameters. The clear spacing between two layers of bars should be not less than 1 inch. The use of more than two layers is not recommended, unless the layers are tied together by adequate metal connections, particularly at and near points where bars are bent up or bent down. Where more than one layer is used at least all bars above the lower layer should be bent up and anchored beyond the edge of the support.

The Joint Committee also recommends [4] that:

> Metal reinforcement in fire-resistive construction shall be protected by not less than 1 in. of concrete in slabs and walls, and not less than 2 in. in beams, girders, and columns, provided aggregate showing an expansion not materially greater than that of limestone or trap-rock is used; when impracticable to obtain aggregate of this grade, the protective coating shall be 1 in. thicker and shall be reinforced with metal mesh, having openings not exceeding 3 in., placed 1 in. from the finished surface.
>
> In structures where the fire hazard is limited, the metal reinforcement shall not be placed nearer the exposed surface than $\frac{3}{4}$ in. in slabs and walls, or $1\frac{1}{2}$ in. in beams, girders, and columns.

The lateral spacing of carrying bars in slabs should not exceed two and one-half times the depth of the slab.

123. Design of Beams.—The methods of applying formulas and tables in the design of rectangular beams is illustrated in the following examples:

(8) Design a rectangular beam to have a span of 25 feet and carry a uniform load of 600 pounds per linear foot, in addition to its own weight using working stresses recommended by the Joint Committee for concrete of 2000 lb./in.^{2} compressive strength.

Solution.—From Table XXII (p. 199), for $n=15$, $f_s=16{,}000$ and $f_c=650$, we find $R=108$, $p=.0077$, $j=.874$.

Assume weight of beam $=300$ pounds per linear foot.

[3] Proceedings, American Society of Civil Engineers, December, 1916.

[4] Ibid., October, 1924.

Then

$$M=\frac{wl^2}{8}=\frac{(600+300)(25)^2\times 12}{8}=843750 \text{ in.-lb.}$$

$$bd^2=M/R=843750/108=7812, \text{ and for } b=12,\ d=25.5,$$

for $b=14$, $d=23.6$. Taking $b=14$, and total depth, $d_1=25.5$, weight of beam $=14\times 25.5\times 150/144=372$ pounds per linear foot. The assumed load is too small. Assume weight of beam $=400$ pounds per linear foot.

$$M=\frac{(600+400)(25)^2\times 12}{8}=937500 \text{ in.-lb., and } bd^2=\frac{937500}{108}=8681.$$

For $b=14$, $d=\sqrt{\frac{8681}{14}}=24.9$. Using $b=14$ and $d=25$, make $d_1=27$.

Then weight of beam $=\dfrac{27\times 14\times 150}{144}=394$ pounds per linear foot, which agrees with the assumption.

Horizontal steel, $A_s=pbd=.0077\times 14\times 25=2.73$ in.^{2}

From Table XXVI (p. 203),

seven $\frac{5}{8}$-in. square bars give $A_s=2.73$ in.^{2}
five $\frac{3}{4}$-in. square bars give $A_s=2.81$ in.^{2}
six $\frac{3}{4}$-in. round bars give $A_s=2.65$ in.^{2}

Seven $\frac{5}{8}$-inch square bars, spaced $1\frac{7}{8}$ inches c. to c. or six $\frac{3}{4}$-inch round bars, spaced $2\frac{1}{4}$ inches c. to c. might be placed in the width of 14 inches, meeting the requirement of spacing 3 diameters c. to c. We will use five $\frac{3}{4}$-inch square bars, spaced $2\frac{1}{2}$ inches c. to c. and 2 inches from side of beam.

Maximum Shear, $V=\dfrac{W}{2}=\dfrac{25\times 1000}{2}=12500$ pounds

$$v_m=\frac{V}{bjd}=\frac{12500}{14\times .874\times 25}=40.6 \text{ lb./in.}^2$$

The section is sufficient for shear, and no diagonal tension reinforcement is necessary.

Bond Stress, Table XXVI, for five $\frac{3}{4}$-in. bars, $\Sigma o=5\times 3.00=15$ in.^{2} and (19)

$$u=\frac{bv}{\Sigma o}=\frac{14\times 40.6}{15}=37.9 \text{ lb./in.}^2$$

which is less than the allowable stress.

9. A simple beam of 18-foot span, center to center of bearings, is to carry a load of 700 pounds per linear foot in addition to its own weight.

Design the beam, assuming $n=15$, $f_s=15{,}000$ lb./in.^{2}, $f_c=750$ lb./in.^{2}, safe value of unit shear $=120$ lb./in.^{2} and safe unit diagonal tension 40 lb./in.^{2}

Solution.—Assuming the weight of beam as 240 pounds per linear foot, the total load is, $(700+240)18=16{,}920$ pounds.

$$M=\frac{Wl}{8}=\frac{16920\times 18\times 12}{8}=456{,}840 \text{ in.-lb.}$$

From Table XXII (p. 199), for $f_s=15{,}000$, $f_c=750$, and $n=15$,

$$R=138,\ j=0.857,\ p=0.0107.$$

$$bd^2=\frac{M}{R},\quad \text{or}\quad d^2=\frac{M}{bR}.$$

Assuming $b=12$,

$$d^2=\frac{456840}{12\times 138}=276 \text{ in.}^2$$

From Table XXV (p. 202),

$$d=16.75 \text{ inches}$$

$$A_s=pbd=.0107\times 12\times 16.75=2.15 \text{ in.}^2$$

From Table XXVI (p. 203),

five $\frac{3}{4}$-in. round bars $=2.21$ in.^{2}

four $\frac{7}{8}$-in. round bars $=2.40$ in.^{2}

three 1-in. round bars $=2.36$ in.^{2}

The three 1-in. round bars will be used, spaced 3.5 inches center to center, and 2.5 inches from center of steel to sides of beam.

If the centers of the bars are placed 2.25 inches from the bottom surface of the beam, the total depth of beam, d_1, will be 19 inches, and the weight, $12\times 19\times 150/144=238$ pounds, almost exactly as assumed.

Maximum Shear,

$$V=\frac{W}{2}=\frac{16920}{2}=8460 \text{ pounds}$$

$$v=\frac{8460}{bjd}=\frac{8460}{12\times .857\times 16.75}=50 \text{ lb./in.}^2$$

This is less than 120 lb./in.^{2}, and the dimensions of the beam are sufficient.

Reinforcement for Diagonal Tension is needed between the support and the section where $v=40$ lb./in.^{2}, or from Formula 14 (p. 218),

$$x=\frac{(160-v_m)l}{6v_m}=\frac{(160-50)216}{6\times 50}=79 \text{ inches}$$

Reinforcement is required for a distance, $108-79=29$ inches from the support.

Vertical Stirrups.—Assuming $s=0.45d=0.45\times16.75=7.5$ inches as recommended by the Joint Committee.

$$A_v=\frac{vbs}{f_v}=\frac{(50-40)\times12\times7.5}{15000}=0.06 \text{ in.}^2$$

For U-shaped stirrup, the section of rod required will be one-half of this, or 0.015 in.^{2} Three-eighth inch round bars will give more than sufficient area, and five stirrups spaced 3.5, 11.0, 18.5, 26.0, and 33.5 inches from the center of the support will be used.

Bond Stress.—For the horizontal steel, Table XXVI (p. 203), gives, $\Sigma o=3\times3.14=9.42$ in.^{2}, and Formula 20 (p. 223) gives,

$$u=\frac{bv}{\Sigma o}=12\times50/9.42=64 \text{ lb./in.}^2,$$

which is less than the allowable bond stress and no anchoring is necessary, but hooked ends are recommended.

For the vertical stirrups, Formula 22 (p. 226) gives,

$$l_b=\frac{f_s i}{4u}=\frac{15000\times3}{4\times80\times8}=17.6 \text{ inches}$$

or stirrups would need 17.6 inches above a plane $0.3d$ from the lower for anchorage. They must, therefore, have hooked ends.

ART. 29. T-BEAMS WITH TENSION REINFORCEMENT

124. Flexure Formulas.—In a rectangular reinforced concrete beam, in which the steel carries all the tension, the area of concrete below the neutral axis does not affect the resisting moment of the beam. The office of this concrete is to hold the steel in place and carry the shear, thus connecting the steel with the compression area of concrete.

In a T-beam, the flange carrying the compression is connected with a narrow web which holds the steel, as shown in Fig. 60. When the neutral axis is in the flange, such a beam may be computed by the formulas and tables used for a rectangular beam, using the width of the flange, b, as the width of the beam.

When the neutral axis is below the bottom of the flange of the T-beam, the compression area is less than that of the rectangular beam, and special formulas are necessary. Figure 60 represents a beam of this kind. The amount of compression on the web is usually

very small and may be neglected without material error, thus greatly simplifying the formulas.

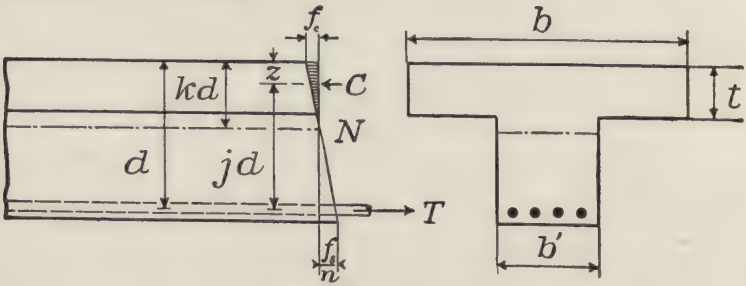


FIG. 60.—T-Beam with Tension Reinforcement.

The same notation will be employed as in the rectangular beam, letting b = width of flange;

b' = width of web;

t = thickness of flange.

The position of the neutral axis in terms of the unit stresses may be found as in the rectangular beam, giving

$$\frac{f_s}{f_c}=\frac{n(1-k)}{k}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and

$$k=\frac{nf_c}{f_s+nf_c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The average unit compression on the flange is the half sum of the compressions at the top and bottom of the flange, or

$$\frac{1}{2}\left(f_c+f_c\frac{kd-t}{kd}\right)=f_c\frac{2kd-t}{2kd}.$$

The total compression on the concrete is

$$C=f_c\frac{2kd-t}{2kd}\,bt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

This is then equal to the total tension on the steel,

$$T=f_sA_s=f_spbd. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

From the equality of (25) and (26) we find

$$p=\frac{(2k-t/d)}{2n(1-k)}\cdot\frac{t}{d}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and

$$k=\frac{pn+\frac{1}{2}(t/d)^2}{pn+t/d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The distance of the centroid of compression from the upper face of the beam is

$$\frac{3k-2t/d}{2k-t/d}\cdot\frac{t}{3},$$

therefore

$$jd=d-\frac{3k-2t/d}{2k-t/d}\cdot\frac{t}{3}. \quad \ldots\ldots\ldots \quad (29)$$

The resisting moment of the beam is

$$M=Tjd=A_sf_sjd=pf_sjbd^2, \quad \ldots\ldots \quad (30)$$

or

$$M=Cjd=f_c\frac{2k-t/d}{2k}\cdot btjd. \quad \ldots\ldots \quad (31)$$

Examples.—The use of these formulas in the solution of problems arising in the design or investigation of T-beams is illustrated in the following examples:

10. A T-beam has the following dimensions, $b=48$ inches, $t=4$ inches, $d=22$ inches, $b'=10$ inches. The steel reinforcement consists of six $\frac{3}{4}$-inch round bars. If the safe unit stresses of steel and concrete are 15,000 and 600 lb./in.^{2} respectively, and $n=15$, what is the safe resisting moment of the beam?

Solution.—From Table XXVI (p. 203),

$$A_s=2.65 \text{ in.}^2, \text{ and } p=\frac{2.65}{22\times48}=.0025;$$

formula (28) (p. 232) gives

$$k=\frac{.0025\times15+\frac{1}{2}(\frac{4}{22})^2}{.0025\times15+\frac{4}{22}}=.247.$$

Using (29) we find

$$jd=22-\frac{3\times.247-2(\frac{4}{22})}{2\times.247-\frac{4}{22}}\cdot\frac{4}{3}=20.39.$$

From (23) (p. 232),

$$\frac{f_s}{f_c}=\frac{15(1-.247)}{.247}=45.7.$$

If $f_c=600$ lb./in.^{2}, $f_s=600\times45.7=27{,}420$ lb./in.^{2}

This is greater than the safe unit stress on steel, and the safe moment will be that which causes a stress of 15,000 lb./in.^{2} on the steel, or from (30),

$$M=2.65\times15000\times20.39=810000 \text{ in.-lb.}$$

11. The flange of a T-beam is 26 inches wide and 4 inches thick. The beam is to carry a bending moment of 520,000 in.-lb. The safe unit stresses for concrete and steel are 600 and 16,000 lb./in.^{2} respectively. What area of steel and depth of beam are needed.

Solution.—By Formula 24 (p. 232),

$$k=\frac{15\times600}{16000+15\times600}=360.$$

Find d by assuming values and testing their suitability. Try $d=18$; from Formula 29 (p. 233),

$$jd=18-\frac{3\times.360-2(\frac{4}{18})}{2\times.360-\frac{4}{18}}\cdot\frac{4}{3}=16.3.$$

Formula 30 (p. 233), gives $C=M/jd=520000/16.3=31900$.

From Formula 25 (p. 232), $f_c=\dfrac{C}{\dfrac{2kd-t}{2kd}\cdot bt}=440$ lb./in.^{2}. This is a safe value, but a less depth will answer. Trying 15 inches, $C=38{,}000$ pounds, and $f_c=580$ lb./in.^{2}; 15 inches is, therefore, approximately the minimum value for d. For this value of d, Formula 26 (p. 232), gives,

$$A_s=T/f_s=38000/16000=2.375 \text{ in.}^2$$

The following are the recommendations of the Joint Committee for T-beams:

Flange Width.—Effective and adequate bond and shear resistance shall be provided in beam-and-slab construction at the junction of the beam and slab; the slab shall be built and considered an integral part of the beam; the effective flange width to be used in the design of symmetrical T-beams shall not exceed one-fourth ($\frac{1}{4}$) of the span length of the beam, and its overhanging width on either side of the web shall not exceed eight (8) times the thickness of the slab nor one-half ($\frac{1}{2}$) the clear distance to the next beam.

For beams having a flange on one side only, the effective flange width to be used shall not exceed one-tenth ($\frac{1}{10}$) of the span length of the beam, and its overhanging width from the face of the web shall not exceed six (6) times the thickness of the slab nor one-half ($\frac{1}{2}$) the clear distance to the next beam.

Transverse Reinforcement.—Where the principal slab reinforcement is parallel to the beam, transverse reinforcement, not less in amount than 0.3 per cent of the sectional area of the slab, shall be provided in the top of the slab and shall extend across the beam and into the slab not less than two-thirds ($\frac{2}{3}$) of the width of the effective flange overhang. The spacing of the bars shall not exceed 18 in.

Compressive Stress at Supports.—Provision shall be made for the compressive stress at the support in continuous T-beam construction.

Shear.—The flange of the beam shall not be considered as effective in computing the shear and diagonal tension resistance of T-beams.

Isolated Beams.—Isolated beams, in which the T-form is used only for the purpose of providing additional compression area, shall have a flange thickness of not less than one-half ($\frac{1}{2}$) the width of the web and a total flange width of not more than four (4) times the web thickness.

125. Shear and Bond Stresses.—Stresses due to shear in the concrete and bond stresses between the steel and concrete in T-beams are found by the same methods that are used for rectangular beams. The shearing and diagonal tension stresses must be carried by the web of the beam, the area of flange not being considered in finding unit shear. Using the same notation as for rectangular beams and letting b' represent the width of the web of the T-beam, the formulas as applied to T-beams become:

For shear,

$$v=\frac{V}{b'jd},$$

and

$$b'd=\frac{V}{vj}. \qquad (32)$$

For vertical stirrups,

$$A_v=\frac{vb's}{f_v}=\frac{Vs}{f_vjd},$$

or

$$s=\frac{A_vf_v}{vb'}. \qquad (33)$$

For diagonal steel,

$$A_d\doteq\frac{vb's}{f_v\sqrt{2}}=\frac{Vs}{f_vjd\sqrt{2}},$$

or

$$s=\frac{A_df_v\sqrt{2}}{vb'}. \qquad (34)$$

For section where it is allowable to turn up steel,

$$A-A_x=\frac{Ax^2}{(l/2)^2},$$

and

$$x=\frac{l}{2}\sqrt{\frac{A-A_x}{A}}. \qquad (35)$$

For bond stress,

$$u=\frac{b'v}{\Sigma o}=\frac{V}{\Sigma ojd},$$

and

$$\Sigma o=\frac{b'v}{u}=\frac{V}{ujd}. \qquad (36)$$

For length of bar to prevent slipping,

$$l_b=\frac{f_si}{4u}. \qquad (37)$$

The Width of the Web (b') must be sufficient to provide proper area for carrying shear, as shown in (32), and also to allow for properly spacing the steel, as explained in Section 122. Usually b' should not be taken at less than $d/3$, except in heavy beams where a thickness of $d/4$ may be allowable. The value of j, when not known, may be assumed as $\frac{7}{8}$ without material error, and the value of vj in (32) may be taken as $\frac{7}{8}$ of the allowable unit shear.

126. T-Beam Diagrams.—The labor of T-beam computations may be considerably lessened by tabulation of some of the terms which enter into the formulas. Some of these tabulations are here given in the form of diagrams.

If we place $Q=f_c j\cdot\dfrac{2k-t/d}{2k}$, Q will be constant for any particular values of unit stresses and t/d. Substituting in Formula (31) (p. 233), we obtain $M=Qbtd$ and $M/bt=Qd$ or

$$Q=M/btd. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

In Diagram XIV (p. 237), values of f_c and p are given in terms of various values of d/t and Q for $n=15$ and $f_s=16{,}000$. This diagram may be used in design of beams when these units are to be employed, or similar diagrams may easily be prepared for other values of f_s and n.

Diagram XV (p. 238), gives values of p and j in terms of f_s/f_c and d/t, when $n=15$. This diagram may be used in reviewing a beam of known dimensions and reinforcement, or in design when values of f_s other than that used in Diagram XIV are to be employed.

Examples.—The following examples illustrate the use of these diagrams and formulas in computation:

12. A T-beam has dimensions as follows: $b=45$ inches, $t=4$ inches, $d=20$ inches, $b'=9$ inches. It is reinforced with ten $\frac{3}{4}$-inch round steel bars. If the safe unit stresses of steel and concrete are 16,000 and 650 lb./in.^{2} respectively, what is the safe resisting moment for the beam?

Solution.—Table XXVI (p. 203), $A_s=.4418\times10=4.418$ in.^{2}, and $p=\dfrac{4.418}{45\times20}=.0049$. From Diagram XV, for $p=.0049$ and $d/t=5$, we find $f_s/f_c=29$, and $j=.914$. If $f_c=650$, $f_s=650\times29=18{,}850$. This is more than is allowable, and the safe resisting moment is that giving a stress of 16,000 on the steel, or using (30) (p. 233),

$$M=A_s f_s jd=4.418\times16000\times.914\times20=1{,}292{,}800 \text{ in.-lb.}$$

13. The dimensions of a T-beam are, $b=36$ inches, $t=3$ inches, $d=$ 13 in. The beam is reinforced with six $\frac{3}{4}$-inch round steel bars.

If this beam is subjected to a bending moment of 550,000 in.-lb., what are the stresses in the steel and concrete respectively?

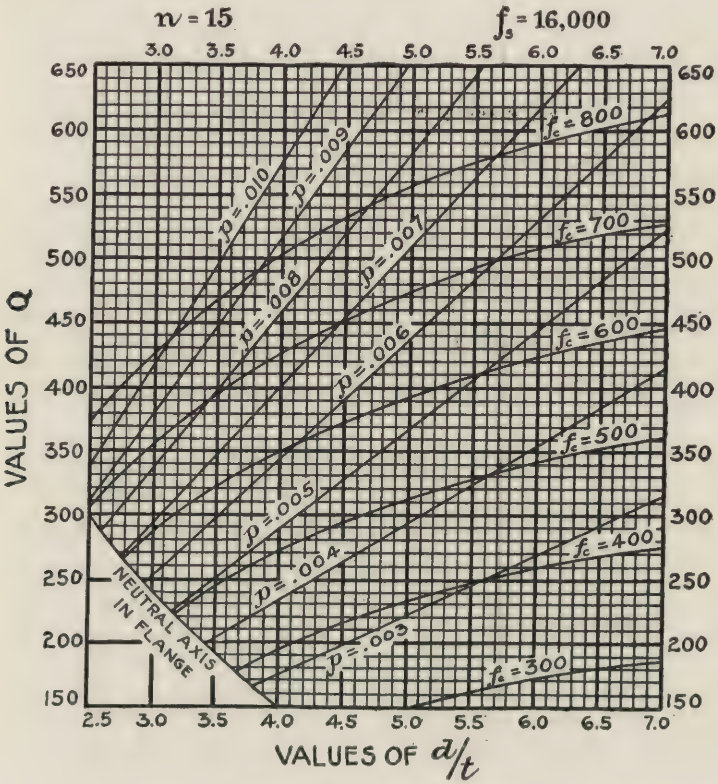


Diagram XIV.—For T-Beam Design.
$M/bt = Qd.$

Solution.—$A_s = .4418 \times 6 = 2.65$ in.², and $p = \frac{2.65}{36 \times 13} = .0057.$ $d/t = 13/3 = 4.33$. From Diagram XV (p. 238), we find $f_s/f_c = 27.5$ and $j = .903$. Formula 30 (p. 233) now gives $f_s = \frac{550000}{2.65 \times .903 \times 13}$ $= 18270$ lb./in.², from which $f_c = 18270/27.5 = 660$ lb./in.²

14. The flange of a T-beam is to be 30 inches wide and 5 inches thick. The beam is to sustain a bending moment of 930,000 in.-lbs. and a maximum shear of 14,500 pounds. The safe unit stresses, on steel and concrete are 16,000 and 650 lb./in.², and maximum unit shear 120 lb./in.² What dimensions of web and area of steel are required?

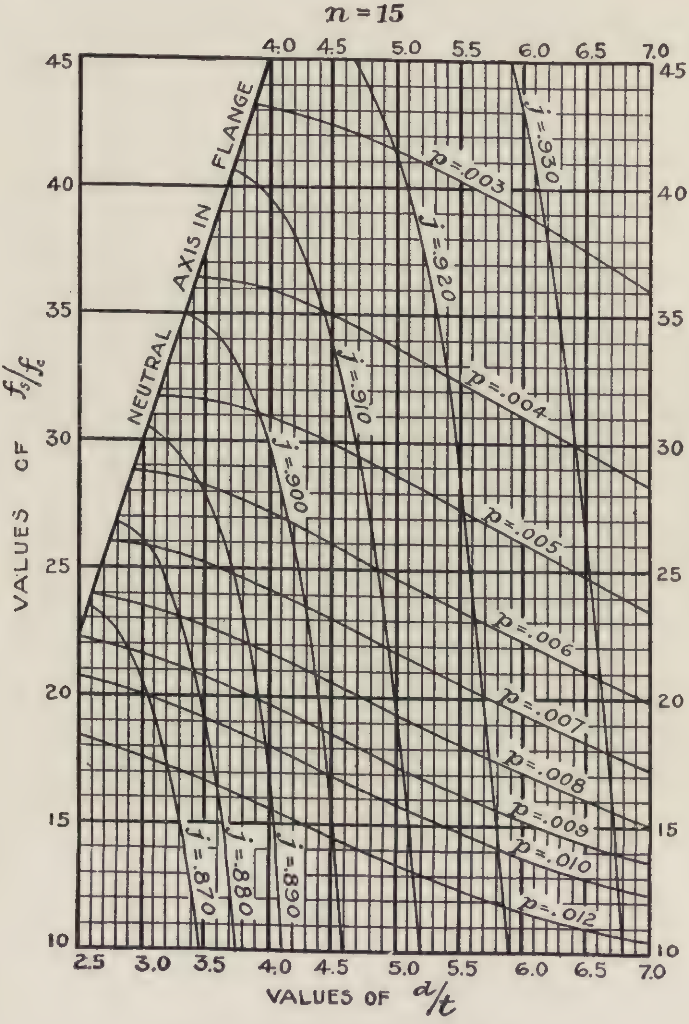


DIAGRAM XV.—For Review of T-Beam.

Solution.—Assuming $j=\frac{7}{8}$, $vj=105$, and from (32) (p. 235) $b'd=\frac{V}{vj}=\frac{14500}{105}=138$ in.[2] For $b'=8$, $d=18$ or for $b'=7$, $d=20$ inches. Either of these values would give proper form to the web. The deeper beam will require less steel and may be used provided it gives sufficient width for placing the steel, and if the stress upon the concrete is satisfactory. Assume $d=20$ inches. Then $d/t=4$ and Formula 38

(p. 236) $Q=\frac{930000}{30\times5\times20}=310$. For these values Diagram XIV (p. 237) gives $f_c=540$ lb./in.² and $p=.0054$. $A_s=pbd=.0054\times20\times30=3.24$ in.² and Formula 21 (p. 223), $\Sigma o=\frac{V}{ujd}=\frac{14500}{80\times\frac{7}{8}\times20}=10.35$ in.

From Table XXVI (p. 203) for

six $\frac{3}{4}$-inch square bars, $A_s=3.37$ in.², $\Sigma o=18.0$ in.
four $\frac{15}{16}$-inch square bars, $A_s=3.52$ in.², $\Sigma o=15.0$ in.
four 1-inch round bars, $A_s=3.14$ in.², $\Sigma o=12.56$ in.

The four $\frac{15}{16}$-inch bars could be placed in the 7-inch width of web in two rows (see Section 122). The six $\frac{3}{4}$-inch bars need a width of at least $7\frac{1}{2}$ inches and could be used in two rows by increasing the width of web by $\frac{1}{2}$ inch.

If d be made 21 inches, the steel needed would be $A_s=3.09$ in.² and the four 1-inch round bars could be used in two rows in the 7-inch width. At ordinary prices, the saving in steel would more than pay for the increased amount of concrete, and this would make the cheapest beam.

127. T-Beam Tables.—The work of designing T-beams with tension reinforcement is considerably shortened by the use of tables in which certain constants have been computed once for all.

Table XXXVI (p. 240) gives coefficients for different ratios of t to d for computing lever arms jd.

Table XXXVII (p. 241) gives coefficients for different ratios of t to d for computing widths of flange b.

Examples.—15. A state highway bridge has a span of 30 feet center to center of end bearings and a clear width of roadway of 18 feet. The depth of roadway fill is 9 inches and the floor slab has a total thickness of 8 inches. The live load consists of a 15-ton tractor with 10 tons on one axle and 5 tons on the other, preceded or followed by 10-ton trailers having 5 tons on each axle. Axles are spaced 10 feet, and wheels, 6 feet center to center. An impact of 20 per cent of the live load is also to be provided for. The allowed unit stresses on steel and concrete are 16,000 and 650 lb./in.² respectively.

Assume a depth, d, of the T-beam of 30 inches, and a width of stem, b', of 14 inches.

The total dead load, liveload, and impact bending moment is 3,600,000 in./lb.

From Table XXXVI (p. 240), for $t/d=8/30=0.27$, $C_1=0.377$, and $z=C_1\times t=0.377\times8=3.00$ in., $jd=d-3=30-3=27$ in.

$$A_s=\frac{M}{f_sjd}=\frac{3600000}{16000\times27}=8.33\text{ in.}^2$$

TABLE XXXVI.—FOR COMPUTING THE LEVER ARMS OF T-BEAMS

$jd = d - z. \qquad z = \frac{2k - 2t/d}{2k - t/d} \cdot t/3 = C_1 t$

f_s	f_c	k	Values of C_1 — Ratio t/d: .10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30	.32	.34	.36	.38	.40	.42
16,000	500	.319	.469	.461	.453	.444	.434	.424	.412	.400	.385	.369	.352						
	550	.339	.471	.464	.456	.448	.440	.430	.420	.409	.396	.382	.368	.351					
	600	.358	.473	.466	.460	.453	.445	.435	.425	.415	.405	.393	.380	.366	.350				
	650	.378	.475	.468	.463	.455	.448	.440	.431	.422	.412	.402	.391	.378	.364	.349			
	700	.397	.476	.470	.465	.458	.451	.443	.436	.428	.419	.410	.398	.387	.375	.362	.346		
	750	.414	.478	.472	.466	.460	.453	.447	.440	.432	.423	.415	.405	.395	.384	.372	.358	.344	
	800	.429	.478	.473	.468	.463	.456	.449	.442	.435	.428	.419	.410	.400	.390	.379	.368	.354	.340
18,000	500	.294	.465	.457	.448	.437	.425	.414	.400	.385	.369	.349							
	550	.314	.467	.459	.451	.442	.431	.421	.409	.397	.382	.366	.348						
	600	.333	.470	.463	.456	.448	.440	.430	.420	.409	.396	.381	.366	.346					
	650	.351	.473	.465	.458	.451	.443	.433	.425	.414	.402	.389	.376	.360	.344				
	700	.369	.473	.467	.460	.454	.445	.438	.430	.420	.409	.398	.385	.372	.358				
	750	.385	.475	.470	.463	.456	.449	.442	.433	.425	.415	.405	.394	.381	.368	.354			
	800	.400	.476	.470	.465	.458	.451	.443	.436	.428	.419	.410	.398	.387	.375	.362	.343		
20,000	500	.272	.463	.453	.443	.430	.417	.403	.387	.369	.348								
	550	.292	.465	.457	.448	.437	.425	.414	.400	.385	.369	.349							
	600	.311	.467	.459	.451	.442	.431	.421	.409	.397	.382	.366	.348						
	650	.328	.470	.462	.454	.446	.436	.416	.416	.404	.388	.376	.361						
	700	.344	.471	.464	.457	.449	.442	.432	.421	.410	.398	.385	.371	.355					
	750	.359	.473	.466	.460	.453	.445	.435	.425	.415	.405	.393	.380	.366	.350				
	800	.374	.475	.468	.463	.455	.448	.440	.430	.421	.411	.400	.389	.375	.361	.345			

Credit for this table is due to Mr. M. F. Marks.

TABLE XXXVII.—FOR COMPUTING WIDTH OF FLANGE b OF T-BEAMS

$$b=\frac{C_2A_s}{t}. \qquad C_2=\frac{f_s/f_c}{1-\frac{t/d}{2k}}. \qquad t=\text{thickness of flange}$$

f_s	f_c	k	Values of C_2 — Ratio t/d .10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30	.32	.34	.36	.38	.40	.42
16,000	500	.319	38.0	39.4	41.0	42.8	44.6	46.5	48.8	51.3	53.9	57.0	60.4						
	550	.339	34.1	35.4	36.7	38.1	39.6	41.3	43.0	45.0	47.2	49.6	52.1	55.1					
	600	.358	31.0	32.1	33.1	34.4	35.6	37.0	38.5	40.3	41.9	43.8	45.9	48.2	50.8				
	650	.378	28.4	29.3	30.2	31.2	32.3	33.4	34.8	36.0	37.5	39.1	40.8	42.6	44.8	47.0			
	700	.397	26.2	27.0	27.8	28.7	29.6	30.6	31.6	32.8	34.0	35.3	36.8	38.4	40.0	41.8	43.9		
	750	.414	24.3	25.0	25.7	26.4	27.2	28.1	29.1	30.1	31.0	32.2	33.4	34.8	36.2	37.8	38.4	41.3	
	800	.429	22.6	23.3	23.9	24.6	25.3	26.1	26.9	27.8	28.7	29.7	30.8	31.9	33.1	34.5	35.9	37.5	39.2
18,000	500	.294	43.2	45.1	47.0	49.2	51.5	54.3	57.0	60.1	63.8	67.6							
	550	.314	38.8	40.5	42.0	43.9	45.9	48.0	50.3	53.0	55.8	59.0	62.5						
	600	.333	35.3	36.6	38.0	39.5	41.1	42.9	44.8	46.9	49.2	51.8	54.5	57.6					
	650	.351	32.4	33.5	34.6	35.8	37.3	38.7	40.3	42.1	43.9	46.0	48.4	52.0	53.6				
	700	.369	29.7	30.7	31.8	32.8	34.0	35.3	36.6	38.2	39.7	41.4	43.3	45.4	47.8	50.2			
	750	.385	27.6	28.5	29.3	30.3	31.3	32.4	33.6	34.9	36.3	37.7	39.2	41.0	43.0	45.1	47.4		
	800	.400	25.7	26.5	27.3	28.1	29.0	30.0	31.0	32.1	33.4	34.6	36.0	37.5	39.1	40.9	42.9	45.0	
20,000	500	.272	49.0	51.5	53.9	56.7	59.8	63.4	67.2	71.7	76.6								
	550	.292	44.0	46.0	47.9	50.1	52.6	55.4	58.5	61.9	65.5	70.0							
	600	.311	39.7	41.3	43.0	44.8	46.8	49.1	51.5	54.0	57.2	60.5	64.2						
	650	.328	36.4	37.7	39.2	40.7	42.4	44.3	46.4	48.6	51.0	53.7	56.7	60.0					
	700	.344	33.5	34.7	35.9	37.3	38.8	40.4	42.1	43.9	46.0	48.1	50.7	53.5	56.5				
	750	.359	30.9	32.0	33.0	34.2	35.5	36.8	38.4	40.0	41.7	43.5	45.7	48.0	50.5				
	800	.374	28.8	29.8	30.8	31.8	33.0	34.1	35.4	36.8	38.3	40.0	41.7	43.6	45.9	48.2			

Credit for this table is due to Mr. M. F. Marks.

From Table XXVI (p. 203), it is found that six $1\frac{1}{4}$-inch square bars have an area of 9.45 in.^{2}, and these will be used.

17. The width of flange of the T-beam of Example 12 may be checked by Table XXXVII (p. 241).

The area of steel to be used was found to be $A_s = 9.45$ in.^{2} The thickness, t, of the slab was 8 inches and the depth, d, of the T-beam was 30 inches. As before, the ratio of t to d is 0.27. From Table XXXVII, $C_2 = 38.3$, and $b = C_2A_s/t = 40.25$ inches, which is satisfactory.

ART. 30. BEAMS REINFORCED FOR COMPRESSION

128. Flexure Formulas.—It is frequently necessary to place steel in the compression as well as the tension side of a beam. When the size of a rectangular beam is limited, so that the concrete area is insufficient to carry the stress, steel may be used to take the surplus compression. In this case the concrete and steel act together, and the stress upon the steel must be limited to such an amount as will not overtax the compressive strength of the concrete

In this discussion, the following notation will be used, in addition to that employed for rectangular beams:

A'_s = area of cross-section of compression steel;
p' = ratio of compression steel area to effective area of beam ($p' = A'_s/bd$);
d' = depth of center of gravity of compression steel below compression face of beam;
f'_s = unit stress in compression steel;
C' = total compression on steel.

The same principles apply in this case as in that of the beam reinforced for tension only, and the concrete is supposed to carry compression, but no tension. It is easily seen that

$$f_s = f_c n \frac{(1-k)}{k}, \qquad (39)$$

and that

$$f'_s = nf_c \frac{kd-d'}{kd} = f_c \frac{n(k-d'/d)}{k}. \qquad (40)$$

Compression on concrete,

$$C = \tfrac{1}{2} f_c kbd, \qquad (41)$$

and compression on steel,

$$C' = A_s' f'_s = f'_s p'bd \qquad (42)$$

Tension on steel,

$$T = C + C' = A_s f_s = f_s pbd \qquad (43)$$

Substituting (41) and (42) in (43) and combining with (39) and (40) we find

$$k=\sqrt{2n\left(p+p'\frac{d'}{d}\right)+n^2(p+p')^2}-n(p+p'). \quad . \quad . \quad (44)$$

Taking moments about the tension steel, we find the resisting moment of the beam,

$$M=Cjd+C'(d-d'). \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Example.—The use of these formulas in design will be illustrated by the following example:

17. A beam whose dimensions are $b=12$ in., $d=22$ in., $d'=2$ in., is to carry a bending moment of 1,100,000 in.-lb. The safe unit stresses are 700 and 16,000 lb./in.^{2} for concrete and steel respectively, $n=15$. Find the areas of steel required.

Solution.—For the given stresses, Table XXII (p. 199) gives $j=.868$. Formula (41) (p. 242) gives

$$C=\frac{700}{2}\times.397\times12\times22=36680 \text{ pounds.}$$

From (45),

$$C'=\frac{1{,}100{,}000-36680\times.868\times22}{20}=19980.$$

By Formula (40) (p. 242),

$$f'_s=15\times700\times\frac{.397-\frac{2}{22}}{.397}=8085 \text{ lb./in.}^2$$

and Formula 42 (p. 242).

$$A's=19980/8085=2.47 \text{ in.}^2$$

Area of tension steel, Formula 43 (p. 242).

$$A_s=\frac{T}{f_s}=\frac{36680+19980}{16000}=3.54 \text{ in.}^2$$

129. Tables.—The labor of computation may be materially lessened by the use of tables, which may be made in several ways, of which the following seem most convenient for use.

Table XXXVIII (p. 244). Transposing the terms of Formula (43) we have

$$A_s=\frac{C}{f_s}+\frac{C'}{f_s}.$$

TABLE XXXVIII.—BEAMS WITH COMPRESSION REINFORCEMENT

Values of $f's$ in terms of f_s, f_c and d'/d

$n = 15$

f_s	f_c	R	p_1	Values of d'/d.							
				.06	.08	.10	.12	.14	.16	.18	.20
14,000	500	77	.0062	6210	5780	5350	4920	4490	4060	3630	3200
	600	102	.0084	7620	7160	6700	6240	5780	5320	4860	4400
	700	128	.0107	9030	8540	8050	7560	7070	6580	6090	5600
	800	157	.0133	10440	9920	9400	8880	8360	7840	7320	6800
15,000	500	74	.0055	6150	5700	5250	4800	4350	3900	3450	3000
	600	99	.0075	7560	7080	6600	6120	5640	5160	4680	4200
	700	125	.0097	8970	8460	7950	7440	6930	6420	5910	5400
	800	151	.0118	10380	9840	9300	8760	8220	7680	7140	6600
16,000	500	72	.0050	6090	5620	5150	4680	4210	3740	3270	2800
	550	83	.0059	6745	6310	5825	5340	4855	4370	3885	3400
	600	95	.0068	7500	7000	6500	6000	5500	5000	4500	4000
	650	108	.0078	8205	7690	7175	6660	6145	5630	5115	4600
	700	121	.0087	8910	8380	7850	7320	6790	6260	5730	5200
	750	134	.0097	9615	9070	8525	7980	7435	6890	6345	5800
	800	147	.0107	10320	9760	9200	8640	8080	7520	6960	6400
	900	174	.0128	11730	11140	10550	9960	9370	8780	8190	7600
18,000	600	88	.0055	7380	6840	6300	5760	5220	4680	4140	3600
	700	113	.0072	8790	8220	7650	7080	6510	5940	5370	4800
	800	139	.0089	10200	9600	9000	8400	7800	7200	6600	6000
	900	165	.0107	11610	10980	10350	9720	9090	8460	7830	7200
20,000	600	83	.0046	7260	6680	6100	5520	4940	4360	3780	3200
	700	106	.0060	8670	8060	7450	6840	6230	5620	5010	4400
	800	132	.0075	10080	9440	8800	8160	7520	6880	6240	5600
	900	157	.0091	11490	10820	10150	9480	8810	8140	7470	6800

$$A'_s = \frac{M - Rbd^2}{f'_s(d-d')}, \qquad A_s = p_1 bd + \frac{A'_s f'_s}{f_s}.$$

This may be placed in the form,

$$A_s = p_1bd + \frac{A_s'f'_s}{f_s}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

in which p_1 is the ratio of steel for a beam with the same unit stresses and without compression steel.

Formula 45 (p. 243) may be put in the form

$$M = Rbd^2 + f'_sA'_s(d-d'),$$

or solving for A'_s

$$A'_s = \frac{M - Rbd^2}{f'_s(d-d')}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Values of R, p_1 and f'_s, in terms of various values of f_s, f_c, and d'/d for $n=15$, are given in Table XXXVIII (p. 244). This table may be used to find the areas of steel required when a beam of given dimensions must carry a bending moment too great to be resisted by tension reinforcement only.

Table XXXIX (p. 246). Combining (41), (42) and (45) we have

$$M = \tfrac{1}{2}f_cjkbd^2 + f'_sp'(1-d'/d)bd^2,$$

from which

$$M/bd^2 = \tfrac{1}{2}f_cjk + f'_sp'(1-d'/d) = G, \quad . \quad . \quad . \quad (48)$$

in which G is constant for definite values of unit stresses and steel ratios. In Table XXXIX, values of p' and p are given directly for various values of f_c and G when $n=15$ and $f_s=16{,}000$ lb./in.^{2}

To use this table in design, it is only necessary to find G by dividing the bending moment M by bd^2 for the proposed beam and take the required ratios of steel directly from the table.

Tables XL (p. 247) and XLI (p. 248). If Formulas (39) and (44) be combined a value for f_s/f_c in terms of p, p' and d'/d may be found. These values are tabulated in Table XXXIX.

If the values of f'_s from (40) (p. 242) be substituted in (48), it becomes

$$\frac{M}{bd^2} = \frac{1}{2}f_cjk + f_cnp'\left(\frac{k-d'/d}{k}\right)\left(1-\frac{d'}{d}\right),$$

and making

$$\tfrac{1}{2}jk + np'\left(\frac{k-d'/d}{k}\right)(1-d'/d) = N,$$

we have

$$\frac{M}{bd^2} = Nf_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Combining the above value of N with (39) (p. 242), we find that the value of N depends upon n, f_s/f_c, p' and d'/d. In Table XLI, values

TABLE XXXIX.—BEAMS WITH COMPRESSION STEEL

Values for p'.

$f_s = 16{,}000$. $n = 15$. $G = M/bd.^2$

f_c	G	p	Values of d'/d							
			.06	.08	.10	.12	.14	.16	.18	.20
500	72	.0050	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	80	.0056	.0014	.0015	.0017	.0019	.0022	.0025	.0030	.0036
	100	.0070	.0049	.0054	.0060	.0068	.0077	.0089	.0104	.0125
	120	.0085	.0084	.0093	.0103	.0116	.0132	.0153	.0189	.0214
	140	.0100	.0119	.0132	.0146	.0165	.0188	.0216	.0264	.0306
	160	.0115	.0154	.0171	.0190	.0214	.0243	.0280	.0338	.0397
	180	.0130	.0189	.0210	.0233	.0263	.0298			
600	95	.0068	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	100	.0072	.0007	.0008	.0009	.0010	.0011	.0012	.0014	.0016
	120	.0086	.0035	.0039	.0043	.0047	.0053	.0060	.0068	.0078
	140	.0100	.0064	.0070	.0077	.0085	.0093	.0107	.0122	.0141
	160	.0114	.0091	.0101	.0111	.0123	.0136	.0155	.0176	.0203
	180	.0127	.0120	.0132	.0145	.0161	.0178	.0202	.0230	.0266
	200	.0140	.0148	.0163	.0179	.0199	.0220	.0250	.0284	.0328
	220	.0155	.0176	.0194	.0214	.0237	.0262	.0298	.0338	.0391
650	108	.0078	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	120	.0086	.0015	.0017	.0019	.0021	.0023	.0025	.0028	.0033
	140	.0099	.0041	.0045	.0050	.0055	.0061	.0067	.0076	.0087
	160	.0112	.0067	.0074	.0081	.0089	.0099	.0109	.0124	.0141
	180	.0125	.0093	.0102	.0112	.0123	.0137	.0151	.0172	.0196
	200	.0139	.0119	.0130	.0143	.0157	.0174	.0194	.0219	.0250
	220	.0152	.0145	.0159	.0174	.0191	.0212	.0236	.0267	.0304
	240	.0165	.0171	.0187	.0205	.0225	.0250	.0278	.0315	.0358
	260	.0179	.0197	.0215	.0236	.0259	.0288	.0320	.0363	.0413
700	121	.0087	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	140	.0100	.0023	.0024	.0026	.0029	.0032	.0035	.0040	.0046
	160	.0114	.0046	.0050	.0054	.0060	.0066	.0073	.0082	.0094
	180	.0128	.0070	.0076	.0083	.0091	.0100	.0111	.0125	.0142
	200	.0143	.0094	.0102	.0111	.0122	.0135	.0149	.0167	.0190
	220	.0157	.0118	.0128	.0140	.0153	.0169	.0187	.0210	.0230
	240	.0172	.0142	.0154	.0168	.0184	.0203	.0227	.0252	.0286
	260	.0186	.0165	.0180	.0197	.0215	.0237	.0265	.0295	.0334
	280	.0200	.0189	.0206	.0225	.0246	.0272	.0303	.0337	.0382
	300	.0215	.0213	.0232	.0254	.0277	.0306	.0341	.0380	.0430
800	147	.0107	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	160	.0116	.0013	.0014	.0016	.0017	.0019	.0021	.0023	.0025
	180	.0131	.0034	.0036	.0040	.0043	.0048	.0052	.0058	.0064
	200	.0144	.0055	.0058	.0064	.0069	.0076	.0084	.0093	.0103
	220	.0159	.0075	.0080	.0088	.0095	.0105	.0115	.0128	.0143
	240	.0173	.0096	.0103	.0112	.0122	.0134	.0147	.0163	.0182
	260	.0188	.0116	.0125	.0136	.0148	.0163	.0179	.0198	.0221
	280	.0203	.0137	.0147	.0160	.0174	.0192	.0211	.0233	.0260
	300	.0217	.0158	.0169	.0184	.0200	.0220	.0242	.0268	.0299
	320	.0232	.0178	.0192	.0208	.0227	.0249	.0274	.0303	.0338

TABLE XL.—BEAMS WITH COMPRESSION STEEL

Values of f_s/f_c in terms of p and p'

$n = 15$

$\frac{d'}{d}$	p'	Values of p										
		.008	.009	.010	.011	.012	.014	.016	.018	.020	.022	.024
.06	.004	28.1	26.0	24.3	22.8	21.3	19.3	17.7	16.3	15.1	14.1	13.1
	.006	30.1	27.8	26.0	24.3	22.9	20.5	18.8	17.3	16.1	15.0	14.0
	.008	32.1	29.7	27.6	25.9	24.4	21.7	19.8	18.3	17.0	15.8	14.9
	.010	34.2	31.6	29.4	27.4	25.7	23.2	21.1	19.5	.17.9	16.7	15.7
	.012	36.3	33.4	31.0	29.1	27.3	24.5	22.2	20.4	18.9	17.6	16.5
	.014	38.4	35.3	32.6	30.6	28.7	25.8	23.4	21.6	19.8	18.4	17.3
	.016	40.5	37.2	34.3	32.2	30.2	27.0	24.6	22.8	20.7	19.3	18.2
	.018	42 3	39.0	36.1	33.8	31.7	28.3	25.9	23.8	21.7	20.1	18.9
	.020	44.1	40.8	37.9	35.3	33.2	29.6	27.2	24.8	22.7	21.0	19.7
.10	.004	27.5	25.6	23.9	22.4	21.1	19.1	17.5	16.1	15.0	14.0	13.1
	.006	29.2	27.2	25.3	23.7	22.4	20.2	18.5	17.0	15.8	14.8	13.8
	.008	30.8	28.7	26.7	24.9	23.7	21.3	19.4	18.0	16.7	15.5	14.6
	.010	32.6	30.1	28.1	26.3	25.0	22.4	20.4	18.9	17.6	16.4	15.4
	.012	34.3	31.5	29.5	27.7	26.2	23.5	21.5	19.7	18.4	17.2	16.1
	.014	36.0	33.1	30.9	29.0	27.4	24.6	22.5	20.6	19.2	18.0	16.9
	.016	37.6	34.6	32.3	30.2	28.6	25.8	23.5	21.5	20.0	18.7	17.6
	.018	38.1	36.1	33.7	31.6	29.8	26.9	24.4	22.4	20.8	19.5	18.3
	.020	40.6	37.7	35.1	32.9	30.9	27.9	25.4	23.4	21.6	20.2	18.9
.14	.004	26.9	25.1	23.4	22.0	20.8	18.8	17.3	16.0	14.8	13.8	12.9
	.006	28.4	26.4	24.6	23.2	21.9	19.8	18.2	16.8	15.5	14.5	13.6
	.008	29.8	27.6	25.8	24.3	23.0	20.8	19.0	17.5	16.3	15.2	14.3
	.010	31.2	28.9	27.0	25.4	24.1	21.8	19.9	18.3	17.1	16.0	15.0
	.012	32.5	30.2	28.2	26.5	25.1	22.7	20.7	19.1	17.8	16.7	15.7
	.014	33.7	31.4	29.4	27.6	26.1	23.6	21.6	19.9	18.5	17.4	16.3
	.016	34.9	32.5	30.5	28.7	27.1	24.5	22.4	20.6	19.2	18.0	16.9
	.018	36.1	33.7	31.6	29.8	28.1	25.4	23.2	21.3	19.9	18.7	17.5
	.020	37.1	34.9	32.6	30.8	29.0	26.2	24.0	22.0	20.6	19.3	18.1
.18	.004	26.1	24.6	23.1	21.7	20.5	18.6	17.1	15.7	14.6	13.6	12.8
	.006	26.9	25.6	24.1	22.7	21.4	19.4	17.8	16.4	15.3	14.3	13.4
	.008	28.6	26.6	25.0	23.6	22.3	20.2	18.5	17.1	15.9	14.9	14.0
	.010	29.6	27.6	25.9	24.5	23.2	21.0	19.3	17.8	16.5	15.5	14.6
	.012	30.6	28.6	26.8	25.3	24.0	21.8	20.0	18.5	17.2	16.1	15.2
	.014	31.6	29.6	27.7	26.1	24.8	22.6	20.7	19.2	17.9	16.7	15.8
	.016	32.6	30.5	28.6	27.0	25.6	23.3	21.4	19.8	18.5	17.3	16.3
	.018	33.6	31.4	29.4	27.8	26.4	24.1	22.1	20.4	19.1	17.9	16.9
	.020	34.5	32.2	30.3	28.6	27.2	24.8	22.8	21.0	19.7	18.4	17.4

TABLE XLI.—BEAMS WITH COMPRESSION STEEL

Values of N in Formula, $Nf_c = M/bd$;

$n = 15$

$\frac{f_s}{f_c}$	$\frac{d'}{d}$	Values of p'									
		.002	.004	.006	.008	.010	.012	.014	.016	.018	.020
16	.06	.228	.252	.277	.302	.327	.351	.376	.401	.425	.450
	.10	.224	.246	.267	.289	.310	.331	.353	.374	.395	.417
	.14	.221	.240	.258	.276	295	.313	.331	.349	.368	.386
	.18	.219	.234	.250	.265	.281	.297	.312	.328	.343	.359
18	.06	.217	.241	.266	.290	.315	.339	.364	.388	.413	.437
	.10	.213	.234	.255	.276	.297	.318	.339	.360	.381	.402
	.14	.210	.228	.246	.264	.281	.299	.317	.335	.353	.370
	.18	.207	.222	.237	.252	.266	.281	.296	.311	.326	.340
20	.06	.208	.232	.256	.281	.305	.329	.354	.378	.402	.426
	.10	.205	.225	.246	.267	.287	.308	.329	.349	.370	.391
	.14	.201	.219	.236	.253	.271	.288	.306	.323	.340	.357
	.18	.198	.212	.227	.241	.254	.269	.283	.297	.311	.326
22	.06	.199	.223	.247	.271	.295	.319	.343	.367	.391	.415
	.10	.195	.216	.236	.256	.277	.297	.317	.337	.358	.378
	.14	.192	.209	.226	.243	.260	.276	.293	.310	.327	.344
	.18	.187	.202	.216	.229	.243	.257	.270	.284	.297	.311
24	.06	.191	.215	.239	.263	.286	.310	.334	.353	.381	.405
	.10	.187	.207	.227	.247	.267	.287	.307	.327	.347	.367
	.14	.184	.200	.217	.233	.249	.266	.282	.299	.315	.332
	.18	.181	.193	.207	.220	.233	.246	.259	.272	.285	.295
26	.06	.183	.207	.230	.254	.277	.301	.324	.348	.375	.395
	.10	.179	.199	.218	.238	.257	.277	.296	.316	.335	.355
	.14	.176	.192	.207	.223	.239	.255	.271	.287	.303	.319
	.18	.172	.185	.197	.209	.222	.234	.246	.259	.271	.284
28	.06	.177	.201	.224	.247	.271	.294	.317	.340	.364	.387
	.10	.173	.193	.212	.231	.250	.270	.284	.308	.327	.346
	.14	.170	.185	.201	.216	.231	.247	.263	.278	.293	.309
	.18	.166	.178	.190	.202	.214	.226	.237	.249	.261	.273
30	.06	.172	.195	.218	.241	.264	.287	.310	.334	.357	.380
	.10	.168	.187	.205	.224	.243	.262	.281	.300	.319	.338
	.14	.164	.179	.194	.209	.224	.238	.254	.269	.284	.299
	.18	.160	.171	.183	.194	.205	.217	.228	.239	.251	.262
32	.06	.166	.188	.211	.233	.256	.278	.301	.323	.346	.368
	.10	.161	.180	.199	.217	.236	.254	.273	.292	.310	.329
	.14	.157	.172	.186	.201	.215	.230	.244	.259	.273	.289
	.18	.154	.165	.175	.186	.197	.208	.219	.229	.240	.251

of N are tabulated for various values of f_s/f_c, p' and d'/d when $n=15$.

These tables may be used in the investigation of beams of known dimensions and reinforcement, for the purpose of finding the safe resisting moment, or the unit stresses under given bending moment.

Examples.—The use of these tables will be best illustrated by a few examples.

18. Solve Problem 17 (p. 243) by the use of the tables.

Solution.—As $n=15$ and $f_s=16{,}000$ lb./in.², Table XXXIX (p. 246) may be used. $d'/d=.09$, $G=M/bd^2=\dfrac{1100000}{12\times22\times22}=190.$ From Table XXXIX (p. 246), with $f_c=700$, $G=190$ and $d'/d=.09$, we find directly that $p=.0136$ and $p'=.0093$, from which,

$$A_s=.0136\times12\times22\times3.59 \text{ in.}^2,$$

and
$$A'_s=.0093\times12\times22=2.46 \text{ in.}^2$$

19. A rectangular beam has the following dimensions; $b=10$ in., $d=18$ in., $d'=1.5$ in., and is to carry a bending moment of 550,000 in./lb. The safe unit stresses are 600 and 14,000 lb./in.² for concrete and steel respectively. $n=15$. Find the areas of steel required.

Solution.—$d'/d=1.5/18=.083$. From Table XXXVIII (p. 244) for $f_s=14{,}000$, $f_c=600$, and $d'/d=.083$, we find $R=102$, $p_1=.0084$, $f'_s=7090$ lb./in.²; substituting these values in (47) (p. 245) there results

$$A'_s=\frac{550000-102\times10\times18\times18}{7090\times16.5}=1.88 \text{ in.}^2$$

and (46) (p. 245)

$$A_s=.0084\times10\times18+\frac{7090}{14000}\times1.88=2.46 \text{ in.}^2$$

20. A rectangular beam in which $b=10$ in., $d=22$ in. and $d'=2$ in., is reinforced with 2.6 in.² of steel in tension and the same amount in compression. The beam carries a bending moment of 850,000 in.-lb. What are the maximum unit stresses upon the steel and concrete respectively?

Solution.—$p=p'=\dfrac{2.6}{10\times22}=.0118.$ $d'/d=.09$. For these values Table XL (p. 247) gives $f_s/f_c=26.5$ and Table XLI (p. 248) $N=.280$. Then formula (49) (p. 245)

$$f_c=\frac{850000}{.280\times10\times22\times22}=627 \text{ lb./in.}^2$$

$$f_s=627\times26.5=16620 \text{ lb./in.}^2$$

21. A rectangular beam has $b=10$ in., $d=16$ in., $d'=2$ in., $A_s=$ 2404 in.², $A'_s=2.25$ in.² If the safe unit stresses on steel and con-

crete are 16000 and 650 lb./in.^{2} respectively, what is the safe resisting moment for the beam?

Solution.—$p=\frac{2.4}{10\times16}=.015,\quad p'=\frac{2.25}{10\times16}=.014,\quad d'/d=.125.$ From Table XL (p. 247), for these values $f_s/f_c=24.6$ and (p. 246) Table XLI (p. 248), $N=.298$. If $f_s=16,000$, $f_c=16,000/23=696$ lb./in.^{2}, which is greater than is allowable. The safe moment will therefore be that which produces a stress of 650 lb./in.^{2} in the concrete. Substituting in (49) (p. 245),

$$M=.298\times650\times10\times16\times16=495,870 \text{ in.-lb.}$$

ART. 31. DIRECT STRESS AND FLEXURE

130. General.—In designing certain reinforced concrete sections which are subject to direct stress, and flexure due either to forces perpendicular to the axis or axial forces eccentrically applied, the use of the " transformed section " is convenient. Such problems are met with in the design of beams which are acted upon by axial forces, and in the design of the sections of elastic arches.

131. Transformed Section.—In order that the compressive deformation of steel and concrete may be equal, the allowed unit stress on

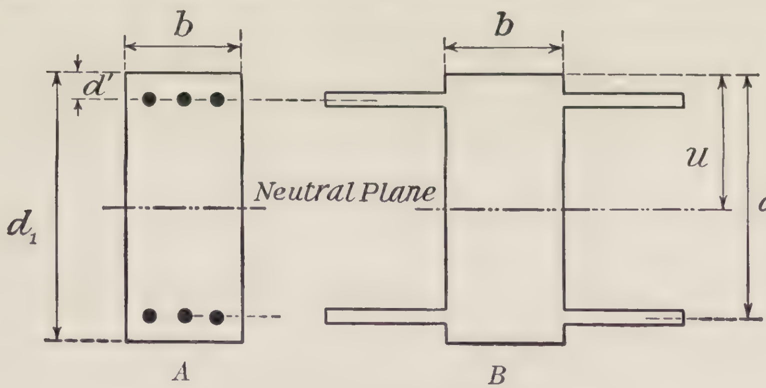


Fig. 61.—Transformed Section.

the steel must be limited to n times that specified for the concrete. In arch sections it is customary to use symmetrical reinforcement and sections with symmetrical reinforcement only will here be considered.

Figure 61 (a) shows a concrete arch section with steel reinforcement symmetrically disposed, and Fig. 61 (b) shows the transformed section with the steel replaced by n times its sectional area of concrete.

It has been shown in Section 54 (p. 85) that in order to preserve compression over the whole section when no reinforcement is used, the permissible eccentricity of the axial force must be limited to $\frac{1}{6}$ of the total depth of the section. The use of reinforcement increases the permissible eccentricity and the amount of permissible eccentricity is dependent upon the percentage of reinforcement, the embedment of the steel, and the value of n. With the center of symmetrical steel embedded $\frac{1}{10}$ of the total depth from each face and the value of $n=15$, the permissible eccentricities for wholly compressive stress in the section are approximately $\frac{2}{11}$, $\frac{1}{5}$, $\frac{2}{9}$, and $\frac{1}{4}$ of the total depth for steel at each surface of $\frac{1}{2}$, 1, $1\frac{1}{2}$, and 2 per cent respectively.

For convenience the face having the greater stress may be designated as the "greater compressive face," and the opposite face as the "lesser compressive face."

The method of procedure is similar to that given in Sec. 54 (p. 85). In addition to the notations used in Sections 54, 116, and 128,

Let u = distance from greater compressive face to centroid of transformed section $\left(=\frac{d_1}{2}\right.$, for symmetrically reinforced section);

$A_c = bd_1$ = area of concrete;

A_s = area of steel near lesser compressive face;

A'_s = area of steel near greater compressive face ($=A_s$ for symmetrically reinforced section);

A_t = area of transformed section [$=A_c+(n-1)2A_s$ for symmetrically reinforced section];

$I_c = \frac{bd_1^3}{12}$ = moment of inertia of concrete;

I_s = moment of inertia of steel [$=2A_s\left(\frac{d_1}{2}-d'\right)^2$ for symmetrically reinforced section];

$I_t = I_c + nI_s = \frac{bd_1^3}{12} + (n-1)2A_s\left(\frac{d_1}{2}-d'\right)^2$;

f = unit stress due to concentric axial force, F;

$f' = \frac{Mu}{I_t} = \frac{Fed_1}{2I_t}$ = unit fiber stress due to flexure;

f_c = unit fiber stress on concrete at greater compressive face;

f''_c = unit fiber stress on concrete at lesser compressive face;

Then
$$f_c = f + f' = \frac{F}{A_t} + \frac{Fed_1}{2I_t}, \tag{50}$$

and
$$f_c''=f-f'=\frac{F}{A_t}-\frac{Fed_1}{2I_t}. \tag{51}$$

The moment of inertia of the steel about the axes through the center of the steel is negligible.

Both the value of f_c and f_c'' must be positive (or f_c'' may be zero) for compression over the whole section, and the value of f_c must be within the specified limits for the concrete.

While usually it is not economical to use steel with such low unit stresses in place of concrete, in arch construction the gain in permissible eccentricity, the greater reliability of the results, and the prevention of cracking of the concrete more than offset the slight additional cost.

Example.

22. A certain arch section has a width, $b=12$ inches and a total depth, $d_1=28$ inches. The reinforcement is 1 per cent above and an equal amount below the neutral plane. The axial force, F, is 115,000 pounds and its eccentricity of application is 5 inches, while the permissible eccentricity for wholly compressive stress at the section is 5.6 inches. The steel is embedded $\frac{1}{10}$ of the total depth from each face and the value of n is 15. The allowed unit stress on the concrete is 600 lb./in.², and on the steel is $15\times600=9000$ lb./in.² Compute the fiber stresses in the concrete and the steel at both surfaces.

Solution.

$$A_t=A_c+(n-1)2A_s=bd_1+(n-1)2A_s$$
$$=336+94=430 \text{ in.}^2,$$

from which

$$f=\frac{F}{A_t}=\frac{114000}{430}=268 \text{ lb./in.}^2,$$

$$I_t=I_c+nI_s=\frac{bd_1{}^3}{12}+(n-1)2A_s\left(\frac{d_1}{2}-d'\right)^2$$
$$=21950+11900=33,850 \text{ in.}^4$$

$$f'=\frac{Fed_1}{2I_t}=\frac{115000\times5\times28}{2\times33850}=240 \text{ lb./in.}^2,$$

from Formula 50 (p. 251),

$$f_c=f+f'=268+240=508 \text{ lb./in.}^2,$$

and from Formula 51,

$$f_c''=f-f'=268-240=\ 28 \text{ lb./in.}^2$$

As there is compression over the whole section, the neutral plane will be below the lesser compressive face, and the distance of the neutral plane below the greater compressive face will be,

$$kd_1 = (kd_1 - d_1)\frac{f_c}{f_c''}$$

$$= (kd_1 - 28)\frac{508}{28} = 29.63 \text{ in.}$$

As the center of the reinforcement which is subject to the greater compression is 2.8 inches from the greater compressive face,

$$f'_s = \frac{26.83}{29.63} \times 508.15 = 6900 \text{ lb./in.}^2$$

The center of reinforcement which is subject to the lesser compression is 2.8 inches from the lesser compressive face, and

$$f_s = \frac{4.43}{29.63} \times 508 \times 15 = 1140 \text{ lb./in.}^2$$

132. Tension in Part of the Section.—While some designers prefer to proportion arch sections so that the stresses in the section are wholly compressive, others favor the utilization of the reinforcing steel for tension. When the eccentricities are greater than the limits given in Sec. 131, tension will exist in part of the section. For convenience the more highly stressed face may be designated as the "compression face," and the opposite face, as the "tension face."

For finding the unit stresses in the concrete and in the steel due to the concentric axial force, the transformed section may be used, but for finding the flexural unit stresses due to the eccentric application of the force, the procedure used for beams reinforced for compression will be followed. The tension in the concrete between the neutral plane and the tension face will be neglected. The total unit stress in the concrete will be the sum of the unit stresses due the direct force and flexure, and the same will be true of the total compressive unit stress in the steel near the compression face; the total tensile unit stress in the steel near the tension face will be the algebraic sum of the unit stresses due to the direct force and flexure. The total unit stresses in all cases must of course be within the limits prescribed by the specifications.

Example.

23. A certain arch section has a width, b, of 12 inches and a total depth, d_1, of 20 inches. The reinforcement is 1 per cent above and an equal amount below the neutral plane, being embedded $\frac{1}{10}$ of the total depth in each case. The value of n is 15. The allowed

compressive unit stress on the concrete is 650 lb./in.^{2} and on the steel, 9750 lb./in.^{2}; and the allowed tensile unit stress on the steel is 16,000 lb./in.^{2} The force, F, normal to the section is 88,000 pounds and its eccentricity of application is 5 inches. As, under the conditions given, the permissible eccentricity for wholly compressive stress in the section is 4 inches, there will be tension in part of the section. Various values are as follows: $d=18$ inches, $d'=2$ inches, $A_c=240$ in.^{2}, $A_s=A'_s=2.4$ in.^{2}

Compute the actual fiber stresses in the concrete, and in the steel at both the compression and tension faces.

Solution.—The area of the transformed section is

$$A_t=A_c+(n-1)2A_s=240+67.2=307.2 \text{ in.}^2,$$

from which

$$f=\frac{F}{A_t}=\frac{88000}{307.2}=286 \text{ lb./in.}^2$$

For $f_s=16{,}000$ and $f_c=650$ lb./in.^{2}, Table XXII (p. 199) gives $k=0.379$ and $j=0.874$.

$$jd=.874\times 18=16.73 \text{ inches}$$

The bending moment due to eccentricity is

$$M=Fe=88000\times 5=440000 \text{ in.-lb.}$$

and the total compression in the section due to bending is

$$C=\frac{M}{jd}=\frac{440000}{15.73}=28000 \text{ pounds}$$

The stress on the concrete due to the concentric axial force is $f=286$ lb./in.^{2}, and the available unit stress for bending is $f_c'''=(f_c-f)=650-286)=364$ lb./in.^{2}

If C'' is the total available value of concrete for compression in the section,

$$C''=\tfrac{1}{2}bkd\times f_c'''=40.93\times 364=14900 \text{ pounds}$$

and the total compression on the steel will be

$$C'=28000-14900=13100 \text{ pounds}$$

As the area of the compression steel is 2.4 in.^{2}, the compressive unit stress on the steel due to flexure is $f_s''=\dfrac{13100}{2.4}=5460$ lb./in.^{2}

The unit stress on the compression steel due to the concentric axial force is $nf=15\times 286=4290$ lb./in.^{2} and the total unit compression on this steel is

$$f_s'=f_s''+nf=5460+4290=9750 \text{ lb./in.}^2\text{, as allowed.}$$

The total tension on the steel due to bending is

$$T=\frac{M}{jd}=\frac{440000}{15.73}=28000 \text{ pounds},$$

and the tensile unit stress due to bending is

$$f_s'''=\frac{28000}{2.4}=11666 \text{ lb./in.}^2$$

The unit compressive stress due to the concentric axial force is

$$nf=15\times 286=4290 \text{ lb./in.}^2$$

making the net unit tensile unit stress

$$f_s=f_s'''-nf=11666-4290=7376 \text{ lb./in.}^2,$$

which is satisfactory.

ART. 32. SLAB AND BEAM DESIGN

133. Bending Moments and Shears.—Structural forms in which slabs of concrete are supported by T-beams are very common in reinforced concrete structures. In this type of construction, the slab is commonly made continuous over the T-beam and forms the flange of the T-beam (see Fig. 62, p. 258), being built with the beam and a part of it. In determining the bending moments and shears in such construction, the loads may usually be taken as uniform, and the slabs and beams as fully or partly continuous, depending upon the method of support.

Fully Continuous Beams. If a slab which passes over one or more cross-beams is firmly held at the ends by being built into and tied by reinforcement to a wall or heavy beam, it may be considered as fully continuous, and when uniformly loaded, the positive moments of the middle of the spans are $\frac{1}{24}wl^2$ and the negative moments at supports $\frac{1}{12}wl^2$. The shear at each end of span in such a beam is $\frac{1}{2}wl$. If the movable load covers some of the spans leaving others unloaded, these moments may be somewhat increased. For slabs of this type, it is conservative practice to use $\frac{1}{12}wl^2$ for both positive and negative bending moments and $\frac{1}{2}wl$ for maximum vertical shear.

Supported Ends.—The ends of continuous beams, resting upon side walls or end columns, cannot be considered as fixed, and are to be taken as simply supported. Such a beam, or a slab the ends of which are not fixed, has greater positive moments in the end spans and greater negative moments at the first supports from the ends than fully continuous beams. These moments are usually taken as $\frac{1}{10}wl^2$ for beams of more than two spans. The shear in the end span next the first support may be greater than one-half the load on the

span and should be taken as $.6wl$. For beams of two spans, the negative moment at the middle support is taken as $\frac{1}{8}wl^2$, and the positive moment as $\frac{1}{10}wl^2$.

The moments for continuous beams of unequal spans, or with concentrated and uneven loading should be carefully determined for each individual case.

The Joint Committee makes the following recommendations in its 1924 report:

Span Length.—The span length, l, of freely supported beams and slabs shall be the distance between centers of the supports, but shall not exceed the clear span plus the depth of beam or slab. The span length for continuous or restrained beams built to act integrally with supports shall be the clear distance between faces of supports. Where brackets having a width not less than the width of the beam and making an angle of 45° or more with the horizontal axis of a restrained beam are built to act integrally with the beam and support, the span shall be measured from the section where the combined depth of the beam and bracket is at least one-third ($\frac{1}{3}$) more than the depth of the beam, but no portion of such a bracket shall be considered as adding to the effective depth of the beam. Maximum negative moments are to be considered as existing at the ends of the span, as defined above.

Slightly Restrained Beams of Equal Span.—Beams and slabs of equal spans built to act integrally with beams, girders, or other slightly restraining supports and carrying uniformly distributed loads shall be designed for the following moments at critical sections:

(*a*) Beams and slabs of one span,

Maximum positive moment near center,

$$M = \frac{wl^2}{8} \quad \text{. . . . .} \quad (12)^*$$

(*b*) Beams and slabs continuous for two spans only,

1. Maximum positive moment near center,

$$M = \frac{wl^2}{10} \quad \text{. . . . .} \quad (13)^*$$

2. Negative moment over interior support,

$$M = \frac{wl^2}{8}. \quad \text{. . . . .} \quad (14)^*$$

(*c*) Beams and slabs continuous for more than two spans,

1. Maximum positive moment near center and negative at support of interior spans,

$$M = \frac{wl^2}{12}. \quad \text{. . . . .} \quad (15)^*$$

* Joint Committee formulae numbers.

2. Maximum positive moment near centers of end spans and negative moment at first interior support,

$$M=\frac{wl^2}{10} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (16)^*$$

(*d*) Negative moment at end supports for Cases (*a*), (*v*), and (*c*) of this section,

$$M=\text{not less than } \frac{wl^2}{16}. \quad . \ . \ . \ . \ . \ . \ . \ . \quad (16a)^*$$

Beams Built into Brick or Masonry Walls.—Beams or slabs built into brick or masonry walls in a manner which develops partial end restraint shall be designed for a negative moment at the support of,

$$M=\text{not less than } \frac{wl^2}{16}. \quad . \ . \ . \ . \ . \ . \ . \ . \quad (17)^*$$

Freely Supported Beams of Equal Spans.—Beams and slabs of equal spans freely supported and assumed to carry uniformly distributed loads shall be designed for the moments specified for "Slightly Restrained Beams of Equal Span," except that no reinforcement for negative moment need be provided at end supports where effective measures are taken to prevent end restraint. The span shall be taken as defined under "Span Length" for freely supported beams.

Restrained Beams of Equal Span.—Beams and slabs of equal span built to act integrally with columns, wall, or other restraining supports and assumed to carry uniformly distributed loads, shall be designed (except as provided under "Slightly Restrained Beams of Equal Spans") for the following moments at critical sections:

(*a*) Interior spans,

1. Negative moment at interior supports except the first,

$$M=\frac{wl^2}{12}. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (18)^*$$

2. Maximum positive moment near centers of interior spans,

$$M=\frac{wl^2}{16}. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (19)^*$$

(*b*) End spans of continuous beams and beams of one span in which $\frac{I}{l}$ is less than twice the sum of the values of $\frac{I}{h}$ for the exterior columns above and below which are built into the beams:

1. Maximum positive moment near center of span and negative moment at first interior supports,

$$M=\frac{wl^2}{12}. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (20)^*$$

* Joint Committee formula number.

2. Negative moment at exterior supports,

$$M = \frac{wl^2}{12}. \qquad \text{(21)*}$$

(c) End spans of continuous beams, and beams of one span, in which $\frac{I}{l}$ is equal to or greater than twice the sum of the values of $\frac{I}{h}$ for the exterior columns above and below which are built into the beam:

1. Maximum positive moment near center of span and negative moment at first interior support,

$$M = \frac{wl^2}{10}. \qquad \text{(22)*}$$

2. Negative moment at exterior support,

$$M = \frac{wl^2}{16}. \qquad \text{(23)*}$$

Continuous Beams of Unequal Spans or with Non-Uniform Loading.—Continuous beams with unequal spans, or with other than uniformly distributed loading, whether freely supported or retrained, shall be designed for the actual moments under the conditions of loading and restraint.

Provision shall be made where necessary for negative moment near the center of short spans which are adjacent to long spans, and for the negative moment at the end supports, if restrained.

Unsupported Flange Length.—The distance between lateral supports of the compression area of a beam shall not exceed twenty-four (24) times the least width of the compression flange.

134. Loading of Slabs, Beams and Girders.—Slabs are commonly used as continuous beams passing over a number of T-beams, of which the slab forms the flange as shown in Fig. 62.

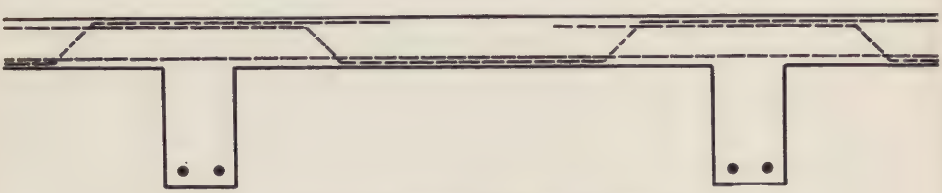


FIG. 62.—Reinforced Slab and T-Beam.

They are reinforced for tension in one direction, perpendicular to the T-beams, and in computation are considered as rectangular beams one foot in width. The T-beams supporting such slabs frequently rest

* Joint Committee formula number.

upon girders, which are used to widen the interval between columns, and permit the T-beams to be spaced close enough for economical design of slab. The load upon a T-beam in such a system is uniformly distributed and consists of the weight of a half span of the slab and its load, on each side of the beam. The loads upon the girders are concentrated at the points where the T-beams cross, but may usually be taken as uniformly distributed without material error.

Double Reinforced Slabs.—Slabs of long span and nearly square in plan may be supported on all four sides and reinforced in both

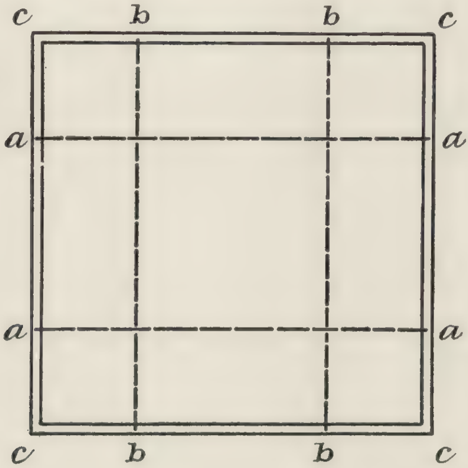


FIG. 63.—Double-reinforced Slabs.

directions. It is not feasible to make an accurate analysis of the distribution of loadings in such a slab. When the length and width of slab are equal, it is assumed that the reinforcement in each direction carries one-half the load as uniformly distributed. The loads carried by the mid-sections (*aaaa, bbbb,* Fig. 63) are, however, greater than those carried by the sections next the supports, and the reinforcement should be spaced closer in the middle than at the sides. It is suggested that the mid-half area (*aaaa, bbbb,* Fig. 63) of the slab be considered as carrying $1\frac{1}{3}$ times the average load, and the side sections (*acca, bccb*) two-thirds of the average load per square foot of slab.

When the slabs are not square, the reinforcement parallel to its shorter dimension carries the greater part of the load. The Joint Committee makes the following recommendation concerning the division of the loads in such slabs:

Floor slabs having the supports extending along the four sides should be designed and reinforced as continuous over the supports. If the length of the

slab exceeds 1.5 times its width the entire load should be carried by transverse reinforcement.

For uniformly distributed loads on square slabs, one-half the live and dead load may be used in the calculations of moment to be resisted in each direction. For oblong slabs, the length of which is not greater than one and one-half times their width, the moment to be resisted by the transverse reinforcement may be found by using a proportion of the live and dead load equal to that given by the formula $r=\frac{l}{b}-0.5$, where l = length and b = breadth of slab. The longitudinal reinforcement should then be proportioned to carry the remainder of the load.

In placing reinforcement in such slabs account may well be taken of the fact that the bending moment is greater near the center of the slab than near the edges. For this purpose two-thirds of the previously calculated moments may be assumed as carried by the center half of the slab and one-third by the outside quarters.

An interesting discussion of the distribution of stresses in double reinforced slabs may be found in a paper by Mr. A. C. Janni in Transactions of the American Society of Civil Engineers, 1916.

135. Problems in Design.—The use of the formulas and tables which have been given, in designing slab and beams, will be illustrated by the solution of a few problems. In these examples, the working stresses recommended by the Joint Committee for 2000 pounds concrete will be used.

Example 24.—A concrete slab is to be supported by T-beams 6 feet apart c. to c., and to carry a live load of 250 pounds per square foot. The T-beams have a clear span of $17\frac{1}{2}$ feet and are built into brick walls at the ends. Design the slab and beams.

Solution.—Assume the weight of slab as 50 pounds per square foot, giving a total load of 300 pounds per linear foot for a section of slab 12 inches wide. Taking the slab as fully continuous.

$$M=\frac{wl^2}{12}=\frac{300\times6\times6\times12}{12}=10800 \text{ in.-lb.}$$

From Table XXII (p. 199), for $f_s=16000$ and $f_c=650$, $R=108$, $p=.0077$ and $j=.874$. Formula 9 (p. 197) gives $12d^2=10,800/108=100$, and $d=2.9$ inches, use 3 inches. $A_s=pbd=3\times12\times.0077=.277$ in.^{2} From Table XLI (p. 248), we select $\frac{3}{8}$-inch round bars spaced 4.5 inches apart, $A_s=.29$ in.^{2}

If concrete extends $\frac{3}{4}$ inch below steel, the thickness of slab is $3\frac{3}{4}$ inches, and the weight of slab is $150\times3.75/12=47$ pounds per square foot, which agrees with the assumed load.

Reinforcement for negative moment over the supports should be the same as for positive moment at mid-span and will be provided by turning up every alternate bar at the quarter point on each side of the

support and continuing them over the support to the one-third point. Transverse reinforcement to prevent cracks will be provided by using $\frac{3}{8}$-inch bars spaced 12 inches apart.

Unit shear at ends of slab,

$$v=\frac{V}{bjd}=\frac{300\times 3}{12\times .874\times 3}=28.6 \text{ lb./in.}^2$$

No diagonal tension reinforcement is necessary.

T-beam.—Assuming the weight of the web of the T-beam as 150 pounds per linear foot, the total load on the T-beam is 6(250+47) +150=1930 pounds per linear foot. Taking the bearing upon the wall as 6 inches the effective length of T-beam between centers of bearings is 17.5+.5=18 feet.

The maximum shear $V=1930\times 9=17{,}370$ pounds. The area required for shear, assuming $j=\frac{7}{8}$, $b'd=\frac{V}{vj}=\frac{17370}{105}=165$ in.^{2}

For $b'=8$, $d=21$ or for $b'=9$, $d=18.5$. Take $b'=9$, and $d=18.5$.

$$M=\frac{wl}{8}=\frac{1930\times 18\times 18\times 12}{8}=938{,}000 \text{ in.-lb.}$$

Width of flange $b=2\times 6\times 3\frac{3}{4}+9=54$ inches, and by (31)

$$Q=\frac{M}{btd}=\frac{938000}{54\times 3.75\times 18.5}=250;\ \frac{d}{t}=4.95;$$

from Diagram XIV (p. 237), $f_c=430$ lb./in.^{2}, and $p=.0035$, then

$$A_s=pbd=.0035\times 54\times 18.5=3.50 \text{ in.}^2,$$

and from Table XXVI (p. 203), six $\frac{7}{8}$-inch round bars in two rows, 2 inches c. to c., spaced 2.75 inches apart in the rows and 1.75 inches from side of web. $A_s=3.61$ in.^{2}

As the ends of the beam are built into the walls, some negative moment may be developed at the supports, which might cause cracks to occur unless reinforced. The upper layer of reinforcement will therefore be turned up, two rods at the quarter point and the other midway between the quarter point and support, and extend to the end of the beam (see Fig. 64, p. 262), all having hooked ends.

If the concrete extend 2 inches below the steel, the weight of web below the slab is $9(18.5+2.75-3.75)\times 150/144=164$ pounds per linear foot. This is a little greater than the assumed value, but would add less than 1 per cent to the total load and need not be redesigned. For the three bars in bottom of beam at the support, Table XXVI gives $\Sigma o=3\times 2.75=8.25$, and the unit bond stress $u=b'v/\Sigma o=9\times$

120/8.25 = 131 lb./in.² This is too great for safety, and the bars should be bent into hooks at the ends.

As the beam is designed for a unit shear of 120 lb./in.², it must be reinforced for diagonal tension for 120 − 40 = 80 lb./in.² from the support to the section where $v = 40$ lb./in.² The shear at the center of the span must be assumed as one-fourth ($\frac{1}{4}$) of the maximum end shear, or 17370/4 = 4340 pounds. Formula 14 (p. 218)

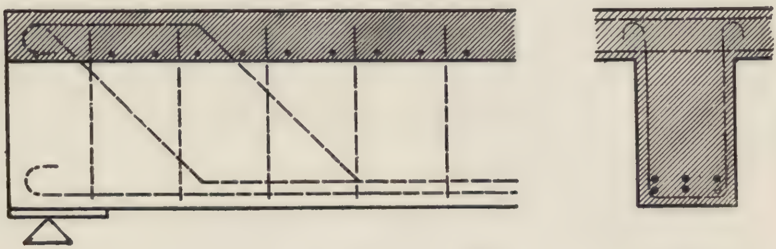


FIG. 64.—T-Beam Design.

gives the distance from the center of the span to the section where the stirrups may be discontinued,

$$x = \frac{(160 - v_m)l}{6 \times v_m} = 40 \times 18 \times 12/6 \times 120 = 12 \text{ inches,}$$

and stirrups will be needed to within 12 inches of the center of the span. The maximum spacing for vertical stirrups is, $.45d = .45 \times 18.5 = 8.32$ inches. Using $\frac{1}{2}$-inch round bar U-stirrups, the spacing next the support, by Formula 33 (p. 235), must be

$$s = A_v f_v / vb' = .39 \times 16000/80 \times 9 = 8.7 \text{ inches.}$$

The arrangement of the stirrups may then be, one space of 4 inches, and the balance of 8 inches, measuring from the center of the support toward the center of the span.

Example 25.—A reinforced concrete slab, to carry a live load of 200 pounds per square foot, is to rest upon a series of T-beams 5 feet apart c. to c. The T-beams are to be continuous for three spans over girders 15 feet c. to c. The girders are supported by walls at the ends and have a span of 20 feet. Design the slab and beams.

Solution.—Assume the weight of slab at 40 pounds per square foot; then $M = \frac{240 \times 5 \times 5 \times 12}{12} = 6000$ in.-lb. From Table XXII (p. 199) $R = 108$, $p = .0077$, $j = .874$, $12d^2 = 6000/108 = 55.5$ and $d = 2.15$. Take $d = 2.25$ in. $A_s = 2.25 \times 12 \div .0088 = .2$ in.²

If concrete extends $\frac{3}{4}$-inch below steel, the total depth of slab is 3 inches, and weight of slab is $150 \times \frac{3}{12} = 37.5$ pounds per square foot.

From Table XXVII (p. 204) $\frac{3}{8}$-inch round bars spaced 6 inches apart give $A_s = .22$ in.^{2} Negative moment at supports will be provided for by bending these up at the quarter points. For lateral reinforcement to prevent cracks, $\frac{3}{8}$-inch round bars spaced 18 inches c. to c. will be used.

T-beams.—Assuming weight of web of T-beam as 125 pounds per linear foot, load upon T-beam is $5(200+40)+125=1325$ pounds per linear foot and total span load is $1325 \times 15 = 19{,}875$ pounds.

Maximum shear in end span next girder is $V = 19{,}875 \times .6 = 11{,}925$ pounds, and $b'd = V/jd = 11{,}925/105 = 113$ in.^{2} Either 7×16 or 8×14 might be used. Try 7×16, then $M = Wl/10 = 19{,}875 \times 15 \times 12/10 = 357{,}750$ in.-lb. Taking overhang of flange as six times its depth, $b = 2 \times 6 \times 3 + 7 = 43$ inches and Formula (38) (p. 236).

$$Q = \frac{357750}{43 \times 3 \times 16} = 173, \; d/t = 5.3.$$

From diagram XIV (p. 237), $f_c = 325$ lb./in.^{2} and $p = .0024$. $A_s = 1.65$ in.^{2} Table XXVI (p. 203), six $\frac{5}{8}$-inch round bars, $A_s = 1.84$ in.^{2} in two rows, $1\frac{3}{4}$ inches apart and spaced 2 inches c. to c. and 1.5 inches from side of web.

If concrete extends 2 inches below steel, the weight of web below slab is $7 \times 16 \times 150/144 = 117$ pounds per linear foot, which is within the assumed load.

The negative moment at crossing of girder is equal to the positive moment already found. Turn up the upper row of bars on each side to provide for tension at top of beam and run the lower ones through the bottom to provide compression reinforcement as shown in Fig. 65 (p. 264). We now have a beam with compression reinforcement, in which $b = 7$, $d = 16$, $d' = 3$, $A_s = A_s = 1.84$, $p = p' = 1.84/112 = .0164$, $d'/d = .18$.

Formula (48) (p. 245) gives $G = \frac{M}{bd^2} = \frac{357750}{7 \times 16 \times 16} = 200$, and from Table XXXIX (p. 244) for $f_s = 16{,}000$, $f_c = 650$, $G = 200$, and $d'/d = .18$, we find that $p = .0139$ and $p' = .0219$ are required. The area of steel in compression ($p' = .0164$) is not sufficient and we must either increase the area of compression steel in the bottom of beam or increase the area of concrete section over the support. Try making $d = 17$ inches. Then $p = p' = 1.841/19 = .0157$, $d'/d = 2.875/17 = .17$ and $G = \frac{357750}{7 \times 17 \times 17} = 177$. Now from Table XXXIX, we find that

$p=.0123$ and $p'=.0156$ are required. The reinforcement is now sufficient and we will increase the depth to 17 inches at the girder, sloping the haunches as shown in Fig. 65.

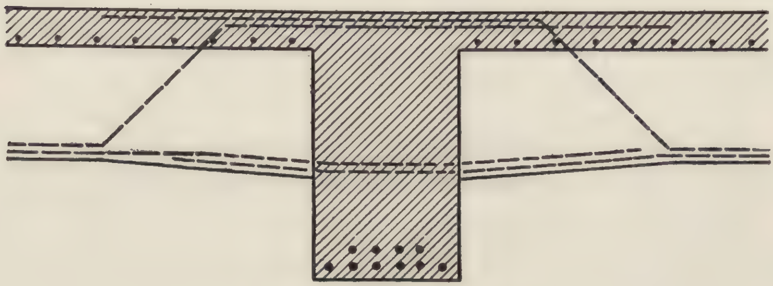


FIG. 65.—T-Beam and Girder.

Diagonal Tension.—The diagonal tension must be taken care of by the web of the T-beam. The unit shear at the support, assuming $j=.86$, is $v=11925/7\times.86\times17=117$ lb./in.[2] Using $\frac{3}{8}$-inch round vertical U-stirrups, Formula 33 (p. 235) gives, $s=A_v f_v/vb'=.22\times 16000/77\times7=6.53$ inches. The maximum allowed spacing for vertical stirrups is, $s=.45d=.45\times16=6.5$ inches. It is required that the shear at the section where the computed shear is zero shall be taken as one-fourth of the maximum shear at the support, or $11925/4=2980$ pounds. The distance from the center of the span (approximate section of zero shear) to the section where the unit shear is 40 lb./in. is, by Formula 14 (p. 218), $x=\dfrac{(160-v_m)l}{6v_m}=43\times180/792=11$ inches. The arrangement of stirrups may then be, one space of 3 inches, and the balance of 6 inches, measured from the center of the support toward the center of the span.

Girders.—The girders are simple rectangular beams carrying three concentrated loads at the middle and quarter points. Each load is 1.1 times a span load of the T-beam, or $1.1\times19875=21862$ pounds; assuming that the girder weighs 800 pounds per linear foot, the reaction or shear at the support is $1.5\times21862+800\times10=40793$ pounds and the maximum bending moment $M=40{,}793\times(120{,}29{,}862)\times60=3{,}103{,}440$ in.-lb.

Table XXX (p. 207) gives a beam with a width of 20 inches and a depth of 38 inches together with the area of the necessary steel, 5.86 in.[2] Table XXVI (p. 203) gives ten $\frac{7}{8}$-inch round bars with an area of 6.01 in.[2] These will be used and arranged in two tiers 2 inches apart on centers, six bars being placed in the lower and four

in the upper tier. Two bars of the upper tier may be turned up at 30 inches, and two at 60 inches from the support. Making the distance from the centers of the lower bars to the underside of the girder $2\frac{1}{2}$ inches, the total depth will be $41\frac{1}{2}$ inches, and the weight of the girder below the slab will be $20\times38.5\times150/144=800$ pounds, as assumed.

The maximum end shear is 40,739 pounds and Diagram XI (p. 222) shows that for a 20+38-inch beam the unit shear is 62 lb./in.^{2} Diagonal tension reinforcement will be needed from the support to the first load. The shear $V_d=20\times38\times(62-40)=16{,}720$ pounds. Diagram XII (p. 223) shows that for the given shear, $\frac{3}{8}$-inch round vertical U-stirrups spaced, $0.3d=.3\times38=11.4$ inches will answer. As the shear is uniform, the first stirrup will be spaced 5 inches from the center of the support and the balance 10 inches c. to c.

The shear between the quarter-point and the center is 10,931 pounds, and no stirrups are needed.

The bond stress on six horizontal bars at the end of the girder is, $bv/\Sigma_0=20\times62/6\times2.75=75$ lb./in.^{2} as against 80 lb./in.^{2} allowed.

Example 26.—A reinforced concrete slab, divided into panels 12 ft.$\times$14 feet, by T-beam supports is to carry a live load of 150 pounds per square foot. The T-beams are supported by columns at the corners of the panels; their ends resting upon side walls. Design the slab and beams.

Solution.—Assume the weight of slab at 75 pounds per square foot. The proportion of load carried by the 12-foot span is 14/12—0.5=.67 (see Section 134). The load on the slab in the 12-foot length is $(150+75)\times.67=150$ pounds per square foot and in the 14-foot length $220\times.34=75$ pounds per square foot. If 4/3 of the average load per square foot be borne by the mid-section, the load to be carried by the 12-inch width will be $150\times4/3=200$ pounds per linear foot.

$$M=\frac{wl^2}{12}=\frac{200\times12\times12\times12}{12}=28{,}800, \text{ in.-lb.}$$

Table XXVII (p. 204) gives a slab with a depth, $d=4.75$ inches and an area of steel, $A_s=0.44$ in.^{2} Table XLI (p. 248) gives $\frac{1}{2}$-inch round bars spaced 5 inches on centers, and these will be used. If the concrete extends $1\frac{1}{4}$ inch below the center of the steel, the total depth of slab, d_1, will be 6 inches, and the weight of the slab per square foot will be 75 pounds, as assumed.

Alternate bars in each span will be turned up at the quarter-points for negative moments at the supports.

The side-sections of the shorter span will carry one-half as much

bending moment as the mid-sections, and the distance center to center of the reinforcing bars may be increased to 6, 7, and 8 inches.

For the longer span (14 feet) the load upon the mid-section will be $75\times\frac{3}{4}=100$ pounds per linear foot, and the bending moment will be

$$M=\frac{wl^2}{12}=\frac{100\times14\times14\times12}{12}=19,600 \text{ in.-lb.}$$

The reinforcement in the 14-foot direction will be placed on top of that in the shorter span, and the effective depth will be $\frac{1}{2}$ inch less, or $d=4.75-0.5=4.25$ inches. $R=M/bd^2=19,600/12\times4.25\times4.25=90$. Table XXII (p. 199) gives for $f_s=16,000$ and $R=90$, $p=.0064$ and $f_c=580$. $A_s=.0064\times12\times2.25=0.33$ in.^{2} Table XLI (p. 248) gives $\frac{1}{2}$-inch round bars spaced 7 inches center to center. For the side-sections, use bars of the same size with gradually increasing spaces from 7 to 10 inches.

T-Beams.—Assuming the longer T-beam to weigh 225 pounds per linear foot, the total load will be $200\times12\times14+225\times14=36,750$ pounds. The maximum shear will be $36,750\times.6=22,050$ pounds, and the section needed for shear will be $b'd=\frac{V}{vj}=22050/105-210$ in.^{2}

Use $b'=10$ in. and $d=21$ inches.

The load at the middle of the beam is greater than that at the ends but the error will not be greater than 2 per cent if the load is taken as uniformly distributed.

$$M=\frac{Wl}{10}=36,750\times14\times12/10=617,400 \text{ in./lb.}$$

Taking the width of flange as one-fourth of the span,

$$b=42 \text{ inches.}$$

Formula 38 (p. 236) gives

$$Q=\frac{M}{btd}=\frac{617400}{24\times6\times21}=117,\ \frac{d}{t}=21/6=3.5.$$

Diagram XIV (p. 237) shows that the neutral axis is in the flange and the beam is to be designed as a rectangular section.

$$R=\frac{M}{bd^2}=\frac{617400}{42\times21\times21}=33.$$

Table XXII (p. 199), for $f_s=16,000$ and $R=33$, shows that f_c is less than 350 lb./in.^{2}, and the steel needed is $p=.0023$ ($p=7R/100,000$, approximately), or $A_s=.0023\times42\times21=2.03$ inches. Four

$\frac{3}{4}$-inch square bars with an area of 2.25 in.^{2} will be used, and two of them will be turned up at the quarter-point on each side of the support to provide for tension due to negative moment.

The result of the design is a beam with double reinforcement at the support, in which $b=10$ inches, and $d=21$ inches.,

$$A_s=A'_s=2.25 \text{ in.}^2,\ p=p'=2.252/10=.0107$$

$$d'=2+\text{in.},\ d'/d=2/21=0.95, \text{ and } G=\frac{617400}{10\times21\times21}=140.$$

From Table XXXVIII (p. 244), for $f_s=16{,}000$, $f_c=650$, $G=140$, and $d'/d=.095$, it is found that $p=.0099$ and $p'=.0049$ are required. The reinforcement for both tension and compression is ample.

As the beam was designed for a unit shear of 120 lb./in.^{2}, diagonal tension reinforcement is needed. For the maximum shear at the support, $V=22{,}050$ pounds, and $V_d=22{,}050-40\times10\times21\times\frac{7}{8}=$ 14,700 pounds. Diagram X (p. 221) shows that $\frac{3}{8}$-inch round bar vertical U-stirrups should be spaced 0.33 $d=.33\times21=7.00$ inches c. to c. The maximum spacing near the center of the span must be not more than $.45d=.45\times21=9.45$ inches. The requirement that the shear at the section where the computed shear is zero must be assumed as one-fourth the maximum at the support practically necessitates the use of stirrups for the full length of the span. The arrangement of the stirrups may then be, one space of $3\frac{1}{2}$ inches, three spaces of 7 inches, and three spaces of 8 inches, measured from the center of the support toward the middle of the span. and the balance of the spacing about 9 inches c. to c.

The loads upon the shorter beams, assuming the beam below the slab to weigh 85 pounds per linear foot, are $100\times14\times12+85\times12$ $=17{,}820$ pounds. The maximum shear at the support is $17{,}820\times.6$ $=10{,}390$ pounds, and the section necessary to take care of this, assuming $v=120$ lb./in.^{2}, is $b'd=\frac{V}{vj}=\frac{10390}{105}=99$ in.^{2} Use $b'=7.5$ inches and $d=14$ inches.

If the distance from the center of the steel to the under surface of the beam is made $2\frac{1}{2}$ inches, the total depth, d_1, is 16.5 inches and the weight of the web below the slab is $7.5\times10.5\times150/144=82$ pounds per linear foot, practically as assumed.

Then

$$M=\frac{Wl}{10}=\frac{17820\times12\times12}{10}=256{,}610 \text{ in.-lb.}$$

Taking

$$b=\frac{l}{4}=\frac{12\times 12}{4}=36 \text{ inches},$$

$$Q=\frac{M}{btd}=\frac{256610}{36\times 6\times 14}=85,\ \frac{d}{t}=\frac{14}{6}=2.33.$$

Diagram XIV (p. 237) shows that the neutral axis is in the flange and the beam is to be designed as rectangular.

$$R=\frac{M}{bd^2}=\frac{256610}{36\times 14\times 14}=36.3.$$

Table XXII (p. 199) gives f_c less than 350 lb./in.^{2}, and the steel needed is $p=.0025$, or $A_s=36\times 14\times .0025=1.25$ in.^{2} Table XXVI (p. 203) gives three $\frac{3}{4}$-in. square bars with an area of 1.69 in.^{2} These will be used and two of them will be turned up at the quarter-point on one side of the support and one on the other side to provide for the tension due to negative moment.

Then

$$G=\frac{M}{bd^2}=\frac{256610}{7.5\times 14\times 14}=175,\ \frac{d'}{d}=\frac{2.5}{14}=.16.$$

Table XXXIX (p. 246) gives, $p=.0121$ and $p'=.0140$, as against $1.69/7.5\times 14=.0161$. This shows that the beam is amply safe at the supports.

The beam was designed for a unit shear of 120 lb./in.^{2} and reinforcement for diagonal tension is needed. The requirement of an assumed shear of one-fourth the maximum at the section where the computed shear is zero calls for stirrups for the full span length. For the maximum end shear of $V=10390$ pounds, and $V_d=10390-40\times 7.5\times \frac{7}{8}\times 14=6715$ lb. Diagram XII (p. 223) shows that $\frac{1}{4}$-inch round bar vertical U-stirrups may be spaced $.33d=.33\times 14=4.5$ inches c. to c. The maximum spacing allowed is $.45d=.45\times 14=6.3$ inches. The arrangement of stirrups may then be, one space of $2\frac{1}{4}$ inches, three spaces of $4\frac{1}{4}$ inches, and three spaces of 5 inches, measured from the center of the support toward the center of the span and the balance of the spaces about 6 inches c. to c.

ART. 33. FLAT SLAB CONSTRUCTION

136. Flat Slabs for Floors and Roofs.—Flat slab construction is a form of construction in which a flat plate of reinforced concrete of uniform or nearly uniform thickness is supported directly upon reinforced concrete columns and built monolithically with them.

It is growing in favor over the usual form of slab, beam, and girder construction.

The columns are usually enlarged at the top, and in some cases the thickness of the slab is increased above and somewhat beyond the column capital forming what is known as a " drop panel."

Various types of flat slab construction, such as the *two-way*, *three-way*, *four-way*, and *circumferential systems*, have been suggested and used.

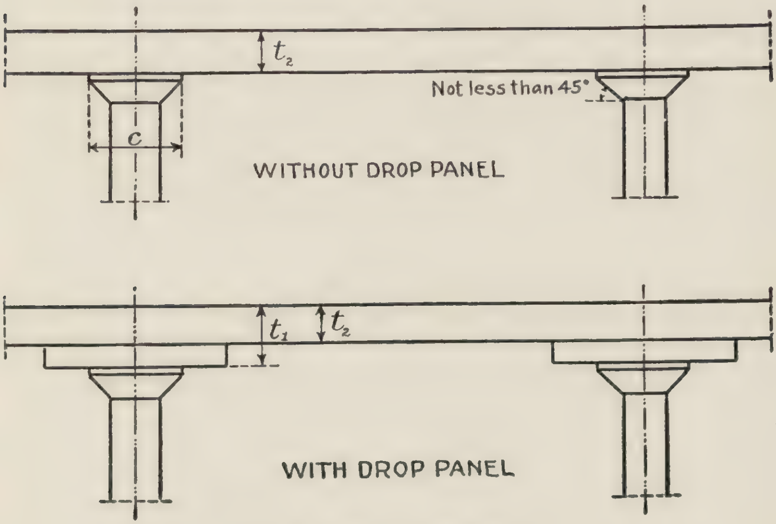


FIG. 66.—Types of Flat Slabs.

The three-way system, with the columns at the apexes of equilateral triangles and reinforcing bars parallel to the sides of the triangles, is especially adapted for use in car-barn and garage construction.

The two-way and four-way systems are constructed either with or without " drop panels." Figure 66.

The two-way type is perhaps the favorite from the standpoint of design and erection.

In designing flat slabs it is customary to deal with the total moment (the numerical sum of the negative and positive moments) caused by the load distributed over a whole panel, and to state the moments at the critical sections in terms of this total moment, M. (See Joint Committee Table 6, p. 272.)

The following is from the Joint Committee Report of October, 1924:

F.—Flat Slabs.

(Two-way and Four-way Systems with Rectangular Panels.)

142. *Moments in Interior Panels.*—The moment coefficients, moment distribution, and slab thicknesses specified herein are for slabs which have three (3) or more rows of panels in each direction, and in which the panels are approximately uniform in size. Slabs with paneled ceiling or with depressed paneling in the floor shall be considered as coming under the requirements herein given. The symbols used in Formulas (36) * to (41) * are defined in Section 105, except as indicated in Sections 142, 145, and 155.

In flat slabs in which the ratio of reinforcement for negative moment in the column strip is not greater than 0.01, the numerical sum of the positive and negative moments in the direction of either side of the panel for which tension reinforcement must be provided, shall be assumed as not less than that given by Formula (36)*:

$$M_0 = 0.09Wl\left(1 - \frac{2c}{3l}\right)^2, \quad \ldots \ldots \ldots \quad (36)^*$$

in which,

M_0 = sum of positive and negative bending moments † in either rectangular direction, at the principal design sections of a panel of a flat slab;

c = base diameter of the largest right circular cone which lies entirely within the column (including the capital) the vertex angle of which is 90° and the base of which is $1\frac{1}{2}$ inches below the bottom of the slab or the bottom of the dropped panel (see Fig. 67, p. 271);

l = span length ‡ of flat slab, center to center of columns, in the rectangular direction in which moments are considered;

l_1 = span length ‡ of flat slab, center to center of columns, perpendicular to the rectangular direction in which moments are considered; and

W = total dead and live load uniformly distributed over a single panel area.

143. *Principal Design Sections.*—In computing the critical moments in flat slabs subjected to uniform load the following principal design sections shall be used:

* Joint Committee formula number.

† The sum of the positive and negative moments provided for by this equation is about 72 per cent of the moment found by rigid analysis based on the principles of mechanics. Extensive tests and experience with existing structures have shown that the requirements here stated will give adequate strength. See "Statical Limitations upon the Steel Requirement in Reinforced Concrete Flat Slab Floors," by John R. Nichols, *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 1670, and "Moments and Stresses in Slabs," by Messrs. H. M. Westergaard and W. A. Slater, *Proceedings*, Am. Concrete Inst., Vol. XVII (1921).

‡ The column strip and the middle strip to be used when considering moments in the direction of the dimension, l, are located and dimensioned as shown in Fig. 68, p. 271. The dimension, l_1, does not always represent the short length of the panel. When moments in the direction of the shorter panel length are considered, the dimensions, l and l_1, are to be interchanged and the strips corresponding to those shown in Fig. 68, but extending in the direction of the shorter panel length, are to be considered.

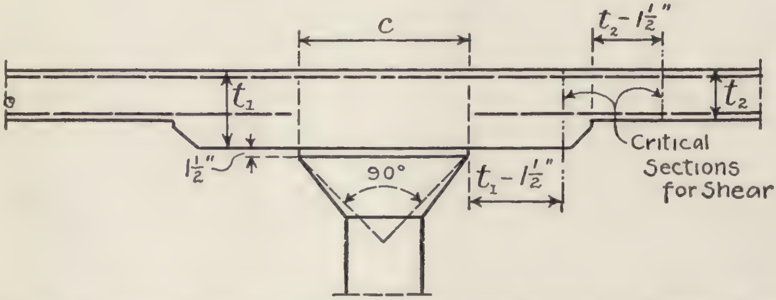


FIG. 67.—Typical Column Capital and Sections of Flat Slab with Dropped Panel. (Courtesy of the American Society for Testing Materials.)

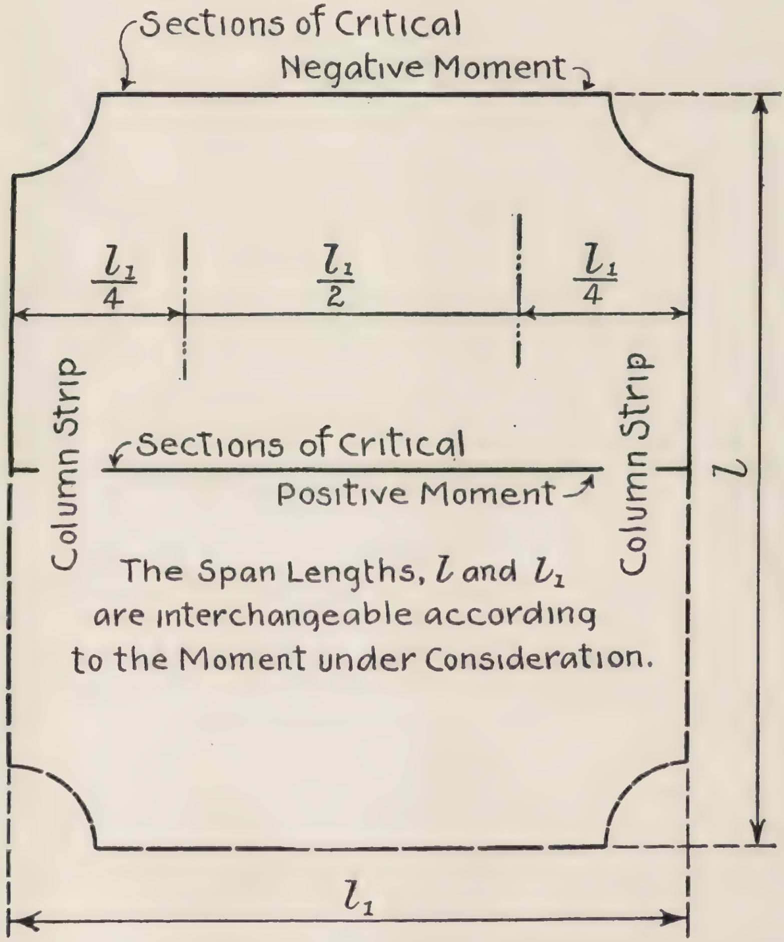


FIG. 68.—Principal Design Sections of Flat Slab. (Courtesy of the American Society for Testing Materials.)

(a) Section for Negative Moment in Middle Strip: The section beginning at a point on the edge of the panel, $\frac{l_1}{4}$, from the column center and extending in a rectangular direction a distance, $\frac{l_1}{2}$, toward the center of the adjacent column on the same panel edge (see Fig. 68, p. 271).

(b) Section for Negative Moment in Column Strip: The section beginning at a point on the edge of the panel, $\frac{l_1}{4}$, from the center of a column and extending in a rectangular direction toward the column to a point, $\frac{c}{2}$, therefrom and thence along a one-quarter ($\frac{1}{4}$) circumference about the column center to the adjacent edge of the panel.

(c) Section for Positive Moment in Middle Strip: The section of a length, $\frac{l_1}{2}$, extending in a rectangular direction across the center of the middle strip.

(d) Section for Positive Moment in Column Strip: The section of length, $\frac{l_1}{4}$, extending in a rectangular direction across the center of the column strip.

144. *Moments in Principal Design Sections.*—The moments in the principal design sections shall be those given in Table 6, except as follows:

(a) The sum of the maximum negative moments in the two column strips may be greater or less than the values given in Table 6 by not more than $0.03M_0$.

(b) The maximum negative moment and the maximum positive moments in the middle strip and the sum of the maximum positive moments in the two column strips may each be greater or less than the values given in Table 6 by not more than $0.01M_0$.

TABLE 6.—MOMENTS TO BE USED IN DESIGN OF FLAT SLABS.*

Strip.	Flat Slabs without Dropped Panels.		Flat Slabs with Dropped Panels.	
	Negative.	Positive.	Negative.	Positive.
Slabs with Two-way Reinforcement.				
Column strip	$0.23M_0$	$0.11M_0$	$0.25M_0$	$0.10M_0$
Two column strips	$0.46M_0$	$0.22M_0$	$0.50M_0$	$0.20M_0$
Middle strip	$0.16M_0$	$0.16M_0$	$0.15M_0$	$0.15M_0$
Slabs with Four-way Reinforcement.				
Column strip	$0.25M_0$	$0.10M_0$	$0.27M_0$	$0.095M_0$
Two column strips	$0.50M_0$	$0.20M_0$	$0.54M_0$	$0.190M_0$
Middle strip	$0.10M_0$	$0.20M_0$	$0.08M_0$	$0.190M_0$

* These are approximately the values which should be obtained by considering one-third ($\frac{1}{3}$) of the total moment, M_0, as positive and two-thirds ($\frac{2}{3}$) of it as negative moment.

145. *Thickness of Flat Slabs and Dropped Panels.*—The total thickness,* t_1, of the dropped panel, in inches, or of the slab if a dropped panel is not used, shall be not less than:

$$t_1 = 0.038\left(1 - 1.44\frac{c}{l}\right)l\sqrt{Rw'\frac{l_1}{b_1}} + 1\tfrac{1}{2}, \quad . \quad . \quad . \quad . \quad . \quad (37)\ \dagger$$

in which

R = ratio of negative moment in the two column strips to M_0;
w' = uniformly distributed dead and live load per unit of area of floor; and
b_1 = dimension of the dropped panel in the direction parallel to l_1.

For slabs with dropped panels the total thickness,* in inches, at points beyond the dropped panel shall be not less than

$$t_2 = 0.02l\sqrt{w'} + 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)\ \ddagger$$

The slab thickness, t_1 or t_2, shall in no case be less than $\frac{l}{32}$ for floor slabs and not less than $\frac{l}{40}$ for roof slabs. In determining minimum thickness by Formulas (37) and (38), the value of l shall be the panel length, center to center of the columns, on the long side of the panel, l_1 shall be the panel length on the short side of the panel, and b_1 shall be the width or diameter of the dropped panel in the direction of l_1, except that in a slab without a dropped panel, b_1 shall be $0.5l_1$.

146. *Minimum Dimensions of Dropped Panels.*—The dropped panel shall have a length or diameter in each rectangular direction of not less than one-third ($\frac{1}{3}$) the panel length in that direction, and a thickness not greater than $1.5t_2$.

147. *Wall and Other Irregular Panels.*—In wall panels and other panels in which the slab is discontinuous at the edge of the panel, the maximum negative moment one panel length away from the discontinuous edge and the maximum positive moment between shall be increased as follows:

(*a*) Column strip perpendicular to the wall or discontinuous edge, 15 per cent greater than that given in Table 6 in Section 144.

(*b*) Middle strip perpendicular to wall or discontinuous edge, 30 per cent greater than that given in Table 6 in Section 144.

In these strips the bars used for positive moments perpendicular to the discontinuous edge shall extend to the edge of the panel at which the slab is discontinuous.

148. *Panels with Marginal Beams.*—In panels having a marginal beam on one edge or on each of two adjacent edges, the beam shall be designed to carry at least the load superimposed directly upon it, exclusive of the panel load. A beam which has a depth greater than the thickness of the dropped panel into which it frames, shall be designed to carry, in addition to the load superimposed upon it, at least one-fourth ($\frac{1}{4}$) of the distributed load for which the adjacent panel or panels are designed, and each column strip adjacent to and parallel with

* The thickness will be in inches regardless of whether l and w' are in feet and pounds per square foot, or in inches and pounds per square inch.

† The values of R used in this formula are the coefficients of M_0 for negative moment in two column strips in Table 6 (Section 144).

‡ Joint Committee formula number.

the beam shall be designed to resist a moment at least one-half ($\frac{1}{2}$) as great as that specified in Table 6 (Section 144) for a column strip.*

Each column strip adjacent to and parallel with a marginal beam which has a depth less than the thickness of the dropped panel into which it frames shall be designed to resist the moments specified in Table 6 for a column strip. Marginal beams on opposite edges of a panel and the slab between them shall be designed for the entire load and the panels shall be designed as simple beams.

149. *Discontinuous Panels.*—The negative moments on sections at and parallel to the wall, or discontinuous edge of an interior panel, shall be determined by the conditions of restraint.†

150. *Flat Slabs on Bearing Walls.*—Where there is a beam or a bearing wall on the center line of columns in the interior portion of a continuous flat slab, the negative moment at the beam or wall line in the middle strip perpendicular to the beam or wall shall be taken as 30 per cent greater than the moment specified in Table 6 for a middle strip. The column strip adjacent to and lying on either side of the beam or wall shall be designed to resist a moment at least one-half ($\frac{1}{2}$) of that specified in Table 6 for a column strip.

151. *Point of Inflection.*—The point of inflection in any line parallel to a panel edge in interior panels of symmetrical slabs without dropped panels shall be assumed to be at a distance from the center of the span equal to 0.30 of the distance between the two sections of critical negative moment at opposite ends of the line; for slabs having dropped panels, the coefficient shall be 0.25.

152. *Reinforcement.*—The reinforcement bars which cross any section and which fulfill the requirements given in Section 153 may be considered as effective in resisting the moment at the section. The sectional area of a bar multiplied by the cosine of the angle between the direction of the axis of the bar and any other direction may be considered effective as reinforcement in that direction.

153. *Arrangement of Reinforcement.*—The design shall include adequate provision for securing the reinforcement in place so as to take not only the critical moments, but the moments at intermediate sections. Provision shall be made for possible shifting of the point of inflection by carrying all bars in rectangular or diagonal directions, each side of a section of critical moment, either positive or negative, to points at least twenty (20) diameters beyond the point of inflection as specified in Section 151. Lapped splices shall not be permitted at or near regions of maximum stress except as described. At least four-tenths ($\frac{4}{10}$) of all bars in each direction shall be of such length and shall be so placed as to provide reinforcement at two sections of critical negative moment and at the intermediate section of critical positive moment. Not less than one-third ($\frac{1}{3}$) of the bars used for positive reinforcement in the column strip shall extend into the dropped panel not less than twenty (20) diameters of the bar, or in case no dropped panel is used, shall extend to a point not less than one-eighth ($\frac{1}{8}$) of the span length from the center line of the column or the support.

154. *Reinforcement at Construction Joints.*—Construction joints made crosswise of a building 100 ft. or more in length, shall have special reinforcement

* In wall columns, brackets are sometimes substituted for capitals, or other changes are made in the design of the capital. Attention is directed to the necessity for taking into account the change in the value of c in the moment formula for such cases.

† The Committee is not prepared to make a more definite recommendation at this time.

placed at right angles to the joint, and extending a sufficient distance on each side of the joint, to develop the strength of the reinforcement by bond. This reinforcement shall be placed near the opposite face of the member from the main tension reinforcement; the cross-sectional area of such reinforcement shall be not less than 0.5 per cent of the section of the members cut by the joint.

155. *Tensile Stress in Reinforcement.*—The tensile stress, f_s, in the reinforcement in flat slabs shall be taken as not less than that computed by Formula (39):

$$f_s=\frac{RM_0}{A_s jd}, \quad \ldots\ldots\ldots \quad (39)^*$$

in which

RM_0 = moment specified in Section 144 for two column strips or for one middle strip; and

A_s = effective cross-sectional area of the reinforcement which crosses any of the principal design sections and which meets the requirements of Section 153. The stress so computed shall not at any of the principal design sections exceed the values specified in Section 194.

156. *Compressive Stress in Concrete.*—The compressive stress in the concrete in flat slabs shall be taken as not less than that computed by Formulas (40) and (41), but the stress so computed shall not exceed $0.4f_c'$.

Compression due to negative moment, RM_0 in the two column strips:

$$f_c=\frac{3.5RM_0}{0.67\sqrt[3]{pnb_1d^2}}\left(1-1.2\frac{c}{l}\right), \quad \ldots\ldots \quad (40)^*$$

in which b_1 is as specified in Section 145.

Compression due to positive moment RM_0 in the two column strips, or negative or positive moment in the middle strip:

$$f_c=\frac{6RM_0}{0.67\sqrt[3]{pnl_1d^2}}. \quad \ldots\ldots\ldots \quad (41)^*$$

In special cases supported by satisfactory engineering analysis, approved by the engineer, compression reinforcement may be used to increase the resistance to compression in accordance with other provisions of these specifications.

157. *Shearing Stress.*—The shearing unit stress in flat slabs shall not exceed the value of v given by Formula (33):

$$v=0.02f_c'(1+r) \quad \ldots\ldots\ldots \quad (33)^*$$

and shall not in any case exceed $0.03f'_c$.

The shearing unit stress shall be computed on:

(*a*) A vertical section which has a depth, in inches, of $\frac{7}{8}(t_1-1\frac{1}{2})$ and which lies at a distance, in inches, of $t_1-1\frac{1}{2}$ from the edge of the column capital; and

(*b*) A vertical section which has a depth, in inches, of $\frac{7}{8}(t_2-1\frac{1}{2})$ and which lies at a distance, in inches, of $t_2-1\frac{1}{2}$ from the edge of the dropped panel.

In no case shall r be less than 0.25. Where the shearing stress computed as in Section (*a*) is being considered, r shall be assumed as the proportional amount of the negative reinforcement, within the column strip, crossing the column capital. Where the shearing stress computed as in Section (*b*) is being consid-

* Joint Committee formula number.

ered, r shall be assumed as the proportional amount of the negative reinforcement, within the column strip, crossing entirely over the dropped panel.* (For typical flat slab and designation of principal design sections, see Figs. 67 and 68, p. 271.)

158. *Unusual Panels.*—For structures having a width of one or two panels, and also for slabs having panels of markedly different sizes, an analysis shall be made of the moments developed in both slab and columns, and the values given in Sections 142 to 157 modified accordingly.

159. *Bending Moments in Columns.*—(See Section 171, p. 283.)

137. Flat Slab Tables.—As in other reinforced concrete designing, the work is greatly facilitated by the use of tables.

Table XLII (p. 278), based on the formula $t=0.2L\sqrt{w}+1$ inch, gives the safe superimposed loads for " drop panel " flat slabs of different spans and thicknesses of slab. This formula is recommended by the Joint Committee, and the American Concrete Institute, and is prescribed by the New York and St. Louis Building Ordinances.

Table XLIII (p. 279) based on the formula $t=\frac{1}{44}\sqrt{W}$ is similar, the formula being that of the Chicago Building Ordinance.

138. Design of Flat Slabs.—In designing flat slabs, certain rules prescribed by City Building Ordinances together with the recommendations of the Joint Committee are followed by conservative designers. The diameter of the column capital is usually made $22\frac{1}{2}$ per cent of the span of the square, or of the mean span of the rectangular panel; and the thickness, t_1, of the " drop panel," is made not less than 1.33 nor more than 1.5 of the thickness, t_2, of the slab.

Example.

27. A " drop panel " two-way system flat floor slab has a rectangular panel 20×22 feet and is to carry a superimposed load of 200 lb./ft.^{2} The allowed values of f_s and f_c are 16,000 and 800 lb./in.^{2} respectively. Design the slab in accordance with the recommendations of the Joint Committee.

Table XLII (p. 278) shows that for a mean span of 21 feet and a superimposed load of 200 lb./ft.^{2}, the total thickness of the slab, t, should be $8\frac{1}{2}$ inches. The table also shows that this thickness meets the requirement for floors, that is, " not less than $\frac{1}{32}$ of the mean span," and gives the weight of the slab as 107 lb./ft.^{2}

Allowing 3 lb./ft.^{2} for a quarter-inch wearing surface, the total load is 310 lb./ft.^{2}

Make the length of the side of the drop panel one-third of the mean span, $21/3=7$ ft., and make the thickness, t_1, $1.33\times8.5=11.33$, or say, 11.5 inches.

* In special cases, where supported by satisfactory engineering analysis, diagonal tension reinforcement may be used and increased shearing stresses allowed in accordance with Sections 127 to 130.

Make the diameter, c, of the column capital $22\frac{1}{2}$ per cent of the mean span, $0.225 \times 21 = 4.72$, or say, 5 feet.

Shearing Stress for Slab.—The allowed shearing unit stress for flat slabs is 60 lb./in.^{2}

$$V = 310\ [20 \times 22 - (8.0)] = 116{,}560 \text{ pounds.}$$

Assuming $1\frac{1}{2}$ inches as the distance from the face of the concrete to the c.g. of the steel,

$$v = \frac{V}{bjd} = \frac{116560}{4 \times 8 \times 12 \times .875 \times 7} = 50 \text{ lb./in.}^2$$

Shearing Stress for Drop Panel.—The Joint Committee allowance for shearing unit stress for drop panels is 60 lb./in.^{2}

$$V = 310\left(20 \times 22 - \frac{\pi \overline{6.66^2}}{4}\right) = 125{,}600 \text{ pounds,}$$

$$v = \frac{125600}{\pi \times 80 \times 10} = 50 \text{ lb./in.}^2$$

Bending Moments.—The total load on the panel is

$$W = [20 \times 22 \times 310 + \tfrac{3}{12}(7 \times 7 \times 150)] = 138{,}240 \text{ pounds,}$$

$$l = 21 \times 12 = 252 \text{ inches,}$$

$$M_c = .09Wl\left(1 - \frac{2c}{3l}\right) = 2{,}194{,}700 \text{ in.-lb.}$$

From Joint Committee Table 6 (p. 272).

Middle Strip:

Positive Moment,	$2{,}194{,}700 \times 0.10 =$	$+219{,}470$ in.-lb.
Negative Moment	$2{,}194{,}700 \times 0.15 =$	$-329{,}200$ in.-lb.

Two-column Strip:

Positive Moment,	$2{,}194{,}700 \times 0.20 =$	$+\ 438{,}940$ in.-lb.
Negative Moment,	$2{,}194{,}700 \times 0.50 =$	$-1{,}097{,}350$ in.-lb.

Reinforcement.

For the middle strip positive moment, assume $\frac{1}{2}$-inch round bars with $\frac{3}{4}$-inch concrete protection below the lower tier of bars. Then d for the upper tier will be $8\frac{1}{2} - 1\frac{1}{2} = 7$ inches, and

$$A_s = \frac{219470}{16000 \times .875 \times 7} = 2.24 \text{ in.}^2$$

From Table XXVI (p. 203), use twelve $\frac{1}{2}$-inch round bars.

For the middle strip negative moment, assume $\frac{1}{2}$-inch round bars with $\frac{3}{4}$-inch protective concrete. In this case, $d = 8\frac{1}{2} - 1 = 7\frac{1}{2}$ inches and

$$A_s = \frac{329200}{16000 \times .875 \times 7.5} = 3.14 \text{ in.}^2$$

TABLE XLII.—SUPERIMPOSED LOADS FOR DROP PANEL FLAT SLABS—POUNDS PER SQUARE FOOT

$t = .02l\sqrt{w} + 1$ in.	w = total load in lb./ft.^{2} t = total slab thickness in inches l = span c. to c. in feet	Upper zig-zag line indicates limiting span for $l''/t'' = 32$ Lower zig-zag line indicates limiting span for $l''/t'' = 40$

Span	Total Slab Thickness—Inches																			
	6	$6\frac{1}{4}$	$6\frac{1}{2}$	$6\frac{3}{4}$	7	$7\frac{1}{4}$	$7\frac{1}{2}$	$7\frac{3}{4}$	8	$8\frac{1}{4}$	$8\frac{1}{2}$	$8\frac{3}{4}$	9	$9\frac{1}{4}$	$9\frac{1}{2}$	10	$10\frac{1}{2}$	11	$11\frac{1}{2}$	12
15′-0″	202	228	253	283	308															
15′-6″	185	208	232	259	287	315														
16′-0″	170	191	214	237	263	290	319													
16′-6′	155	175	195	217	242	268	294	321	349	380	411	443	472	509	546	618				
17-0″	141	160	180	201	224	247	271	298	323	352	381	411	439	471	506	575	653	726	810	895
17-6″	130	147	165	183	205	229	250	275	300	326	352	381	408	440	471	535	605	679	756	836
18-0″	117	135	152	170	189	210	232	254	279	303	327	353	380	408	438	500	565	634	706	781
18-6″	107	124	140	156	175	194	215	236	257	281	306	331	354	381	410	466	529	591	661	731
19-0″	98	113	127	143	162	180	198	218	240	261	282	307	331	355	383	437	494	554	617	681
19′-6″	89	103	117	132	149	165	183	203	221	241	263	286	307	330	356	408	464	519	578	645
20′-0″	81	94	107	121	137	152	170	188	206	225	247	267	287	308	330	381	434	487	544	606
20′-6″		86	98	111	126	141	157	173	191	209	229	248	267	288	309	359	407	457	511	569
21′-0″			90	102	116	131	145	161	178	195	214	231	248	269	289	333	380	428	481	536
21′-6″			82	93	107	119	134	149	166	181	197	215	231	252	271	311	357	404	451	505
22′-0″				85	98	111	124	138	153	168	183	200	218	235	254	293	335	379	427	475
22′-6″				77	89	102	114	128	142	156	169	187	202	219	237	275	314	355	399	447
23′-0″					82	94	106	119	131	145	159	175	189	206	223	257	296	333	376	421
23′-6″						86	97	109	122	134	148	163	176	191	209	241	277	315	354	397
24′-0″						78	89	101	113	125	137	152	165	179	194	226	261	295	335	375
24′-6″							72	92	104	116	127	141	153	167	181	214	245	278	314	352
25′-0″							75	85	96	107	118	131	143	156	170	199	230	262	297	334
Dead Load Sq. Ft.	75	78	82	85	88	91	94	97	100	103	107	109	113	116	119	125	131	138	144	150

Credit for this table due Mr. M. F. Marks.

From Table XXVI, use sixteen $\frac{1}{2}$-inch round bars.

For the two-column strip positive moment, assume $\frac{3}{4}$-inch round bars with $\frac{3}{4}$-inch protection. The value of d is $8\frac{1}{2} - 1\frac{1}{8} = 7\frac{3}{8}$ inches and

$$A_s = \frac{438940}{16000 \times .875 \times 7.37} = 4.25 \text{ in.}^2$$

From Table XXVI, use ten $\frac{3}{4}$-inch round bars.

For the two-column strip negative moment, assume $\frac{3}{4}$-inch bars with $\frac{3}{4}$-inch protection. The value of d for the lower tier of bars is $11\frac{1}{2} - 1\frac{7}{8} = 9\frac{5}{8}$ inches, and

$$A_s = \frac{1097350}{16000 \times .875 \times 9.62} = 8.15 \text{ in.}^2$$

TABLE XLIII.—SUPERIMPOSED LOADS FOR DROP PANEL FLAT SLABS—POUNDS PER SQUARE FOOT

$t = 1/44\sqrt{W}$	W = total load on panel in pounds. t = total slab thickness in inches l = span c. to c. in feet	Upper zig-zag line indicates limiting span for $l''/t'' = 32$ Lower zig-zag line indicates limiting span for $l''/t'' = 40$

Span	Total Slab Thickness—Inches																			
	6	6¼	6½	6¾	7	7¼	7½	7¾	8	8¼	8½	8¾	9	9¼	9½	10	10½	11	11½	12
15′–0	235	258	282	307	333															
15′–6″	215	237	260	284	308	333														
16′–0″	197	218	239	261	283	307	332													
16′–6″	181	200	220	240	261	283	306	330	355	380	407	435	464	494	526	588				
17′–0″	166	184	202	221	241	261	283	306	329	353	378	403	430	458	486	546	609	674	744	816
17′–6″	153	169	186	204	222	241	262	283	305	327	351	375	400	426	452	508	567	628	693	762
18′–0″	140	155	171	188	205	223	242	262	282	303	325	348	371	395	421	473	529	586	648	712
18′–6″	128	143	158	174	190	208	226	245	264	283	304	325	346	368	392	442	494	548	605	666
19′–0″	118	132	146	160	175	191	208	226	244	263	282	302	322	343	366	412	461	512	567	624
19′–6″	108	121	134	148	162	177	193	209	226	244	262	281	300	320	341	385	431	479	531	585
20′–0″	99	111	123	136	149	164	179	194	210	227	244	261	279	298	318	360	403	449	497	548
20 –6″		102	113	125	138	151	165	180	195	211	227	243	260	278	297	336	378	420	466	514
21 –0″			104	115	127	140	153	167	181	196	211	227	243	260	278	315	354	394	438	484
21′–6″			96	106	117	129	142	155	168	182	197	212	227	243	259	294	332	370	411	454
22′–0″				98	108	119	131	143	156	169	183	197	211	226	242	275	310	346	386	427
22′–6″				90	99	110	121	133	145	157	170	183	197	211	226	258	291	325	362	401
23′–0″					91	101	112	123	134	146	158	170	183	197	211	241	273	305	340	378
23′–6″						93	103	113	124	135	147	159	171	184	197	226	256	286	320	355
24′–0″						85	95	105	115	125	136	147	159	171	184	211	240	269	301	335
24′–6″							87	96	106	116	126	137	148	160	172	198	225	253	283	315
25′–0″							80	89	98	108	118	128	138	149	161	185	211	237	266	297
Dead Load Sq. Ft.	75	78	81	84	88	91	94	97	100	103	106	109	113	116	119	125	131	138	144	150

From Table XXVI (p. 203), nineteen ¾-inch round bars will answer.

Fiber Stress in Concrete.—

$$p = \frac{19 \times 0.4418}{84 \times 9.62} = .0010.$$

From Table XXIV (p. 201), $k = .418$ and $j = .861$,

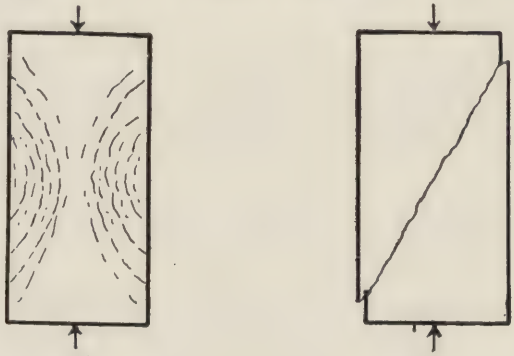
$$f_c = \frac{2 \times 1097350}{.418 \times .86 \times 84 \times 9.62 \times 9.62} = 794 \text{ pounds,}$$

and no revision of the thickness of the drop panel is necessary.

ART. 34. CONCRETE COLUMNS

139. Plain Concrete Columns.—The strength of plain concrete in compression has been discussed in Section 102. The failure of a short

block under compression occurs through lateral expansion and the shearing of the material on surfaces making angles of about 30° with the line of pressure as shown in Fig. 69 (a). As the height of block becomes greater in proportion to its diameter, the resistance of the concrete becomes less certain and plain columns in which the length is more than four times the height frequently fail by shearing diagonally across the column as shown in Fig. 69 (b). This usually occurs where the concrete is of good quality and shows high crushing strength. Weaker concrete usually fails by local crushing.



(a) (b)

FIG. 69.—Crushing of Concrete Columns.

Columns in which the lengths are more than six or eight times the diameters are usually reinforced. The Joint Committee recommends that all columns having an unsupported length of more than forty times the least radius of gyration ($40R$) be reinforced; that the stress on the concrete core of reinforced concrete columns with lateral ties be limited to $f_c = 0.20f_c'$, with the area of longitudinal steel varying from 0.5 per cent to 2 per cent of the area of the core; and that the stress on the concrete core of spiral reinforced concrete columns be limited to $f_c = 300 + (0.10 + 4p)f'_c$, with the area of longitudinal steel varying from 1 per cent to 6 per cent of the concrete core.

The use of concrete rich in cement is nearly always advisable in the construction of columns, on account of the greater reliability of such concrete, as well as because of the economy of reduced section allowable with rich concrete. In reinforced columns, concrete of high compressive strength also admits of more economical use of steel, through employing higher unit stresses than are admissible with less rich concrete. Concrete less rich than 1 to 6 (2000 pounds) mixtures (see Section 102) is undesirable in column work and richer mixtures are commonly preferable.

140. Longitudinal Reinforcement.—Longitudinal bars in the corners of square columns, or near the exterior surfaces of round columns, diminish the uncertainty of action of the columns through preventing the material yielding at points of local weakness. Such reinforcement should always be stayed by light band reinforcement at frequent intervals as shown in Fig. 70 (*a*). This will prevent the longitudinal bars breaking away from the column through bending when loaded.

When a column containing longitudinal steel is loaded, the concrete and steel are shortened by the compression to the same extent and the stress carried by each material is proportional to its modulus of elasticity.

Let A = cross-section of column;
A_s = cross-section of steel;
p = steel ratio $= A_s/A$;
n = ratio of moduli of elasticity $= E_s/E_c$;
P = total load on columns;
f_c = permissible unit compression on concrete;
f'_c = ultimate unit compression on concrete;
f_s = unit compression on steel $= nf_c$.

The total area of concrete is $A(1-p)$, and

$$P = f_cA(1-p) + f_sA_s = f_c(A - pA) + f_cnpA,$$

or

$$P = f_cA[1 + (n-1)p]. \qquad (52)$$

The following is from the October, 1924 report of the Joint Committee on Reinforced Concrete Columns:

160. *Limiting Dimensions.*—The following sections on reinforced concrete columns are based on the assumption of a short column. Where the unsupported length is greater than forty (40) times the least radius of gyration ($40R$), the safe load shall be determined by Formula 47.* Principal columns in buildings shall have a minimum diameter or thickness of 12 in. Posts that are not continuous from story to story shall have a minimum diameter or thickness of 6 in.

161. *Unsupported Length.*—The unsupported length of reinforced concrete columns shall be taken as:

(*a*) In flat-slab construction, the clear distance between the floor and underside of capital.

(*b*) In beam-and-slab construction, the clear distance between the floor and the underside of the shallowest beam framing into the column at the next higher floor level.

(*c*) In floor construction with beams in one direction only, the clear distance between floor slabs.

(*d*) In columns supported laterally by struts or beams only, the clear distance

* Joint Committee formula number.

between consecutive pairs (or groups) of struts or beams, provided that to be considered an adequate support, two such struts or beams shall meet the column at approximately the same level, and the angle between the two planes formed by the axis of the column and the axis of each strut, respectively, is not less than 75 nor more than 105 degrees.

When haunches are used at the junction of beams or struts with columns, the clear distance between supports may be considered as reduced by two-thirds ($\frac{2}{3}$) of the depth of the haunch.

162. *Safe Load on Spiral Columns.*—The safe axial load on columns reinforced with longitudinal bars and closely spaced spirals enclosing a circular core shall not be greater than that determined by Formula 42:*

$$P = A_c f_c + n f_c p A, \quad \text{. . . . .} \quad (42)\ *$$

in which

P = total safe axial load on column the $\frac{h}{R}$ of which is less than 40;

A = area of the concrete core enclosed within the spiral; the diameter of the core (or of the spiral) shall be taken as the distance center to center of the spiral wire;

p = ratio of effective area of longitudinal reinforcement to the area of the concrete core;

$A_c = A(1-p)$ = net area of concrete core;

f_c = permissible compressive stress in concrete

$$= 300 + (0.10 + 4p) f_c'. \quad \text{. . . . .} \quad (43)\ *$$

The longitudinal reinforcement shall consist of at least six (6) bars of minimum diameter of $\frac{1}{2}$ in. and its effective cross-sectional area shall not be less than 1 per cent, nor more than 6 per cent of the core.

163. *Spiral Reinforcement.*—The spiral reinforcement shall not be less than one-fourth ($\frac{1}{4}$) of the volume of the longitudinal reinforcement; it shall consist of evenly spaced continuous spirals held firmly in place and true to line by at least three vertical space bars. The spacing of the spirals shall not be greater than one-sixth ($\frac{1}{6}$) of the diameter of the core and in no case more than three inches. The spiral reinforcement shall meet the requirements of the "Tentative Specifications for Cold-Drawn Steel Wire for Concrete Reinforcement" (Serial Designation A82–21T) of the American Society for Testing Materials.

164. *Protection of Spirally Reinforced Columns.*—Reinforcement shall be protected everywhere by a covering of concrete cast monolithic with the core, which shall have a minimum thickness of $1\frac{1}{2}$ in. in square columns and 2 in. in round or octagonal columns.

165. *Safe Load on Columns with Lateral Ties.*—The safe axial load on columns reinforced with longitudinal bars and separate lateral ties shall be not greater than that determined by Formula 44:

$$P = (A'_c + A_s n) f_c, \quad \text{. . . . .} \quad (44)\ *$$

in which

A_c' = net area of concrete in the column (total column area minus area of reinforcement);

A_s = effective cross-sectional area of longitudinal reinforcement;

f_c = (the permissible compressive stress in concrete) shall not exceed $0.20 f_c'$.

* Joint Committee formula number.

The amount of longitudinal reinforcement considered in the calculations shall not be more than 2 per cent, nor less than 0.5 per cent of the total area of the column. The longitudinal reinforcement shall consist of not less than four (4) bars of minimum diameter of $\frac{1}{2}$ in., placed with a clear distance from the face of the column of not less than 2 in.

166. *Lateral Ties.*—Lateral ties shall be not less than $\frac{1}{4}$ in. in diameter, spaced not more than 8 in. apart.

167. *Bending in Columns.*—Reinforced concrete columns subject to bending stresses shall be treated as follows:

(*a*) With spiral reinforcement: The compressive unit stress on the concrete within the core under combined axial load and bending shall not exceed by more than 20 per cent the value given for axial load by Formula 43.*

(*b*) With lateral ties: Additional longitudinal reinforcement may be used if required, and the compressive unit stress on the concrete under combined axial load and bending may be increased to $0.3f'_c$. The total amount of reinforcement considered in the computations shall be not more than 4 per cent of the total area of the column.

Tension in the longitudinal reinforcement due to bending of the column shall not exceed 16,000 lb. per sq. in.

170. *Long Columns.*—The permissible working load on the core in axially loaded columns which have a length greater than forty (40) times the least radius of gyration of the column core ($40R$) shall not be greater than that determined by Formula 47 *:

$$\frac{P'}{P}=1.33-\frac{h}{120R}, \qquad (47)^*$$

in which

P' = total safe axial load on long column;

P = total safe axial load on column of the same section, the $\frac{h}{R}$ of which is less than 40, determined as in Sections 162 and 165.

R = least radius of gyration of column core.

171. *Bending Moments in Columns.*—The bending moments in interior and exterior columns shall be determined on the basis of loading conditions and end restraint, and shall be provided for in the design. The recognized methods shall be followed in calculating the stresses due to combined load and bending. In spiral columns, the area to be considered as resisting the stress is the area within the spiral.

In Formula (52) (p. 281), if we let $Z=1+(n-1)p=\frac{P}{f_cA}$, and tabulate values of Z (see Table XLIV, p. 284) in terms of n and p, the computation of columns of this type becomes very simple.

Example 28.—A square column is to carry a load of 95,000 pounds, and to be reinforced with 2 per cent of longitudinal steel. If $f_c=400$ lb./in.^{2} and $n=15$, find dimensions for column and steel.

Solution.—From Table XLIV, for $n=15$ and $p=.020$, we find

* Joint Committee formula number.

TABLE XLIV.—COLUMNS WITH LONGITUDINAL REINFORCEMENT

Values of $Z=\frac{P}{f_cA}$, in Terms of n and p

p	$n=10$	$n=12$	$n=15$	p	$n=10$	$n=12$	$n=15$
0.006	1.054	1.066	1.084	0.021	1.189	1.231	1.294
0.007	1.063	1.077	1.098	0.022	1.198	1.242	1.308
0.008	1.072	1.088	1.112	0.023	1.207	1.253	1.322
0.009	1.081	1.099	1.126	0.024	1.216	1.264	1.336
0.010	1.090	1.110	1.140	0.025	1.225	1.275	1.350
0.011	1.099	1.121	1.154	0.026	1.234	1.286	1.364
0.012	1.108	1.132	1.168	0.027	1.243	1.297	1.378
0.013	1.117	1.143	1.182	0.028	1.252	1.308	1.392
0.014	1.126	1.154	1.196	0.029	1.261	1.319	1.406
0.015	1.135	1.165	1.210	0.030	1.270	1.330	1.420
0.016	1.144	1.176	1.224	0.032	1.288	1.352	1.448
0.017	1.153	1.187	1.238	0.034	1.306	1.374	1.476
0.018	1.162	1.198	1.252	0.036	1.324	1.396	1.504
0.019	1.171	1.209	1.266	0.038	1.342	1.418	1.532
0.020	1.180	1.220	1.280	0.040	1.360	1.440	1.560

$Z=1.280$. Then $A=\frac{P}{f_cZ}=186$, and side of column $=13$ inches. $A_s=.020\times186=4.00$ in.^{2} From Table XXVI (p. 203), four 1-inch square bars may be used, $A_s=3.52$ inches.

If 1 to 3 concrete of 3000 pounds compressive strength (see Section 102) were used in the above problem, we would have $f_c=600$, $n=10$, $Z=1.18$, $A=134$ in.^{2} and $A_s=2.68$ in.^{2} The quantities of materials required would be reduced about 25 per cent, while the proportion of cement in the concrete would be about doubled.

Example 29.—A column 12 in.$\times$12 in. section is to carry a 70,000 pounds. If $f_c=400$ and $n=15$ find area of steel required.

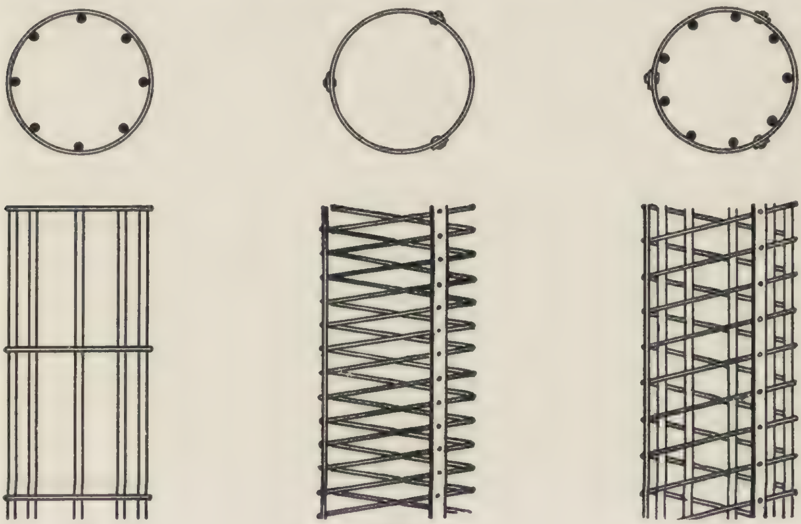
Solution.—$Z=\frac{P}{f_cA}=\frac{70000}{400\times12\times12}=1.21$ and from Table XLIV (p. 284), $p=.015$. Then $A_s=.015\times12\times12=2.16$ in.^{2} Table XXVI (p. 203) gives four $\frac{7}{8}$-inch round bars at the corners with an area of 2.40 in.^{2}, or eight $\frac{5}{8}$-inch round bars at corners and middle of sides with an area of 2.45 in.^{2}

The Joint Committee recommends a minimum of 0.5 per cent of longitudinal steel for columns having a length of more than forty times the least radius of gyration. This gives rigidity to the column,

and security against local yielding in the concrete. High percentages of longitudinal steel are not usually economical, because of the greater cost of steel as compared with concrete for resisting compression, particularly when the stresses in the steel are limited by those in the concrete.

When the concrete is used for fireproofing, the steel should be covered by at least 2 inches of concrete, and about $1\frac{1}{2}$ inches of concrete on the exterior of the column should not be considered in determining the strength of the column.

141. Columns with Spiral Reinforcement.—As shown in Section 103, the failure of concrete under compression commonly occurs



(a) (b) (c)

Fig. 70.—Reinforced Concrete Columns.

through shearing due to lateral expansion. If the concrete in the column be held by band or spiral steel (see Fig. 70 (*b*)) from yielding to lateral expansion, the resistance to crushing will be materially increased. Such reinforcement is either formed of steel bars bent to form a spiral or bands of steel spaced at a uniform distance apart, but in either case, the bands should not be spaced more than about one-sixth of the diameter of the column apart, and must be held in place by longitudinal spacing bars.

Experiments upon columns with spiral reinforcement indicate that the deflections under working loads are not decreased by the reinforcement, but the ultimate strength is considerably increased, as

compared with columns without such reinforcement. When spiral reinforcement is used, it is usual to allow a larger unit stress than for plain columns, or those with longitudinal reinforcement only. The effective area of the column is that inside the reinforcement. The concrete outside the spiral is stripped off when a stress is reached at which plain concrete would fail.

Spiral reinforcement prevents crushing of the concrete until a load is reached which stresses the steel to its yield point, but does not stiffen the column longitudinally, and columns so reinforced frequently fail by bending. The reinforcement is commonly combined with longitudinal steel as shown in Fig. 70 (c). The longitudinal steel serves to stiffen the column against bending, and makes the spiral reinforcing more effective. It is not advisable to use spiral reinforcement without longitudinal steel.

Experiments indicate that about 1 per cent of steel in closely spaced spiral reinforcing is sufficient to resist lateral expansion and give increased strength in compression. Larger amounts of steel do not materially increase the resistance.

Let d = effective diameter of column in inches;
a = area of the steel bars to be used, in square inches;
s = longitudinal spacing of the bands or spirals, in inches;
p = ratio of steel to concrete in column.

Then

$$a = pds/4$$

or

$$s = 4a/pd. \qquad \qquad (53)$$

From this we may obtain the size of bars necessary for steel of required spacing, or the spacing required for bars of given size.

Example 30.—A concrete column is to carry a load of 225,000 pounds and be reinforced with 1 per cent of spiral steel and 3 per cent of longitudinal steel. Using the stresses recommended by the Joint Committee for concrete of 2000 pounds compressive strength, find dimensions for concrete and steel.

Solution.—With spiral reinforcement, the value of

$$f_c = 300 + (0.10 + 4p)f'_c = 300 + (0.10 + 0.12)2000 = 740 \text{ lb./in.}^2$$

Using Table XLIV (p. 284), for $p = .03$ and $n = 15$, $Z = 1.42$, from which

$$A = \frac{P}{f_c Z} = \frac{225000}{740 \times 1.42} = 214 \text{ in.}^2,$$

and the diameter of the column is $16\frac{1}{2}$ inches. Longitudinal steel, $A_s = .03 \times 214 = 6.42$ in.^{2} Table XXVI (p. 203) gives seven $1\frac{1}{8}$-in.

round bars with an area of 6.96 in.^{2} These will be used and spaced about the circumference of the column about $7\frac{1}{2}$ inches apart on centers. For the spiral steel, Formula 53 (p. 286) gives for a spacing of $2\frac{1}{4}$ inches, $a=pds/4=.01\times16.5\times2.25/4=.093$ inch,^{2} and $\frac{3}{8}$-inch round steel will be used.

142. Column Tables.—Tables XLV to XLVIII inclusive (pages 288–291) are in accord with the latest (Oct., 1924) recommendations of the Joint Committee. They are suggestive and may be extended, or supplemented by similar ones without great effort, to meet the requirements of individual designers.

143. Eccentrically Loaded Columns.—When the center of gravity of the load upon a column does not coincide with the gravity axis of the column, bending stresses are produced which must be taken into account in designing the column. In some cases, lateral forces may be acting upon a column, producing bending moments, as in wall columns carrying the ends of beams which are firmly attached to the columns. When these conditions exist, the maximum unit compression due to both direct thrust and bending moment at any section must not exceed the safe values for the concrete, and any tensions which may occur must be taken by proper reinforcement.

Let Fig. 71 (p. 292) represent the section of a column under eccentric load.

A = area of section of column;
A_s = area of longitudinal steel in section;
P = longitudinal load on column·
e = eccentricity of load;
I_c = moment of inertia of section about its gravity axis;
I_s = moment of inertia of steel area about same axis;
u = distance gravity axis to most remote edge of section;
M = bending moment on section, Pe;
f_c = maximum unit compression on concrete;
f_c'' = minimum unit compression on concrete.

f_c = is made up of two parts—that due to direct thrust and that due to bending moment, and is

$$f_c=\frac{P}{A+(n-1)A_s}+\frac{Mu}{I_c+(n)I_s}, \qquad (54)$$

and

$$f_c''=\frac{P}{A+(n-1)A_s}-\frac{Mu}{I_c+(n)I_c}. \qquad (55)$$

When the stress due to moment is greater than that due to direct thrust, f''_c becomes negative, showing the stress to be tension.

TABLE XLV.—SAFE LOADS ON SQUARE COLUMNS WITH LATERAL TIES

Table Based on: $P = Af$,

in which P = Safe Load on Column;
A = Area of Total Effective Cross-section of Column;
f = Allowable Average Unit Pressure upon the Reinforced Column;
f_c = Allowable Unit Stress upon the Concrete of the Column = 400 lb./in.²;
$f = f_c[1 + (n-1)p]$,

in which $p = \frac{A_s'}{A}$, and $n = 15$;
A_s' = Area of Vertical Steel.

Effective Size of Column, inches	$p = .005$		$p = .0075$		$p = .01$		$p = .0125$		$p = .015$		$p = .0175$		$p = .02$	
	$f = 428$ lb./in.²		$f = 442$ lb./in.²		$f = 456$ lb./in.²		$f = 470$ lb./in.²		$f = 484$ lb./in.²		$f = 498$ lb./in.²		$f = 512$ lb./in.²	
	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.
9	35	0.40	36	0.61	37	0.81	38	1.02	39	1.22	40	1.42	42	1.62
9½	39	0.45	40	0.68	41	0.90	42	1.13	44	1.36	45	1.58	46	1.80
10	43	0.50	44	0.75	46	1.00	47	1.25	48	1.50	50	1.75	51	2.00
10½	47	0.55	49	0.84	50	1.10	52	1.38	53	1.68	54	1.93	56	2.20
11	52	0.61	54	0.91	55	1.21	57	1.51	59	1.82	59	2.12	62	2.42
11½	57	0.66	59	1.00	60	1.32	62	1.65	64	2.00	64	2.32	68	2.64
12	62	0.72	64	1.08	66	1.44	68	1.80	70	2.16	70	2.52	74	2.88
12½	67	0.78	69	1.17	71	1.56	74	1.95	76	2.34	76	2.74	80	3.12
13	72	0.85	75	1.27	77	1.69	80	2.12	82	2.54	82	2.96	87	3.38
13½	78	0.91	81	1.37	83	1.82	86	2.28	88	2.74	89	3.20	94	3.64
14	84	0.98	87	1.47	90	1.96	92	2.45	95	2.94	95	3.44	100	3.92
14½	90	1.05	93	1.58	96	2.10	99	2.64	102	3.16	105	3.69	108	4.20
15	97	1.12	100	1.69	103	2.25	106	2.82	109	3.38	112	3.94	115	4.50
15½	103	1.20	106	1.81	110	2.40	113	3.02	117	3.62	120	4.22	123	4.80
16	110	1.28	113	1.92	117	2.56	120	3.20	124	3.84	128	4.49	131	5.12
16½	117	1.36	121	2.05	125	2.72	128	3.41	132	4.10	136	4.77	140	5.44
17	124	1.45	128	2.17	132	2.89	136	3.62	140	4.34	144	5.06	148	5.78
17½	131	1.53	136	2.30	140	3.06	144	3.84	148	4.60	153	5.36	157	6.12
18	139	1.62	143	2.43	148	3.24	152	4.05	157	4.86	162	5.67	166	6.48
18½	147	1.71	151	2.57	157	3.42	161	4.29	166	5.14	171	6.00	175	6.84
19	155	1.81	160	2.71	165	3.61	170	4.51	175	5.42	180	6.32	185	7.20
19½	163	1.90	168	2.86	174	3.80	179	4.76	184	5.76	190	6.66	195	7.60
20	171	2.00	177	3.00	183	4.00	188	5.00	193	6.00	200	7.00	205	8.00
20½	180	2.10	186	3.16	192	4.20	198	5.25	204	6.32	210	7.39	216	8.40
21	189	2.21	195	3.31	202	4.41	208	5.50	214	6.62	220	7.73	226	8.82
21½	198	2.31	204	3.47	211	4.62	217	5.76	224	6.94	231	8.09	237	9.24
22	207	2.42	214	3.63	221	4.84	227	6.04	234	7.26	241	8.45	248	9.68
22½	217	2.53	224	3.80	231	5.06	238	6.33	245	7.60	252	8.86	260	10.12
23	226	2.64	234	3.97	241	5.29	249	6.60	256	7.94	263	9.25	271	10.58
23½	236	2.76	244	4.15	252	5.52	260	6.90	267	8.30	275	9.66	283	11.04
24	247	2.83	255	4.33	263	5.76	271	7.20	279	8.66	288	10.10	296	11.52

Credit for this table is due to Mr. M. F. Marks.

TABLE XLVI.—SAFE LOADS ON SPIRAL COLUMNS

Based on: $P = Af_c[1+(n-1)p]$, and $f_c = 300+(0.10+4p)f_c'$,

in which A = Effective Area of Reinforced Column;
$f_c' = 2000$ lb./in.^{2};
$n = 15$,

and

p = Percentage of Vertical Reinforcement.
A_s' = Area of Vertical Steel.

Effective Diameter, inches	Effective Area, sq. in.	p = .01		p = .02		p = .03		p = .04		p = .05		p = .06	
		P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.
9	64	42	0.64	54	1.28	67	1.92	81	2.56	97	3.20	115	3.84
9½	71	47	0.71	60	1.42	74	2.13	91	2.84	108	3.55	128	4.26
10	79	52	0.79	66	1.58	82	2.37	101	3.16	120	3.95	142	4.74
10½	87	57	0.87	73	1.74	91	2.61	111	3.58	132	4.35	156	5.22
11	95	63	0.95	80	1.90	100	2.85	121	3.80	145	4.75	171	5.70
11½	104	69	1.04	87	2.08	109	3.12	132	4.16	159	5.20	187	6.24
12	113	75	1.13	95	2.26	119	3.39	144	4.52	173	5.65	203	6.78
12½	123	81	1.23	103	2.46	129	3.69	157	4.92	188	6.15	220	7.38
13	133	87	1.33	112	2.66	139	3.99	170	5.32	203	6.65	238	7.98
13½	143	94	1.43	121	2.86	150	4.29	183	5.72	218	7.15	257	8.58
14	154	101	1.54	130	3.08	161	4.62	197	6.16	235	7.70	277	9.24
14½	165	109	1.65	139	3.30	173	4.95	211	6.60	252	8.25	297	9.90
15	177	117	1.77	148	3.54	185	5.31	226	7.08	270	8.85	318	10.62
15½	189	125	1.89	158	3.78	197	5.67	241	7.56	288	9.45	339	11.34
16	201	133	2.01	169	4.02	210	6.03	256	8.04	306	10.05	361	12.06
16½	213	141	2.13	180	4.26	223	6.39	272	8.52	326	10.65	384	12.78
17	226	150	2.26	191	4.52	237	6.78	288	9.04	346	11.30	408	13.56
17½	240	159	2.40	202	4.80	252	7.20	306	9.60	367	12.00	433	14.40
18	254	168	2.54	214	5.08	267	7.62	324	10.16	388	12.70	459	15.24
18½	269	178	2.69	226	5.38	282	8.07	343	10.76	410	13.45	486	16.14
19	284	188	2.84	239	5.68	298	8.52	362	11.36	433	14.20	513	17.04
19½	299	198	2.99	252	5.98	314	8.97	381	11.96	456	14.95	540	17.94
20	314	208	3.14	265	6.28	330	9.42	401	12.56	480	15.70	568	18.84
20½	330	218	3.30	278	6.60	346	9.90	422	13.20	504	16.50	597	19.80
21	346	229	3.46	292	6.92	363	10.38	443	13.84	529	17.30	627	20.76
21½	363	240	3.63	306	7.26	381	10.89	464	14.52	555	18.15	657	21.78
22	380	251	3.80	321	7.60	399	11.40	486	15.20	581	19.00	687	22.80
22½	398	263	3.98	336	7.86	418	11.94	509	15.72	608	19.90	719	23.88
23	416	275	4.16	351	8.32	437	12.48	532	16.64	636	20.80	751	24.96
23½	434	287	4.34	366	8.68	456	13.02	555	17.36	664	21.70	784	26.04
24	452	299	4.52	382	9.06	476	13.59	579	18.12	693	22.65	817	27.18

NOTE.—Spiral not less than $p/4$.

Add 3 in. to effective diameter for outside dimensions of square columns.

Add 4 in. to effective diameter for outside dimensions of cylindrical or octagonal columns.

TABLE XLVII.—SAFE LOADS ON SPIRAL COLUMNS

Based on: $P = Af_c[1 + (n-1)p]$, and $f_c = 300 + (0.10 + 4p)f_c'$,
in which A = Effective Area of Reinforced Column;
$f_c' = 2500$ lb./in.^{2};
$n = 12$,
and
p = Percentage of Vertical Reinforcement.
A_s' = Area of Vertical Steel.

Effective Diameter, inches	Effective Area, sq. in.	$p = .01$		$p = .02$		$p = .03$		$p = .04$		$p = .05$		$p = .06$	
		P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.
9	64	46	0.64	58	1.28	71	1.92	87	2.56	103	3.20	121	3.84
9½	71	51	0.71	64	1.42	79	2.13	97	2.84	115	3.55	135	4.26
10	79	56	0.79	71	1.58	88	2.37	107	3.16	127	3.95	149	4.74
10½	87	62	0.87	78	1.74	97	2.61	118	3.58	140	4.35	165	5.22
11	95	68	0.95	86	1.90	107	2.85	130	3.80	154	4.75	181	5.70
11½	104	74	1.04	94	2.08	117	3.12	142	4.16	169	5.20	198	6.24
12	113	81	1.13	103	2.26	127	3.39	154	4.52	184	5.65	215	6.78
12½	123	88	1.23	112	2.46	138	3.69	167	4.92	199	6.15	234	7.38
13	133	95	1.33	121	2.66	149	3.99	181	5.32	216	6.65	253	7.98
13½	143	103	1.43	130	2.86	161	4.29	195	5.72	233	7.15	273	8.58
14	154	111	1.54	140	3.08	173	4.62	210	6.16	250	7.70	293	9.24
14½	165	119	1.65	150	3.30	186	4.95	225	6.60	268	8.25	315	9.90
15	177	127	1.77	161	3.54	199	5.31	241	7.08	287	8.85	337	10.62
15½	189	136	1.89	172	3.78	212	5.67	257	7.56	306	9.45	360	11.34
16	201	145	2.01	183	4.02	226	6.03	274	8.04	326	10.05	383	12.06
16½	213	154	2.13	195	4.26	240	6.39	291	8.52	346	10.65	405	12.78
17	226	163	2.26	207	4.52	255	6.78	309	9.04	368	11.30	432	13.56
17½	240	173	2.40	219	4.80	271	7.20	328	9.60	391	12.00	458	14.40
18	254	183	2.54	232	5.08	287	7.62	347	10.16	414	12.70	485	15.24
18½	269	193	2.69	245	5.38	303	8.07	367	10.76	437	13.45	513	16.14
19	284	204	2.84	259	5.68	320	8.52	387	11.36	461	14.20	541	17.04
19½	299	215	2.99	273	5.98	337	8.97	408	11.96	486	14.95	570	17.94
20	314	226	3.14	287	6.28	355	9.42	429	12.56	511	15.70	599	18.84
20½	330	237	3.30	301	6.60	373	9.90	451	13.20	537	16.50	630	19.80
21	346	249	3.46	316	6.92	391	10.38	474	13.84	563	17.30	661	20.76
21½	363	262	3.63	332	7.26	410	10.89	497	14.52	590	18.15	693	21.78
22	380	274	3.80	348	7.60	429	11.40	520	15.20	618	19.00	726	22.80
22½	398	287	3.98	364	7.86	449	11.94	544	15.72	646	19.90	759	23.88
23	416	300	4.16	380	8.32	469	12.48	568	16.64	675	20.80	793	24.96
23½	434	313	4.34	397	8.68	490	13.02	593	17.36	705	21.70	828	26.04
24	452	326	4.52	414	9.06	511	13.59	619	18.12	736	22.65	864	27.18

NOTE.—Spiral not less than $p/4$.

Add 3 in. to effective diameter for outside dimensions of square columns.
Add 4 in. to effective diameter for outside dimensions of cylindrical or octagonal columns.

TABLE XLVIII.—SAFE LOADS ON SPIRAL COLUMNS

Based on: $P = Af_c[1 + (n-1)p]$, and $f_c = 300 + (0.10 + 4p)f_c'$,
in which A = Effective Area of Reinforced Column;
$f_c' = 3000$ lb./in.^{2};
$n = 10$,
and
p = Percentage of Vertical Reinforcement;
A_s' = Area of Vertical Steel.

Effective Diameter, inches	Effective Area, sq. in.	$p = .01$		$p = .02$		$p = .03$		$p = .04$		$p = .05$		$p = .06$	
		P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.	P 1000 lb.	A_s' sq. in.
9	64	50	0.64	63	1.28	77	1.92	93	2.56	110	3.20	129	3.84
9½	71	55	0.71	70	1.42	86	2.13	104	2.84	123	3.55	144	4.26
10	79	61	0.79	77	1.58	95	2.37	115	3.16	136	3.95	159	4.74
10½	87	67	0.87	85	1.74	105	2.61	127	3.58	150	4.35	176	5.22
11	95	74	0.95	94	1.90	115	2.85	139	3.80	165	4.75	193	5.70
11½	104	81	1.04	103	2.08	126	3.12	152	4.16	180	5.20	211	6.24
12	113	88	1.13	112	2.26	137	3.39	166	4.52	196	5.65	230	6.78
12½	123	96	1.23	121	2.46	149	3.69	180	4.92	213	6.15	249	7.38
13	133	104	1.33	131	2.66	161	3.99	195	5.32	230	6.65	269	7.98
13½	143	112	1.43	141	2.86	174	4.29	210	5.72	248	7.15	291	8.58
14	154	120	1.54	152	3.08	187	4.62	226	6.16	267	7.70	313	9.24
14½	165	129	1.65	163	3.30	201	4.95	242	6.60	287	8.25	335	9.90
15	177	138	1.77	174	3.54	215	5.31	259	7.08	307	8.85	359	10.62
15½	189	147	1.89	186	3.78	230	5.67	277	7.56	328	9.45	383	11.34
16	201	157	2.01	199	4.02	245	6.03	295	8.04	349	10.05	408	12.06
16½	213	167	2.13	212	4.26	260	6.39	314	8.52	372	10.65	434	12.78
17	226	177	2.26	225	4.52	276	6.78	333	9.04	395	11.30	461	13.56
17½	240	188	2.40	238	4.80	293	7.20	353	9.60	418	12.00	489	14.40
18	254	199	2.54	252	5.08	310	7.62	373	10.16	442	12.70	517	15.24
18½	269	210	2.69	266	5.38	327	8.07	394	10.76	467	13.45	546	16.14
19	284	222	2.84	281	5.68	345	8.52	416	11.36	493	14.20	576	17.04
19½	299	234	2.99	296	5.98	364	8.97	438	11.96	519	14.95	607	17.94
20	314	246	3.14	311	6.28	383	9.42	461	12.56	546	15.70	638	18.84
20½	330	258	3.30	327	6.60	402	9.90	484	13.20	574	16.50	670	19.80
21	346	271	3.46	343	6.92	422	10.38	508	13.84	602	17.30	703	20.76
21½	363	284	3.63	360	7.26	442	10.89	533	14.52	631	18.15	738	21.78
22	380	297	3.80	377	7.60	463	11.40	558	15.20	661	19.00	773	22.80
22½	398	311	3.98	394	7.86	484	11.94	584	15.72	692	19.90	808	23.88
23	416	325	4.16	412	8.32	506	12.48	610	16.64	723	20.80	844	24.96
23½	434	339	4.34	430	8.68	528	13.02	637	17.36	755	21.70	882	26.04
24	452	354	4.52	448	9.06	551	13.59	664	18.12	787	22.65	920	27.18

NOTE.—Spiral not less than $p/4$.
Add 3 in. to effective diameter for outside dimensions of square columns.
Add 4 in. to effective diameter for outside dimensions of cylindrical or octagonal columns.

Tensions in columns, if occurring at all, should be very small and need not be specially provided for. The stresses in steel are always less than nf_c, and therefore within safe limits.

If the section is symmetrical about its gravity axis, $u=d/2$, and for rectangular sections, $I_c=\frac{bd^3}{12}$ and $I_s=A_sd_s^2/4$, in which d_s is distance between centers of steel on the two sides of column. For circular sections, $I_c=.049d^4$ and $I_s=.125A_sd_s^2$, where d is the diameter of the column and d_s is diameter of the circle containing the centers of the steel bars.

Example 31.—A wall column, 12×16 inches in section, carries the end of a beam which brings a longitudinal load of 60,000 pounds and a bending moment of 180,000 in.-lb. upon the column. The column is reinforced with four 1-inch square steel bars at the corners, the centers of steel being 2 inches from surfaces of concrete. $n=15$. Find the unit stresses on the concrete.

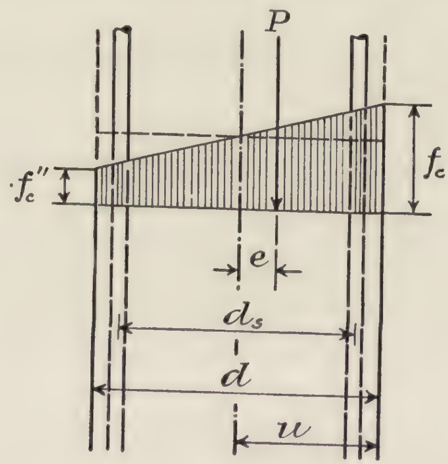


FIG. 71.—Column with Eccentric Load.

Solution.—$I_c=12\times16\times16\times16/12=4096$. $I_s=4\times12\times12/4=144$.

$$f_c=\frac{60000}{192+14\times4}+\frac{180000\times8}{4096+14\times144}=242+230=472 \text{ lb. in.}^2,$$

$$f''_c=242-230=12 \text{ lb./in.}^2$$

Complete discussions of the principles of reinforced concrete design with applications to structures is given in "Concrete, Plain and Reinforced," by Taylor, Thompson and Smulski, and in "Principles of Reinforced Concrete Construction," by Turneaure and Maurer.

CHAPTER VII

RETAINING WALLS

ART. 35. PRESSURE OF EARTH AGAINST A WALL

144. Theories of Earth Pressure.—The lateral pressure of a mass of earth against a retaining wall is affected by so many variable conditions that the determination of its actual value in a particular instance is practically impossible.

Several theories, based in each case upon certain ideal conditions, have been proposed, none of which are more than very rough approximations to the conditions existing in such structures. These theories assume that the earth is composed of a mass of particles exerting friction upon each other but without cohesion, or that the pressure against the wall is caused by a wedge of earth which tends to slide upon a plane surface of rupture, as shown in Fig. 72. Formulas for the resultant thrust against the wall have been produced in accordance with the various theories by several methods, they differ mainly in the direction given to the thrust upon the wall.

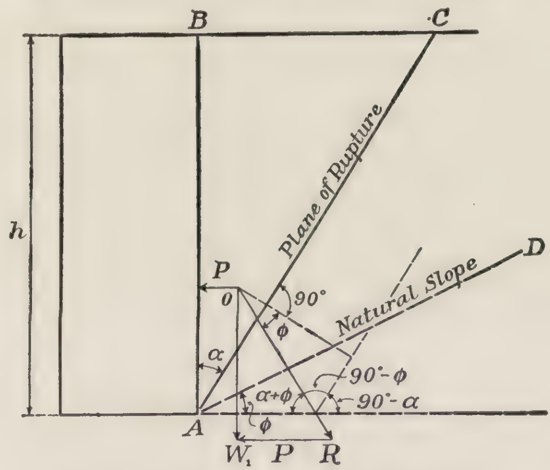


FIG. 72.—Pressure of Earth against a Wall.

Coulomb's Theory.—A formula for computing the lateral thrust against a wall was proposed by Coulomb in 1773. Coulomb assumed

that the thrust was caused by a prism of earth (BAC, Fig. 72) sliding upon any plane AC which produces the maximum thrust upon the wall. There is a certain slope (AD, Fig. 72) at which the material if loosely placed will stand. This is known as the natural slope, and the angle made by this slope with the horizontal as the angle of friction of the earth. On slopes steeper than the natural slope, there is a tendency for the earth to slide down, and if held by a wall, pressures are produced which depend upon the frictional resistance to sliding.

The thrust is assumed by Coulomb to be normal to the wall, and the pressure upon the plane of rupture to be inclined at the angle of friction to the normal to the plane.

The notation recommended in the 1921 edition of the Manual of the American Railway Engineering Association will be used.

Let h = height of wall;
P = resultant pressure upon a unit length of wall;
R = pressure upon the plane of rupture;
W_1 = weight of the wedge of earth;
w = weight of earth per cubic foot;
ϕ = angle of friction of earth;
α = angle between the back of wall and plane of rupture.

If the back of the wall be vertical and the surface of earth horizontal, from Fig. 72 (p. 293),

$$P=W_1 \tan ROW_1=\tfrac{1}{2}wh^2\cdot\frac{\tan \alpha}{\tan (\alpha+\phi)}.$$

For maximum value of P, $\alpha=45^\circ-\frac{\phi}{2}$, and the plane of rupture bisects the angle between the back of the wall and the natural slope.

This is demonstrated as follows:

$$u'=q\frac{\tan \alpha}{\tan (\alpha+\phi)}=q[\tan \alpha \cot (\alpha+\phi)].$$

Deriving and placing the first derivative equal to zero,

$$\tan \alpha_i[-\operatorname{cosec}^2 (\alpha+\phi)]+\cot (\alpha+\phi) \sec^2 \alpha=0,$$

$$\frac{\tan \alpha}{\sin^2 (\alpha+\phi)}=\sec^2 \alpha \cot (\alpha+\phi),$$

$$\frac{\sin \alpha}{\cos \alpha \sin^2 (\alpha+\phi)}=\frac{\cos (\alpha+\phi)}{\cos^2 \sin (\alpha+\phi)}.$$

Multiplying each side of the equation by $\cos\alpha \sin(\alpha+\phi)$,

$$\frac{\sin\alpha}{\sin(\alpha+\phi)}=\frac{\cos(\alpha+\phi)}{\cos\alpha},$$

$$\sin\alpha\cos\alpha=\sin(\alpha+\phi)\cos(\alpha+\phi),$$

$$\frac{\sin 2\alpha}{2}=\frac{\sin 2(\alpha+\phi)}{2},$$

$$\sin 2\alpha=\sin 2(\alpha+\phi),$$

$$\sin(180°-2\alpha)=\sin 2\alpha=\sin 2(\alpha+\phi),$$

$$=\sin(2\alpha+2\phi),$$

Therefore,

$$180°-2\alpha=2\alpha=2\alpha+2\phi,$$

$$180°-2\phi=4\alpha,$$

$$\alpha=45°-\frac{\phi}{2}.$$

for the maximum value of P, and the plane of rupture bisects the angle between the back of the wall and the natural slope.

Substituting this value,

$$P=\tfrac{1}{2}wh^2\tan^2\left(45°-\frac{\phi}{2}\right).$$

P varies as the square of h, and is therefore applied at a distance $h/3$ above the base of the wall. This is the same in all of the theories.

Poncelet's Theory.—In 1840 Poncelet proposed to modify the method of Coulomb by making the thrust upon the wall act at the angle of friction with the normal to the wall.

Before the wall can be overturned about its toe (G, Fig. 73) the back of the wall (AB) must be raised and slide upon the earth behind it, though calling into play the friction of the earth upon the wall as a resistance. As the friction of earth upon a rough masonry wall is greater than that of earth upon earth, a film of earth would be carried with the wall and slide upon the earth behind and the angle of friction is usually taken as equal to the natural slope of the earth.

Let θ = the angle made by the back of the wall with the horizontal;
δ = the angle made by the earth surface with the horizontal.

Following the same procedure as in developing Coulomb's formula, we find the pressure against the wall,

$$P=\tfrac{1}{2}wh^2\frac{\sin^2(\theta-\phi)}{\sin^2\theta\cdot\sin(\theta+\phi)\left(1+\sqrt{\dfrac{\sin(\phi-\delta)\cdot\sin 2\phi}{\sin(\theta-\delta)\cdot\sin(\theta+\phi)}}\right)^2}.$$

For a vertical wall and horizontal earth surface ($\theta = 90°$ and $\delta = 0°$),

$$P = \frac{wh^2}{2} \cdot \frac{\cos \phi}{(1+\sqrt{2} \cdot \sin \phi)^2},$$

which is the formula proposed by Poncelet.

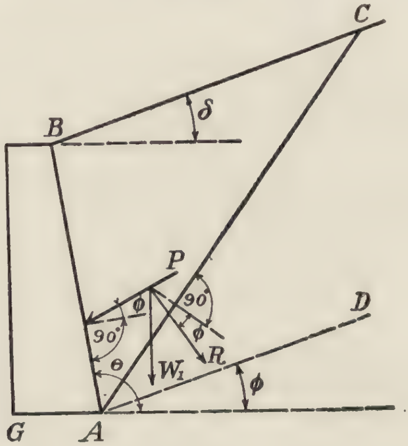


FIG. 73.—Poncelet's Theory of Pressure.

Rankine's Theory.—Rankine considered the earth to be made up of a homogeneous mass of particles, possessing frictional resistance to sliding over each other but without cohesion. He deduced formulas for the pressure upon ideal plane sections through an unlimited mass of earth with plane upper surface, the earth being subject to no external force except its own weight, and determined the direction of the pressure from these assumptions.

Rankine found that the resultant pressure upon any vertical plane section through a bank of earth with plane upper surface is parallel to the slope of the upper surface (see Fig. 74, p. 297).

Let P = the pressure upon the vertical section;
δ = the angle made by the inclination of the upper surface with the horizontal;
ϕ = the angle of friction of the earth;
w = the weight per cubic foot of the earth;
S = the height of vertical section through earth.

Then

$$P = \frac{wS^2}{2} \cos \delta \cdot \frac{\cos \delta - \sqrt{\cos^2 \delta - \cos^2 \phi}}{\cos \delta + \sqrt{\cos^2 \delta - \cos^2 \phi}},$$

is Rankine's formula for earth pressure. This pressure acts upon the vertical section at a distance $S/3$ from its base, and makes an angle δ with the horizontal.

Rankine's formula may be produced in the same manner as Poncelet's by assuming the pressure parallel to the upper slope.

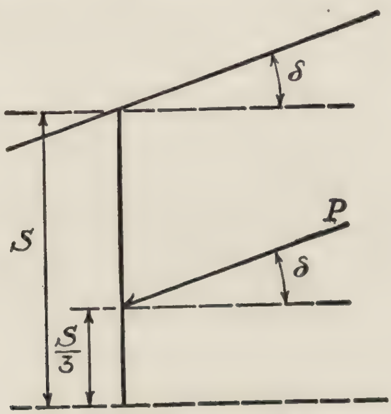


FIG. 74.—Pressure of Earth.

Thus in Fig. 73 (p. 296) if the angle made by P with the normal to the wall be equal to δ, we find

$$P=\frac{wS^2}{2}\cdot\frac{\cos^2\phi}{\cos\delta\left(1+\sqrt{\frac{\sin(\phi-\delta)\sin(\phi+\delta)}{\cos^2\delta}}\right)^2},$$

which may be transformed [1] into Rankine's formula as given above.

Weyrauch's theory is practically the same as Rankine's although produced in a different way.

Cohesion.—In all of the ordinary formulas for earth pressure, the effect of cohesion is neglected. Experiments indicate that this effect is not sufficient to affect very materially the actual pressure upon a wall. It causes the earth to break off and slide upon a concave surface instead of a plane surface. At the upper surface of the earth, the cohesion is sufficient to overcome the lateral thrust and cause the earth to stand in a vertical position, while as the lateral thrust increases with the depth, the cohesion becomes relatively less important and the surface of rupture flattens out. When earth is placed behind a wall after it is constructed cohesion is probably negligible at first, although after the earth has become compacted may develop in some

[1] Wm. Cain, Practical Designing of Retaining Walls, 1914, p. 103.

cases so that practically no pressure comes against the wall. It is so uncertain that no reliance should be placed upon it in designing walls.

Value of Theories.—On account of the variable nature of the material, it is evident that estimates of earth pressures are only rough approximations to the actual pressures. The material assumed as possessing uniform friction and without cohesion does not exist in practice. The general laws developed, however, do give rational methods of reaching reasonable estimates upon which safe designs may be based.

Experiments upon sand pressures, and experience with walls in use, indicate that Coulomb's use of horizontal earth pressures, or Rankine's thrust parallel to earth surface, where the surface is near the horizontal, give thrusts much greater than those actually produced upon walls with vertical backs. For such walls, the use of the Poncelet's formulas, taking into account the friction of the earth on the back of the wall, give results which seem to agree fairly well with experiment and experience.

For walls leaning forward, so that considerable weights of earth rest upon them, Rankine's formulas may be applied to find the thrust upon the vertical section through the earth at the inner edge of the base of the wall. This thrust, combined with the weight of earth resting upon the wall, gives the thrust against the wall.

145. Computation of Earth Thrusts.—When the back of a wall is nearly vertical, the thrust may usually be taken as making the angle of friction with a normal to the surface of the wall, as assumed in the theory of Poncelet. For such walls the thrust may be obtained from the formula already given:

$$P=\frac{wh^2}{2}\cdot\frac{\sin^2(\theta-\phi)}{\sin^2\theta\cdot\sin(\theta+\phi)\left(1+\sqrt{\frac{\sin(\phi-\delta)\cdot\sin 2\phi}{\sin(\theta-\delta)\sin(\theta+\phi)}}\right)^2}.\quad . \quad (1)$$

If we place $P=\frac{wh^2}{2}Q$, values of Q may be tabulated for various slopes and angles of friction as shown in Table XLIX (p. 299). The values of P obtained by this method are supposed to act against the wall at a distance $h/3$ above the base, and make the angle of friction with the normal to the wall.

When a mass of earth rests upon a wall, as in a wall with sloping back or a reinforced concrete wall, the formula of Rankine for pressure upon a vertical section may be applied. This pressure combined

TABLE XLIX.—EARTH PRESSURE AGAINST A WALL

Values of Q in Formula (1), $P=\frac{wh^2}{2}\cdot Q$

Batter of Back of Wall.	Slope of Upper Surface of Earth.		Angle of Friction, ϕ					
	Angle δ.	Vertical to Horizontal.	20°	25°	30°	35°	40°	45°
Vertical $\theta=90°$	33° 40′	1 to $1\frac{1}{2}$	......	......	......	.59	.39	.28
	29° 45′	1 to $1\frac{3}{4}$	......	......	.76	.45	.34	.25
	26° 30′	1 to 2	......	......	.54	.39	.32	.23
	21° 50′	1 to $2\frac{1}{2}$	......	.61	.46	.35	.30	.22
	18° 30′	1 to 3	.72	.52	.40	.33	.28	.21
	14° 00′	1 to 4	.58	.45	.36	.30	.25	.19
	0° 00′	Level	.43	.37	.30	.26	.21	.18
1 in 10 $\theta=95° 40'$	33° 40′	1 to $1\frac{1}{2}$	......	......	......	.72	.50	.37
	29° 40′	1 to $1\frac{3}{4}$	......	......	.90	.56	.44	.35
	26° 30′	1 to 2	......	......	.64	.49	.40	.32
	21° 50′	1 to $2\frac{1}{2}$	......	.70	.56	.44	.36	.30
	18° 30′	1 to 3	.82	.60	.48	.40	.34	.28
	14° 00′	1 to 4	.66	.52	.40	.35	.31	.25
	0° 00′	Level	.48	.40	.34	.30	.26	.22
1 in 5 $\theta=101° 20'$	33° 40′	1 to $1\frac{1}{2}$	......	......	......	.91	.62	.49
	29° 45′	1 to $1\frac{3}{4}$	......	......	1.08	.68	.55	.46
	26° 30′	1 to 2	......	......	.77	.60	.50	.42
	21° 50′	1 to $2\frac{1}{2}$	......	.80	.66	.54	.45	.38
	18° 30′	1 to 3	.93	.68	.57	.48	.42	.36
	14° 00′	1 to 4	.75	.60	.48	.41	.38	.32
	0° 00′	Level	.52	.46	.40	.34	.30	.27

with the weight of the earth resting upon the wall gives the thrust against the wall.

The value of the pressure upon the vertical section is given by Rankine's formula:

$$P=\frac{wS^2}{2}\cdot\cos\delta\cdot\frac{\cos\delta-\sqrt{\cos^2\delta-\cos^2\phi}}{\cos\delta+\sqrt{\cos^2\delta-\cos^2\phi}}=\frac{wS^2}{2}K. \quad . \quad (2)$$

Values of K corresponding to various values of δ and ϕ are tabulated in Table L (p. 300), thus greatly reducing the labor of computing the pressures. P as computed from this formula is supposed to act at a distance $S/3$ from the bottom of the section and to be parallel

TABLE L.—PRESSURES UPON VERTICAL SECTIONS THROUGH EARTH

Values of K in Formula (2) $P=\frac{wS^2}{2}K$

Slope of Upper Surface of Earth.		Angle of Friction.					
Angle δ.	Vertical to Horizontal.	20°	25°	30°	35°	40°	45°
33° 40′	1 to $1\frac{1}{2}$	....	....	....	0.59	0.36	0.26
29° 45′	1 to $1\frac{3}{4}$	....	....	0.76	0.45	0.32	0.23
26° 30′	1 to 2	....	....	0.54	0.39	0.29	0.21
21° 50′	1 to $2\frac{1}{2}$	....	0.60	0.45	0.34	0.27	0.20
18° 30′	1 to 3	0.72	0.52	0.40	0.31	0.24	0.19
14° 00′	1 to 4	0.59	0.45	0.36	0.29	0.23	0.18
0° 00′	Level	0.50	0.40	0.33	0.27	0.22	0.18

to the upper surface of the earth. S in this formula is the height of the earth section and not the height of the wall.

Angle of Friction.—In order to be able to apply any of the formulas for determining earth pressures, it is necessary to know the weight per unit volume and the angle of friction of the earth. These vary with the kind of material to be filled behind the wall and its condition as to compactness and moisture.

The natural slope for the earth is the slope at which the surface of the material will stand when dumped into piles, the frictional resistance keeping the surface layer from sliding or rolling down the slope. The angle of sliding friction of a wedge of earth upon an earth surface may not be the same as the inclination of the natural slope. Values of sliding friction as determined by experiment vary considerably for the same material, and it is possible that much of the variation is due to the methods of testing rather than to differences in the materials. The natural slope of a particular material may usually be approximately determined without difficulty and its use instead of the angle of sliding friction would ordinarily be safe. Table LI (p. 301) gives approximate values of the angle of internal friction, the natural slopes and weights of various materials commonly met in construction.

Surcharged Walls.—The formulas for earth pressure already given assume the earth to carry only its own weight and the upper surface to slope from the top of the wall. When the earth behind the wall carries a load upon its surface, as when supporting a railway track

TABLE LI.—FRICTION ANGLES AND WEIGHTS OF MATERIALS

Kind of Material.	Angle of Friction.	Natural Slope, Horizontal to Vertical.	Weight per Cubic Foot.
Clay, dry	35°	1.5 to 1	110
Clay, damp	40°	1.2 to 1	110
Clay, wet	20°	3.0 to 1	120
Sand, dry	35°	1.5 to 1	100
Sand, moist	40°	1.3 to 1	100
Sand, wet	25°	2.0 to 1	115
Gravel and sand	40°	1.5 to 1	110
Broken rock	45°	1.2 to 1	110

or a pile of material of any sort, the pressure against the wall is increased uniformly over its entire depth.

If w_3 is the weight of the load per unit area of earth surface, Formula (1) becomes,

$$P=\left(\frac{wh^2}{2}+w_3h\right)Q. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The point of application of P is at a distance

$$\frac{1}{3}\cdot\frac{wh+3w_3}{wh+2w_3}h,$$

above the base of the wall.

In the same manner for the pressure on a vertical section through the mass of earth, Formula (2) becomes

$$P=\left(\frac{wS^2}{2}+w_3S\right)K, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and its point of application is at a distance,

$$\frac{1}{3}\cdot\frac{wS+3w_3}{wS+2w_3}S.$$

146. Methods of the American Railway Engineering Association. —In the 1921 edition of the Manual of the A. R. E. A. are various formulas based on Rankine's Theory and on modifications of that theory. Some of these are given below.

Nomenclature:

ϕ = the angle of repose of the filling;

θ = the angle between the back of the wall and a horizontal line passing through the heel of the wall and extending from the back into the fill;

δ = angle of surcharge, which is the angle between a horizontal line and the surface of the filling.

(It is recommended that values of $\delta = 0$ or $\delta = \phi$ be used.)

h = vertical height of wall in feet;

h' = height of surcharge in feet;

l = width of the base of the wall in feet;

e = distance from the center of the base to the intersection of the resultant thrust E and the base;

$a = 2e$ = distance from the toe of the wall to the intersection of the resultant thrust E and the base;

P = the resultant earth pressure per foot length of wall;

E = the resultant of the earth pressure and the weight of the wall;

F = vertical component of resultant, E;

w = the weight of the filling per cubic foot;

w_1 = the weight of the masonry per cubic foot;

W = total weight of wall per foot of length;

W_1 = total weight of wedge of earth 1 foot long;

$W_2 = W + W_1$;

f_c and f_c'' = pressure per square foot on the foundation due to F, at toe and heel, respectively.

Formulas

1. *Vertical Wall, Horizontal Surcharge.* Fig. 75(*a*) (p. 304).

$$P = \tfrac{1}{2}wh^2 \frac{1-\sin\phi}{1+\sin\phi} = \frac{1}{2}wh^2 \tan^2\left(45° - \frac{\phi}{2}\right);$$

For $\phi = 1\tfrac{1}{2}$ to 1 ($\phi = 33°\ 42'$), $P = 0.143\ wh^2$;

For $\phi = 1$ to 1 ($\phi = 45°$, $P = 0.086\ wh^2$;

P is horizontal and applied at a distance $y = \dfrac{h}{3}$ above the base.

2. *Vertical Wall, Sloping Surcharge.* Fig. 75 (*b*).

$$P = \tfrac{1}{2}wh^2 \cos\phi;$$

For $\phi = 1\tfrac{1}{2}$ to 1 ($\phi = 33°\ 42'$), $P = 0.416\ wh^2$;

For $\phi = 1$ to 1 ($\phi = 45°$), $P = 0.35\ wh^2$;

P is parallel to slope of surcharge and applied at a distance $y = \dfrac{h}{3}$ above the base.

3. *Vertical Wall, Loaded Surcharge.* Fig. 75(*c*).

$$P=\tfrac{1}{2}wh(h+2h')\,\frac{1-\sin\phi}{1+\sin\phi};$$

For $\phi=1\tfrac{1}{2}$ to 1 ($\phi=33°\ 42'$), $P=0.143\ wh(h+2h')$;
For $\phi=1$ to 1 ($\phi=45°$), $P=0.086\ wh(h+2h')$;
P is horizontal and applied at a distance

$$y=\frac{h^2+3hh'}{3(h+2h')}\ \text{above the base.}$$

4. *Wall Leaning Forward, Horizontal Surcharge.*—Fig. 75(*d*).

$$P\ =\tfrac{1}{2}wh^2\frac{1-\sin\phi}{1+\sin\phi}=\frac{1}{2}wh^2\tan^2\left(45°-\frac{\phi}{2}\right),\ \text{as in Case 1;}$$

P is horizontal and applied at a distance $y=\dfrac{h}{3}$ above the base.

5. *Wall Leaning Forward, Inclined Surcharge.*—Fig. 75(*e*)

$P=\tfrac{1}{2}wh(h+h')\cos\phi$;
P is parallel to inclined surface and applied at a distance

$$y=\frac{h+h'}{3}\ \text{above the base.}$$

6. *Wall Leaning Forward, Loaded Surcharge.*—Fig. 75(*f*).

$$P=\tfrac{1}{2}wh(h+2h')\,\frac{1-\sin\phi}{1-\sin\phi},\ \text{as in Case 3;}$$

P is horizontal and applied at a distance

$$y=\frac{h^2+3hh'}{3(h+2h')}\ \text{above the base}$$

h' = surcharge per square foot divided by w, equals $\dfrac{w_3}{w}$.

"In calculating the surcharge due to a track, the entire load shall be taken as distributed uniformly over a width of 14 feet for a single track or for tracks spaced more than 14 feet centers, and the distance center to center of tracks where tracks are spaced less than 14 feet.

"In calculating the pressure on a retaining wall where the filling carries permanent tracks or structures, the full effect of the loaded surcharge shall be considered where the edge of the distributed load or structure is vertically above the back edge of the heel of the wall. The effect of the loaded surcharge may be neglected where the edge of the distributed load or structure is at a distance from the vertical line through the back edge of the heel of the wall equal to h, the height of the wall. For intermediate positions, the equivalent

uniform surcharge load is to be taken as proportional. For example, for a track with the edge of the distributed load at a distance,

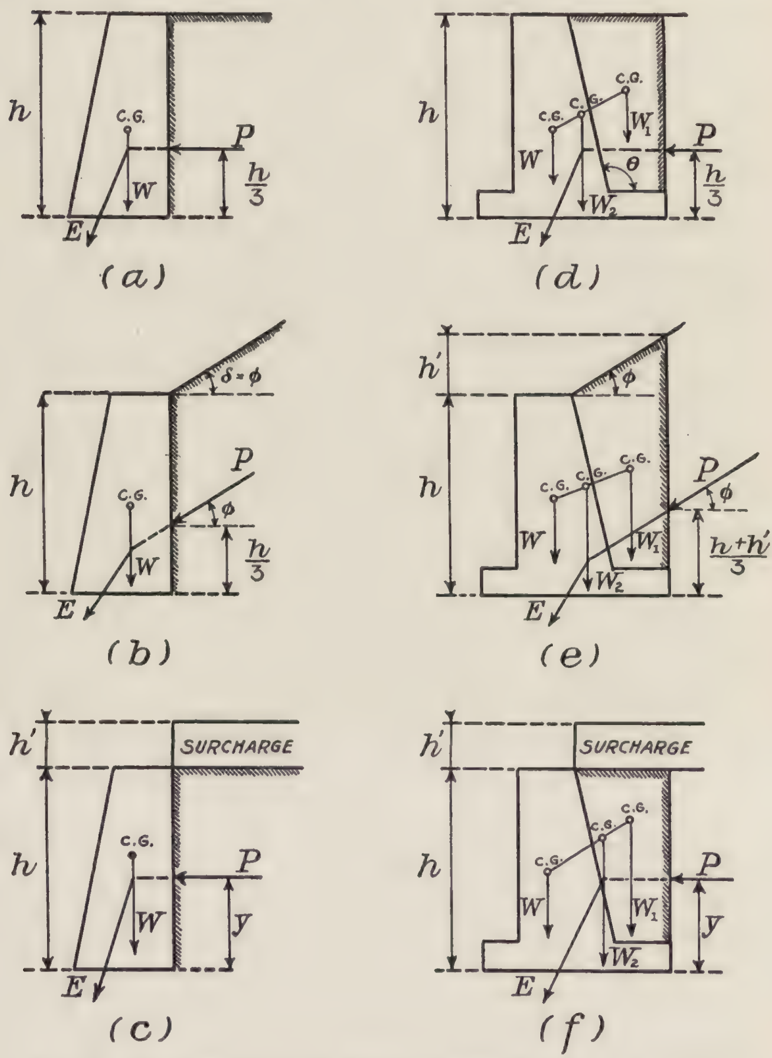


FIG. 75.—Six Typical Gravity Retaining Wall Sections and Loadings. A. R. E. A. 1921 Manual.

$h/2$, from the vertical line through the back edge of the heel of the wall, the equivalent uniform surcharge load is one-half the normal distributed load distributed over the filling.

The wall should be investigated when W_1 includes surcharge, and when surcharge over wedge is omitted.

Detailed discussions of methods of determining earth pressures are given in "Retaining Walls for Earth" by M. A. Howe, New York, 1896, and in "Practical Designing of Retaining Walls," by Wm. Cain, New York, 1914. An interesting paper by E. P. Goodrich in Transactions, American Society of Civil Engineers, December, 1904, gives results of experiments for determination of internal friction and lateral pressure of earth.

ART. 36. SOLID MASONRY WALLS

147. Stability of Walls.—A masonry retaining wall may fail in one of three ways:

1. By overturning or rotating about its toe.
2. By crushing the masonry.
3. By sliding on a horizontal joint.

Insufficient foundation is probably the most common cause of failure of retaining walls. This is not, however, due to failure of the wall itself, but to lack of sufficient footing or other support when placed upon compressible or soft soils or to lack of proper drainage. This is discussed in Art. 36.

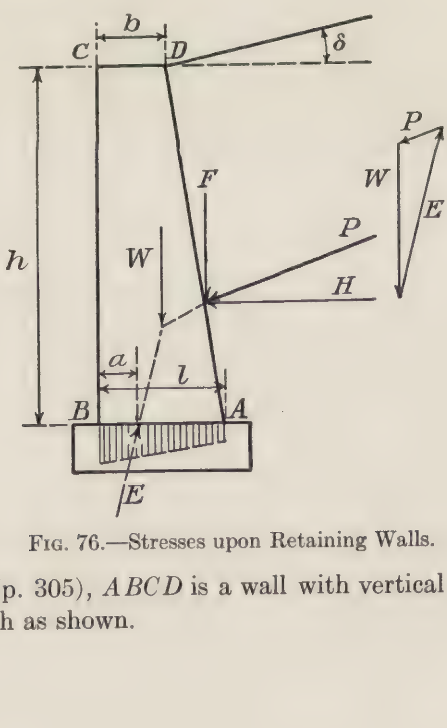


FIG. 76.—Stresses upon Retaining Walls.

In Fig. 76 (p. 305), $ABCD$ is a wall with vertical face supporting a bank of earth as shown.

Let P = thrust of earth against the wall;
F = Vertical component of P;
H = horizontal component of P;
W = weight of wall acting through its center of gravity;
E = resultant pressure on base AB:
l = width of base of wall;
b = width of top of wall;
d = distance from face of wall to its center of gravity;
f_c = unit compression on masonry at toe of wall;
a = distance from toe of wall to point of application of resultant pressure upon the base;
β = angle made by E with normal to base of wall.

Resisting Moment.—The moment of the thrust about the toe of the wall at B is $M_p = H\times\frac{h}{3} - F\times\frac{(2l+b)}{3}$.

This moment tends to overturn the wall by causing rotation about B, and is resisted by the moment of the weight of wall in the opposite direction. This moment is $M_w = Wd$.

When these moments are equal ($M_w - M_p = 0$), the resultant E obtained by combining P and W passes through B and the wall is on the point of overturning. The ratio M_w/M_p is the factor of safety against overturning.

When M_w is greater than M_p, E will cut the base of the wall to the right of B. Placing $M_r = M_w - M_p$ we have

$$(W+F)a = Wd - \frac{Hh}{3} + \frac{F(2l+b)}{3},$$

from which we find the distance of the point of application of E from B:

$$a = \frac{3Wd + F(2l+b) - Hh}{3(W+F)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The resultant E should always cut the base of the wall within its middle third ($a > l/3$) in order that the pressure may be distributed over the whole section of the base and there may be no tendency for the joint to open, or no tensile stress developed at the inner edge (A) of the section.

Crushing of Masonry.—The unit stress at the toe of the wall (B) must not exceed the safe crushing strength of the masonry. The distribution of stress over the section depends upon the position of the point of application of the resultant (E). When $a = l/3$, the

stress at A will be zero, and the stress at B, $f_c=\frac{2(W+F)}{l}$. If a be less than $l/3$, the pressure will be distributed over a distance $3a$ from the toe (B), and the maximum stress, $f_c=\frac{2(W+F)}{3a}$. When a is greater than $l/3$ the maximum compression,

$$f_c=\frac{(W+F)(4l-6a)}{l^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Resistance to Sliding depends upon the development of sufficient friction in any joint through the wall to overcome the pressure parallel to the joint. Thus (Fig. 76, p. 305) in order that no sliding occur at the base of the wall, the frictional resistance in the joint AB must be greater than the horizontal component of the thrust E. This will be the case when E makes an angle (β) with the normal to AB that is less than the angle of friction of masonry sliding upon masonry. Tan $\beta=\frac{H}{W+F}$, must be less than the coefficient of friction of the masonry.

In the construction of heavy walls, resistance to sliding may be increased by breaking joints so that no continuous joint exists through the wall. Joints inclined from the front to the back of the wall are also sometimes used so as to bring the resultant pressure more nearly normal to the joint.

148. Empirical Design.—In the practical designing of retaining walls, engineers have commonly used empirical rules given by certain prominent authorities, or have assumed dimensions based upon their own experiences. The uncertain and conflicting nature of the assumptions used in producing the formulas based upon the various theories, and the lack of satisfactory experimental data have caused the use of dimensions shown by experience to be safe and in very many instances probably quite excessive.

Trautwine's rules have been extensively used for many years, and are as follows [2] for vertical walls:

When the backing is deposited loosely, as usual, as when dumped from carts, cars, etc.,	LENGTH OF BASE EQUALS:
Wall of cut stone, or first-class large ranged rubble in mortar	.35 of its entire vertical height
Wall of good common scabbled mortar-rubble, or brick	.4 of its entire vertical height
Wall of well-scabbled dry rubble	.5 of its entire vertical height

[2] Trautwine's Engineer's Pocket-Book.

With good masonry, however, we may take the height from the ground surface up, instead of the total height as above indicated.

When the wall has a sloping or offset back, the thickness above given may be used as the mean thickness, or thickness at the mid-height.

Baker's Rules.—Sir Benjamin Baker, from an extended experience in the construction of walls under many differing conditions, and after numerous experiments from the thrust of the earth, gives [3] the

Experience has shown that a wall one-quarter of the height in thickness, and battering 1 inch or 2 inches per foot on the face, possesses sufficient stability when the backing and foundation are both favorable. The Author, however, would not seek to justify this proportion by assuming the slope of repose to be about 1 to 1, when it is perhaps more nearly $1\frac{1}{2}$ to 1, and a factor of safety to be unnecessary, but would rather say that experiment has shown the actual lateral thrust of good filling to be equivalent to that of a fluid weighing about 10 pounds per cubic foot, and allowing for variations in the ground, vibrations, and contingencies, a factor of safety of 2, the wall should be able to sustain at least 20 pounds fluid pressure, which will be the case if one-quarter of the height in thickness.

It has been similarly proved by experience that under no ordinary conditions of surcharge or heavy backing is it necessary to make a retaining wall on a solid foundation more than double the above, or one-half of the height in thickness. Within these limits the engineer must vary the strength in accordance with the conditions affecting the particular case.

The rules of Sir Benjamin Baker give walls considerably lighter than those of Trautwine, and the tendency in recent practice has been to somewhat reduce the thicknesses for walls backed with good materials and built under favorable conditions. Where from lack of drainage or other cause, the backing is liable to get into soft condition, it may be necessary to considerably increase thickness.

149. Using Formulas in Design.—The design of a wall to sustain a bank of earth is a comparatively simple matter once the earth pressure has been determined. The difficulties met are those of judging the character of the material and its probable pressure against the wall. It is probable that in most instances the full pressures that theoretically might come upon the wall are not actually developed. The design should be made for the worst conditions which may reasonably be expected to occur, but the construction of heavy walls to provide for bad conditions which are not likely to occur, and which may be met by proper attention to drainage and proper care in placing the backing, is unnecessarily expensive and wasteful.

[3] The Actual Lateral Pressure of Earth, Van Nostrand Science Series, and Proceedings, Institution of Civil Engineers, Vol. LXV, p. 183.

For walls with vertical or nearly vertical backs, Poncelet's formulas, taking into account the friction of the earth on the back of the wall, give thicknesses for walls which agree fairly well with the results of experience and not differing greatly from the rules suggested by Sir Benjamin Baker.

In designing by this method, the pressure of earth is obtained by the use of Formula (1), or from Table XLIX (p. 299), a section of wall is assumed and its sufficiency investigated.

Example 1.—A masonry wall, 22 feet high, is to support a bank of earth whose surface has an upward slope of 1 to 3 from the top of the wall. The backing is ordinary earth whose friction angle may be taken at 35°. Weight of masonry is 150 pounds and of earth 100 pounds per cubic foot. Find proper section for the wall.

Solution.—Try a rectangular wall with thickness of 7.5 feet. From Table XLIX, we find $Q=.33$. Then

$$P=\frac{wh^2}{2}Q=\frac{100\times 22\times 22\times .33}{2}=7986 \text{ pounds per foot of length of wall.}$$

As P makes angle of 35° with normal to back of wall,

$H=P\cos 35°=6770$ pounds and $F=P\sin 35°=4580$ pounds.

$W=22\times 7.5\times 150=24{,}750$ pounds.

From (5), we find

$$a=\frac{3\times 24750\times 3.75+4580\times 3\times 7.5-6770\times 22}{3(24750+4580)}=2.64$$

and the resultant (E) comes just within the middle third of the base.

The crushing stress on the masonry at the toe of the wall is (6)

$$f_c=\frac{(W+F)(4l-6a)}{l^2}=\frac{(24750+4580)(4\times 7.5-6\times 2.64)}{7.5\times 7.5}=7383 \text{ lb./ft.}^2$$

which would be quite safe for any ordinary masonry.

$$\tan\beta=\frac{H}{W+F}=\frac{6770}{24750+4580}=.231, \text{ or } \beta=13°,$$

and sliding could not occur.

Battered Face.—The face of the wall may be battered, so as to diminish the width at top by one-third, using the same width of base without decreasing its stability.

Battered Back.—A wall with battered back may be used. Assume a top thickness, $b=5.5$, and base thickness, $l=9.5$. The angle made by the back of the wall with the horizontal is $\theta=100° 20'$. From

Table XLIX, we find $Q = .47$, then $P = \frac{wh^2Q}{2} = \frac{100 \times 22 \times 22 \times .47}{2}$ $= 11624$ pounds. $H = P \cos(\theta + \phi - 90°) = 8170$ pounds, and $F = P \sin(\theta + \phi - 90°) = 8270$ pounds. $W = \frac{5.5 + 9.5}{2} \times 22 \times 150 = 24750$ pounds.

From (5, p. 306),

$$a = \frac{3 \times 24750 \times 3.83 + 8270(2 \times 9.5 + 5.5) - 8170 \times 22}{3(24750 + 8270)} = 3.11 \text{ feet}$$

E cuts the base practically at one-third its width from the toe.

The crushing stress at the toe is $f_c = \frac{2(W+F)}{l} = 6950$ lb./ft.^{2}, a little less than for the rectangular wall.

$\tan \beta = \frac{8170}{24750 + 8270} = .247$, within safe limits but somewhat more than for the rectangular wall.

Example 2.—A retaining wall 20 feet high is to support a horizontal bank of earth carrying a railway track. The maximum train load is 800 lb./ft.^{2} of surface, and the natural slope of the earth is $1\frac{1}{2}$ to 1. Find the thickness of the wall, using the A. R. E. A. formulas. Masonry weighs 150 and earth 100 lb./ft.^{2}

Solution.—Assume a face batter of 1 inch per foot, a top thickness of 7 feet, and a base thickness of 14 feet.

$$h' = \frac{w_3}{w} = 800 = 8 \text{ feet};$$

$$P = 0.143wh(h + 2h') = 0.143 \times 100 \times 20(20 + 16) = 10{,}296 \text{ pounds.}$$

$$W = 150 \times 20(7 + 14)/2 = 31{,}500 \text{ pounds.}$$

The center of gravity of the wall is 6.2 feet from the toe.
That of the wedge of earth is 12.2 feet from the toe.

$$a = \frac{31500 \times 6.2 + 5333 \times 12.2 - 10296 \times 8.15}{31500 + 5333} = 4.79 \text{ feet.}$$

The resultant thrust cuts the base within the middle third.
The crushing stress at the toe of the wall is, from Formula on p. 81,

$$f_c = \frac{(W+F)(4l - 6a)}{l^2} = 5120 \text{ lb./ft.}^2$$

The minimum thickness allowable for a solid wall is that which causes the resultant thrust (E) to cut the base at a distance $a = l/3$

from the toe of the wall. For a rectangular wall, the width bears a direct ratio to the height for any particular values for weights of materials and angles of friction. Table LII gives minimum values of thickness ratio, by Poncelet's formula for walls in which the weight of masonry is taken as 150 lb./ft.[2] and the weight of earth as 100 lb./ft.[2]

TABLE LII.—MINIMUM THICKNESS OF WALLS BY PONCELET'S FORMULA

Values of l/h, when $w_1 = 150$ lb./ft.[2] and $w = 100$ lb./ft.[2]

Slopes of Earth Surface Vertical to Horizontal.	Angles of Friction.					
	20°	25°	30°	35°	40°	45°
1 to $1\frac{1}{2}$	....	....	....	0.39	0.31	0.25
1 to $1\frac{3}{4}$	....	....	0.46	0.35	0.29	0.24
1 to 2	....	....	0.41	0.34	0.28	0.23
1 to $2\frac{1}{2}$	....	0.36	0.39	0.33	0.27	0.23
1 to 3	0.53	0.43	0.37	0.32	0.27	0.23
1 to 4	0.49	0.41	0.35	0.30	0.26	0.22
Level	0.43	0.38	0.33	0.29	0.25	0.22

For walls battered or stepped on the back the minimum thickness given in the table may be used as the average thickness at the middle of the height. This gives a broader base to the wall and gives a larger factor of saftey against overturning, but requires the same volume of masonry to keep the resultant thrust within the middle third of the base.

Walls computed as rectangular may be battered on the face to an extent which lessens the top thickness by one-third without increasing the base thickness. This slightly decreases the resisting moment, but increases the value of a, lessens the pressure at the toe, and does not impair the stability of the wall.

ART. 37. REINFORCED CONCRETE WALLS

150. Types of Reinforced Concrete Retaining Walls.—There are two types of reinforced concrete retaining walls in common use:

1. The cantilever type and 2, the counterforted type.

Both of these depend upon the weight of earth carried by the base of the wall to prevent overturning. They differ in the way in which the face wall is attached to the base.

A cantilever wall is shown in Fig. 77, consisting of a vertical stem attached to a base, $ACFB$. The weight of the mass of earth $BFEG$, rests upon the base of the wall BF and serves to assist the wall in resisting the overturning moment of the earth thrust. The horizontal pressure of earth on JF is carried by the vertical stem $CDJL$ acting as a cantilever beam. The projecting bases FB and AC are also cantilever beams, the one supporting the weight of earth resting upon it, the other resisting the upward thrust of the foundation at the toe of the wall.

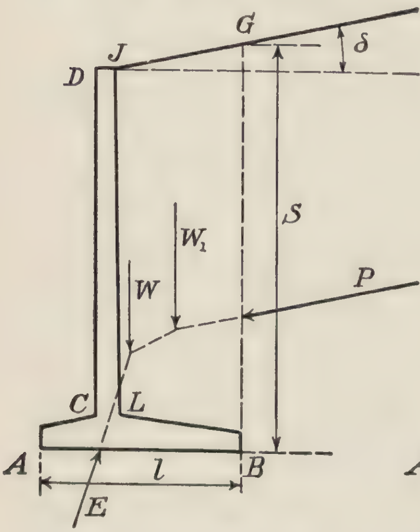


FIG. 77.—Cantilever Wall.

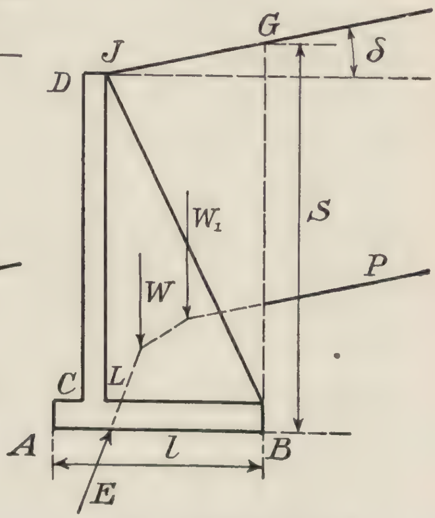


FIG. 78.—Counterforted Wall.

A counterforted wall is shown in Fig. 78. The face wall $CDJL$ is connected with the base $ACLB$ by narrow counterforts JLB, spaced several feet apart. The counterforts are cantilever beams, each carrying the horizontal earth thrust on the face wall JL for a panel length of wall. The face walls $CDJL$ are slabs holding the earth pressure between counterforts and transferring the pressure to the counterforts. The base LB is a slab carrying the weight of earth $LJGB$ between counterforts and holding down the ends of the counterforts. The base AC at the front of the wall is a cantilever carrying the upward thrust of the foundation at the toe of the wall.

The cantilever type is commonly used for moderate heights of wall. For walls more than 20 or 25 feet high, the counterforted wall is usually more economical. The quantities of materials required for a counterforted wall are less and the amount of form work more than for a cantilever wall.

151. Design of Cantilever Wall.—In designing reinforced concrete walls, the thrust in the vertical section of earth passing through the inner edge of the base may be computed by Rankine's formula, as given in Section 144:

$$P=\frac{wS^2}{2}\cdot\cos\delta\cdot\frac{\cos\delta-\sqrt{\cos^2\delta-\cos^2\phi}}{\cos\delta+\sqrt{\cos^2\delta-\cos^2\phi}}=wS^2K. \quad . \quad . \quad (2)$$

Values of K may be taken from Table L (p. 300). This thrust is parallel to the upper surface of the earth and its horizontal and vertical components are

$$H=P\cos\delta=\frac{wS^2}{2}K\cos\delta \quad \text{and} \quad F=P\sin\delta=\frac{wS^2}{2}K\sin\delta.$$

The method of design will be illustrated by numerical examples.

Example 3.—Design a retaining wall to hold a level bank of earth 16 feet high. The base of footing is to be 3 feet below surface of ground and the pressure on the soil is limited to 4000 lb./ft.^{2} The backing is ordinary soil with angle of friction of 35°. Earth weighs 100 pounds and concrete 150 pounds per cubic foot. Unit stresses will be based upon use of 2000-pound concrete and plain bars of medium steel.

Solution.—Assume the base under the wall 15 inches thick. The height of the wall above the base is then 17.75 feet, and the horizontal thrust, taking K from Table L (p. 300), is

$$P=\frac{wS^2K}{2}=\frac{100\times17.75\times17.75\times.27}{2}=4250 \text{ lb. per ft. of length of wall.}$$

The bending moment caused by this thrust upon the section at the top of the base is $M=4250\times5.92\times12=302100$ in.-lb.

From Table XXVIII (p. 205) it is found that d may be 15.25 inches and that the area of steel required is $A_s=1.41$ inches.^{2} From Table XVII (p. 204) may be selected $\frac{7}{8}$-inch round bars, spaced 5 inches on centers. These will be used and embedded $2\frac{3}{4}$ inches. The total thickness of the vertical stem at the top of the base will then be 18 inches.

Taking 10 inches as the minimum thickness at the top of the wall and making the faces of the wall plane surfaces, the thickness at all intermediate points will be greater than required for strength.

For a section 12 feet below the top the value of pressure to be used will be $P=\frac{wS^2K}{2}=100\times12\times12\times.27/2=1944$ lb., and the bending moment will be $M=1944\times4\times12=93213$ in.-lb.

The effective depth of beam is 13 inches. Then

$$R=\frac{93312}{12\times13\times13}=46. \quad R/f_s=46/16000=.0029,$$

and from Table XXIV (p. 201) we find $p=.0032$. The area of steel required is $12\times13\times.0032=0.5$ in.^{2} per foot of length, or about one-third of that at the base. Similarly at a section 6 feet below the top, no steel would theoretically be required.

If all of the bars be carried up 6 feet, every third bar 12 feet and every sixth bar to the top, the reinforcement will be amply strong. The lower ends of these bars should be turned up in the base for anchorage.

The maximum shear in section at base is 4250 pounds, and

$$v=\frac{V}{bjd}=\frac{4250}{12\times.874\times15.25}=27 \text{ lb./in.}^2$$

No diagonal tension reinforcement is necessary.

Overturning Moment.—Assume the width of base at about 45 per cent of the total height, or 8.5 feet. Let the inner surface of the vertical stem be vertical, and place the stem at a distance equal to one-third the width of base ($l/3$) from the toe of the wall. (See Fig. 79, p. 315.)

The moment of the thrust about the toe at A tends to overturn the wall, while the moments of the weights of the wall and earth resting upon it resist overturning.

The weight of the vertical stem is

$$W_s=\frac{10+18}{12\times2}(17.75\times150)=3150 \text{ pounds.}$$

The weight of the base is $W_b=1.25\times8.5\times150=1595$ pounds.

The weight of the earth is $W_1=18\times4.25\times100=7650$ pounds.

The earth thrust,

$$P=\frac{wS^2K}{2}=\frac{100\times19.25\times19.25\times.27}{2}=5000 \text{ pounds.}$$

The moment on the toe at A is

$M=3150\times3.5+1595\times4.25+7650\times6.38-4873\times6.42=35327$ ft.-lb.

The point of application of the resultant on the foundation soil is equal to the moment about A divided by the vertical component of the resultant, or

$$a=\frac{35327}{3150+1595+7650}=2.85 \text{ feet.}$$

This brings the resultant within the middle third of the base.

The factor of safety against overturning is

$$\frac{3150\times 3.5+1595\times 4.25+7650\times 6.38}{4873\times 6.42}=2.12.$$

Pressure on Foundation.—The total vertical load on the foundation is $3150+1595+7650=12,395$ pounds. The pressure at the

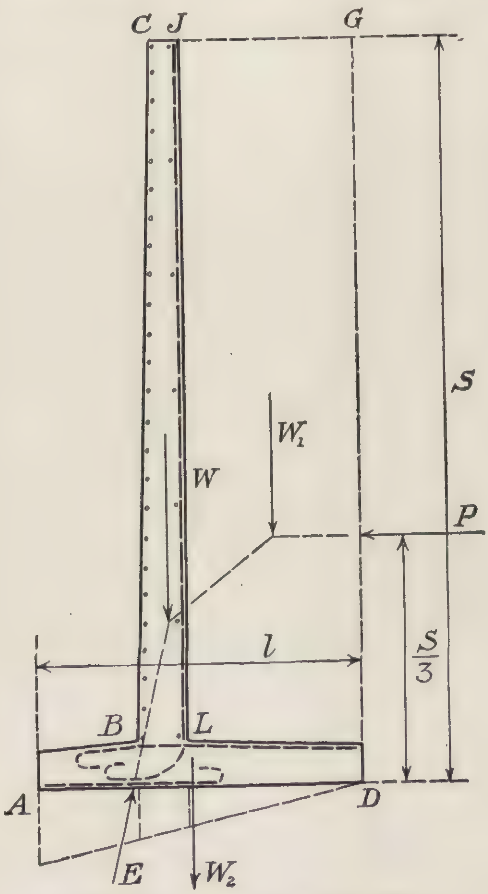


FIG. 79.—Design of Cantilever Wall.

toe is twice the average pressure, $2\times 12,395/8.5=2916$ lb./ft.^{2}, which is within safe limits.

Inner Base Cantilever.—The length of the inner cantilever is 4.25 feet. It is subject to the action of three loads:

(1) The weight of the earth resting upon it (1800 pounds per linear foot);

(2) Weight of the cantilever itself (188 pounds per linear foot.

(3) Upward pressure of the foundation soil (which is 0 at the end D and 1458 pounds where the cantilever joins the vertical wall at L).

The bending moment on the section at L is

$$M=1800\times4.25\times\frac{4.25}{2}+188\times4.25\times\frac{4.25}{2}-\frac{1458\times4.25}{2}\times\frac{4.25}{3}$$

$$=13566 \text{ ft.-lb.} \quad \text{or} \quad 162{,}800 \text{ in.-lb.}$$

From Table XXVIII (p. 205), a depth of $11\frac{1}{4}$ inches is found to be suitable to resist the bending moment, but before deciding definitely, the section should be investigated for shear.

The shear in section at L is

$$V=1800\times4.25+188\times4.25-1458\times4.25/2=5355 \text{ pounds.}$$

Diagram XI (p. 222) shows that for a shear of 5350 pounds, the value of $b\times d$ should be 150 in.^{2} for a unit shear of 40 pounds per inch square. A depth of $d=12\frac{1}{2}$ inches will be used to obviate the necessity of reinforcing for diagonal tension. Then

$$R=162800/12\times12.5\times12.5=87. \quad R/f_s=87/16000=.0054,$$

and from Table XXIV (p. 201), $p=.006$. $A_s=12\times12.5\times.006=0.9$ inch.^{2} Table XXVII (p. 204) gives $\frac{3}{4}$-inch round bars spaced 5 inches on centers (the same spacing as for the bars in the stem) with an area of 1.06 inches,^{2} and these will be used.

These bars should be anchored by bending or by continuing them through the concrete on the front of the base to a length of at least 50 diameters (37.5 inches).

The steel will be embedded $2\frac{1}{2}$ inches from the top of the base, making the total depth, d_1, 15 inches as assumed.

Outer Base Cantilever.—The length of the outer cantilever is 2.75 feet and to provide for the shear at the section B, the total thickness will be made 18 inches. The forces acting upon it are its own weight acting downward and the thrust of the soil acting upward (1916 lb./ft.^{2} at A and 1972 lb./ft.^{2} at B). The shear at the section B is

$$V=\frac{2916+1972}{2}\times2.75-225\times2.75=6100 \text{ lb.}$$

For $V=6100$ pounds and $v=40$ lb./in.^{2}, Diagram XI (p. 222) gives $b\times d=185$ in.^{2} Since b is 12 inches, d must be $15\frac{1}{2}$ inches.

The bending moment at B is

$$M=\frac{1972\times 2.75\times 2.75}{2}+\frac{2916-1972}{2}\times\frac{2\times 2.75\times 2.75}{3}$$

$$=10078 \text{ ft.-lb.}=120936 \text{ in.-lb.}$$

$$R=\frac{M}{bd^2}=\frac{120936}{12\times 15.5\times 15.5}=42.$$

$$\frac{R}{f_s}=\frac{42}{16000}=.0026.$$

From Table XXIV (p. 201), $p=.003$, and $A_s=.003\times 12\times 15.5=$ 0.56 in.^{2} per foot length of wall.

Table XXVII (p. 204) shows that if the 5-inch spacing is adhered to, $\frac{5}{8}$-inch round bars must be used. They must extend into the base a distance of at least 50 diameters (31 inches) past the Section at B. They will be placed $2\frac{1}{2}$ inches above the lower surface of the concrete and the total depth of the outer base will be 18 inches, as assumed.

Horizontal bars should be placed longitudinally through the wall near the exposed face to prevent cracking due to contraction; $\frac{1}{2}$-inch bars spaced 12 inches apart are sufficient for this purpose.

Example 4.—A cantilever wall is to be 17 feet high above ground and to support a bank of earth whose surface has an upward slope of 2 horizontal to 1 vertical from the top of the wall. Angle of friction for backing earth, $\phi=35°$. The soil under the base may be safely loaded with 6000 pounds per square foot. Earth fillings weighs 100 lb./ft.^{3} and concrete 150 lb./ft.^{3} Safe values of $f_c=500$ lb./in.^{2} $f_s=16{,}000$ lb./in.^{2}, and for diagonal tension $v=30$ lb./in.^{2} $n=15$. The base of the wall will extend 4 feet below the surface of the ground and the toe of the wall cannot extend beyond its face.

Solution.—Assume a depth of base of 24 inches and a width of base of 12 feet. (See Fig. 80, p. 319.)

Vertical wall.—The total height of the vertical wall is 19 feet. The thrust on the back of this wall is

$$P=\frac{wh^2K}{2}=\frac{100\times 19\times 19\times .39}{2}=7040 \text{ lb. per foot of wall.}$$

This acts parallel to the surface of the earth and its horizontal component $H=7040 \cos 26° \ 30'=6300$ pounds. The moment of this about the base of the wall is $(6300\times 19/3)\times 12=478{,}800$ in.-lb. From Table XXII (p. 199) $R=72$ and $p=.005$. $12d^2=478800/72=$ 6648, and $d=24$ inches.

The total thickness at base is 26 inches. Take top as 12 inches thick, and make face of wall vertical. At base, $A_s = 24 \times 12 \times .005 = 1.44$ in.^{2} From Table XXVII (p. 204), $\frac{3}{4}$-inch square bars $4\frac{1}{2}$ inches apart will answer. All bars will extend to 12 feet below top, every third bar to 6 feet below top and every sixth bar to top of wall.

Shear at base section is 6300 pounds and

$$v = \frac{6300}{12 \times .9 \times 24} = 25 \text{ lb./in.}^2$$

which is within limits without diagonal tension reinforcement.

Overturning Moment.—The thrust on the vertical section at the inner edge of the base is

$$P = \frac{wS^2}{2}K = \frac{100 \times 26.5 \times 26.5}{2} \times .39 = 13690 \text{ lb.}$$

Its horizontal component is

$$H = 13690 \cos 26° \, 30' = 12250 \text{ lb.}$$

and its vertical component

$$F = 13690 \sin 26° \, 30' = 6100 \text{ lb.}$$

The weight of the base of wall

$$W_b = 12 \times 2 \times 150 = 3600 \text{ lb.}$$

Weight of vertical wall

$$W_v = \frac{1 + 2.2}{2} \times 19 \times 150 = 4560 \text{ lb.}$$

Weight of earth on wall

$$W_1 = \left(\frac{9.8 + 11}{2} \times 19 + \frac{11 \times 5.5}{2}\right) \times 100 = 22785 \text{ lb.}$$

The moment of P about the toe of the wall is

$$M_0 = 12250 \times 26.5/3 - 6100 \times 12 = 35000 \text{ ft.-lb.}$$

The moment of resistance is

$$M_r = 3600 \times 6 + 4560 \times .64 + 22785 \times 7.0 = 184000 \text{ ft.-lb.}$$

The factor of safety against overturning is $184000/35000 = 5.2$.

The distance from toe to point of application of resultant pressure on foundation

$$a = \frac{184000 - 35000}{3600 + 4560 + 22785 + 6100} = 4.03 \text{ ft.}$$

The maximum pressure on the soil at the toe of the wall is twice the average, or

$$f_c=\left(\frac{3600+4560+22785+6100}{12}\right)2=6175 \text{ lb./ft.}^2$$

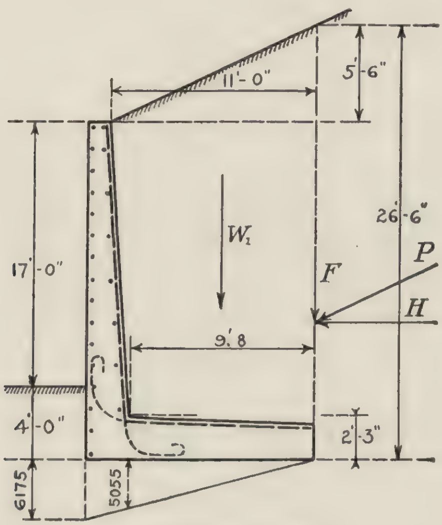


FIG. 80.—Cantilever Wall.

Base Cantilever.—The weight of earth resting upon the inner base is

$$W_1=\left(9.8\times 19.6+\frac{9.8\times 4.9}{2}\right)\times 100=21610 \text{ lb.}$$

The weight of the base is $9.8\times 2\times 150=2940$ lb. The upward thrust of the soil is

$$\frac{5055}{2}\times 9.8=24770 \text{ lb.}$$

The maximum bending moment at junction with vertical wall is

$$M=\left(21610\times 5.1+2940\times 4.9-24770\times\frac{9.8}{3}\right)\times 12=524000 \text{ in.-lb.}$$

$$d=\sqrt{\frac{M}{Rb}}=\frac{524000}{72\times 12}=24.6 \text{ in.} \quad \text{Make full depth 27 in.}$$

$A_s=24.6\times 12\times .005=1.43$ in.^{2}; $\frac{3}{4}$-inch square bars spaced $4\frac{1}{2}$ inches apart as in vertical wall meet the requirement.

For this loading, the point of maximum shear occurs where the

intensity of the downward forces equals that of the upward forces. This occurs at a point distant

$$\frac{5055-(1960+300)}{516+50}=4.94 \text{ ft.}$$

from the back of the vertical wall. The shear at this point is

$$V=\left(\frac{24.5+22.0}{2}\right)\times 4.86\times 100+2\times 4.86\times 150-\frac{2508}{2}\times 4.86=6663 \text{ lb.}$$

and

$$v=\frac{V}{bjd}=\frac{6663}{12\times .9\times 25}=25 \text{ lb./in.}^2$$

The reinforcing bars must be anchored and longitudinal bars introduced to prevent cracking as in Example 3.

152. Design of Counterforted Walls.—In walls of the counterforted type, the vertical curtain wall (see Fig. 81, p. 325) is a slab supported against the horizontal thrust of the earth by the counterforts at frequent intervals. The counterforts are cantilever beams held in place by the base, and each carrying a panel load of the thrust against the vertical slab. The inner base is a horizontal slab, suspended from the counterforts, and carrying the weight of earth resting upon it. The outer base is a cantilever and carries the upward pressure of the soil upon the toe of the wall as in the cantilever wall.

Example.—A wall with counterforts is to support a bank of earth 23 feet high, carrying a double track railway as shown in Fig. 81, (p. 325). The base of the wall will extend 4 feet below the surface of the ground, and the soil is capable of carrying a load of 7000 lb./ft.^{2} The filling is of ordinary earth with natural slope of $1\frac{1}{2}$ to 1, weight of earth is 100 pounds, and of masonry 150 lb./ft.^{3} Maximum allowable stresses are $f_c=650$ lb./in.^{2}, $f_s=16000$ lb./in.^{2} and $v=120$ lb./in.^{2} or without diagonal tension reinforcement, $v=40$ lb./in.^{2}, $n=15$. Design the wall using American Railway Engineering Association formulas.

Solution.—For heavy locomotive loads, the surcharge may be taken as 1000 pounds per square foot of surface, or $h'=10$ feet.

The distance apart of counterforts may vary with different conditions and should be carefully examined in each instance as to its effect upon the cost of the wall. In this problem, assume a distance c. to c. of counterforts of 8 feet. Also try a thickness of counterforts of 18 inches.

Vertical Walls.—Assuming the base to be 2 feet thick, the height of the curtain wall is $23+4-2=25$ feet. Divide the vertical slab

into strips each 1 foot high. The horizontal thrust against the bottom strip is

$$2[0.143W(h+h')]=2[0.143\times100(25+10)]=1000 \text{ lb./ft.}^2$$

This strip then is a horizontal beam supported at intervals of 8 feet, and carrying a uniform load of 1000 pounds per linear foot. Considering it to be a partly continuous beam,

$$M=\frac{wl^2}{10}=\frac{1000\times8\times8\times12}{10}=76800 \text{ in.-lb.}$$

Table XXVIII (p. 205) gives $d=7.75$ in.

Make $d=7.75$ inches, and the total thickness 10 inches. As 10 inches is about the minimum thickness allowable at the top of the wall, make the thickness the same for the whole slab.

For the bottom strip with $d=7.75$ in., $R=\frac{76800}{12\times7.75\times7.75}=110$, $R/f_s=110/16000=.00687$ and from Table XXIV (p. 201), $p=.0076$.

$$A_s=.0076\times12\times7.75=.71 \text{ in.}^2$$

From Table XXVII (p. 204) we find that $\frac{5}{8}$-inch round bars spaced 5 inches apart will answer.

For a strip 16 feet below the top of the wall $M=\frac{744\times8\times8\times12}{10}=$ 57139, $R=\frac{57139}{12\times7.75\times7.75}=79$, $p=.0056$, $A_s=.52$ in.^{2} and the $\frac{5}{8}$-inch bars are needed 7 inches apart.

At 8 feet below the top of the wall $M=\frac{515\times8\times8\times12}{10}=39552$ in.-lb., $A_s=.34$ in.^{2}, and the $\frac{5}{8}$-inch bars may be spaced 10 inches apart.

We will therefore use $\frac{5}{8}$-inch round bars spaced 5 inches apart for the lower 9 feet, 7 inches apart for the next 8 feet, and 10 inches apart in the upper 8 feet of the curtain wall. These bars will be $2\frac{1}{4}$ inches from the face of the wall, and negative moments at the counterforts will be taken care of by short rods of the same diameter and spacing, properly anchored beyond the point of counterflexure.

The span for shear is the clear distance between counterforts. Assuming the counterfort to be 18 inches thick, the maximum shear is $V=(4-0.75)\times1000=3250$ lb., and the unit shear

$$v=\frac{3250}{12\times.874\times7.75}=40 \text{ lb./in.}^2$$

The 7.75-inch thickness is therefore sufficient without reinforcement for diagonal tension.

Resistance to Overturning.—Assume the width of base at about 60 per cent of the total height or 16.5 feet, and place the middle of the vertical wall over a point one-third the width from the toe. Taking a foot of length of wall between counterforts, the

weight of curtain wall,	$150\times 24.75\times 10/12$	$=3095$ lb.
weight of base,	$150\times\ 2.25\times 16.5$	$=\ 5570$ lb.
weight of earth wedge,	$100\times 24.75\times 10.6$	$=26235$ lb.
weight of load upon surface,	1000×10.6	$=10600$ lb.

Using the formulas of the A. R. E. A.

$$P=0.143wh(h+2h')=0.143\times 100\times 27\times 47=18147 \text{ lb.}$$

The distance from the base of the wall to the center of pressure is

$$y=\frac{(h^2+3hh')}{3(h+2h')}=\frac{27\times 27+3\times 27\times 10}{3(27+2\times 10)}=10.9 \text{ ft.}$$

The moment of P about the base of the wall is

$$M_0=18147\times 10.9=197803 \text{ ft.-lb.}$$

and the moment of resistance is

$$M_r=3095\times 5.5+5570\times 8.25+(26235+10600)\times 11.2$$
$$=475525 \text{ ft.-lb.}$$

The distance from the toe of the wall to the point of application of the resultant pressure is

$$a=\frac{475525-197803}{3095+5570+36835}=6.1 \text{ ft.,}$$

and is within the middle third of the base.

Pressure on Soil.—The maximum pressure at the toe of the wall is, as demonstrated in Section 54 (p. 85),

$$f_c=\frac{F}{l^2}(4l-6a)=\frac{3095+5570+36835}{16.5\times 16.5}(4\times 16.5-6\times 6.1)=4910 \text{ lb./ft.,}^2$$

and at the inner edge of the base is

$$f_c''=\frac{F}{l^2}(6a-2l)=\frac{3095+5570+36835}{16.5\times 16.5}(6\times 6.1-2\times 16.5)=600 \text{ lb./ft.}^2$$

Inner Base Slab.—The loading on the horizontal base slab is the difference between the sum of the weights of earth and of the base acting downward, and the soil pressure acting upward. The

maximum load will be at the inner edge, where the upward pressure is a minimum. Taking a foot in width along this edge, the load will be, $1000+24.75\times100+2.25\times150-600=3215$ pounds per linear foot.

The thickness of the base slab will probably be determined by the requirements for shear. The maximum shear at the edge of the counterfort (taking the width of the counterforts as 18 inches) is

$$V=3215(4-0.75)=10450 \text{ lb.}$$

In order to avoid the necessity of reinforcing for diagonal tension, the value of v will be taken at 40 lb./in.^{2}, and

$$d=\frac{V}{bjv}=\frac{10450}{12\times .874\times40}=25 \text{ in.}$$

and the full depth, d_1, will be made 27 inches as assumed.

The bending moment in the base slab is

$$M=\frac{3215\times8\times8\times12}{10}=246910 \text{ in.-lb.}$$

Using the value of $d=25$ in.,

$$R=\frac{246910}{12\times25\times25}=33. \quad \frac{R}{f_s}=\frac{33}{16000}=.0021.$$

Table XXIV (p. 201) gives $p=.0023$, and $A_s=.0023\times12\times25=0.69$ in.^{2}

Table XXVII (p. 204) shows that $\frac{7}{8}$-inch round bars spaced $10\frac{1}{2}$ inches c. to c. are needed.

The negative moments at the counterforts are the same as the positive moments and may be provided for by bending up alternate bars on each side of the support, and extending these across the counterforts to the quarter points in the next panel.

Counterforts.—The counterforts act as cantilevers to carry the horizontal thrust upon the curtain wall for panel lengths of 8 feet. This thrust is

$$P=8[0.143\times100\times24.75(24.75+20)]=126730 \text{ lb.},$$

and it is applied a distance above the top of the base,

$$y=\frac{24.75\times24.75+3\times24.75\times10}{3(24.75+20)}=10.85 \text{ ft.}$$

The bending moment at the top of the base then is

$$M=126730\times10.85\times12=16500000 \text{ in.-lb.}$$

The counterfort will be considered as a rectangular beam, with the center of gravity of the steel $3\frac{1}{2}$ inches from the surface of the con-

crete, and the effective depth measured at right angles to the direction of the longitudinal steel, say, 112 inches. Then

$$R=\frac{16500000}{18\times112\times112}=73. \quad \frac{R}{f_s}=\frac{73}{16000}=.0046.$$

From Table XXIV (p. 201), $p=.0051$ and $A_s=.0051\times18\times112=$ 10.28 in.^{2}

From Table XXVI (p. 203), seven $1\frac{1}{4}$-inch square bars with an area of 10.94 in.^{2} will be selected. These will be arranged in two tiers, four in the outer and three in the inner tier.

For a section 17 feet below the top,

$$P=8[0.143\times100\times17(17+20)]=71960 \text{ lb.},$$

and this is applied at a distance above such section,

$$y=\frac{17\times17+3\times17\times10}{3(17+20)}=7.2 \text{ ft.}$$

The bending moment at the section then is

$$M=71960\times7.2\times12=6217344 \text{ in.-lb.}$$

For this section,

$$d=76.5 \text{ in.},$$

and

$$R=\frac{6217344}{18\times76.5\times76.5}=59. \quad \frac{R}{f_s}=\frac{59}{16000}=.0037.$$

From Table XXIV (p. 201) $p=.0041$ and $A_s=.0041\times18\times76.5=$ 5.64 in.^{2}

The three inside bars may be stopped about 16 feet on the slope from the top, and two of the remaining bars may be stopped about $7\frac{1}{2}$ feet on the slope from the top.

The total shear in the base section of the counterfort is

$$V=126730 \text{ lb.}$$

and

$$v=\frac{126730}{18\times.874\times133}=61 \text{ lb./in.}^2$$

At 11 feet below the top, $v=40$ lb./in.^{2}. Reinforcement for diagonal tension is needed from the base to a section about 11 feet below the top. This may be provided by the bars to be used for bonding the counterforts to the curtain walls and base slabs.

Bonding Bars.—The curtain wall and base slab must be tied to the counterforts by horizontal and vertical bars capable of carrying the reactions at the points of support. These will equal the shears on the two sides of the counterfort. At the bottom of the curtain

wall the load per foot of height is $2(4.00-0.75)1000=6500$ pounds, and the area of steel required is $6500/16000=0.41$ inch.^{2} If these bars are placed in pairs and at the same distance apart as the horizontal bars in the curtain walls, $\frac{3}{8}$-inch round bars will answer. These must be looped around the steel in the face of the curtain wall and extend into the counterfort at least 50 diameters for bond strength.

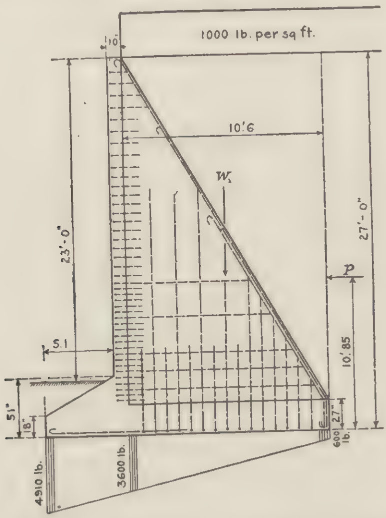


FIG. 81.—Design of Counterforted Wall.

For the base slab, the load upon the bonding bars per foot of width is, 2 (4.00 − 0.75) 3215 = 20897 pounds, and the area of steel required is, $A_s=20897/16000=1.31$ inches.^{2} Pairs of $\frac{7}{8}$-inch round bars spaced $10\frac{1}{2}$ inches on centers meet this requirement. All of the bars should extend upward and be hooked around the tension steel in the counterfort as well as being hooked around the tension steel in the base slab. This will take care of the diagonal tension.

Base Cantilever.—The projection of the base at the toe of the wall is a cantilever, as in the cantilever wall, and carries the upward thrust of the soil. The maximum shear is

$$V=\frac{4910+3600}{2}-\frac{225+637}{2}\times 5.1=19500 \text{ lb.}$$

and Diagram XI (p. 222) gives a depth of 48 inches. The base cantilever will be made 51 inches deep at the face of the curtain wall and will taper to 18 inches deep at the toe of the wall.

The maximum moment is $M=21700\times 2.67-2200\times 2.13=53190$ foot-pounds, or 638280 inch-pounds.

$$R=\frac{638280}{12\times 48\times 48}=24. \quad \frac{R}{f_s}=\frac{24}{16000}=.0015.$$

Table XXIV (p. 201) gives, $p=.0016$, and $A_s=.0016\times 12\times 48=.92$ inch.[2]

From Table XXVII (p. 204) it is found that $\frac{7}{8}$-inch square bars spaced 10 inches on centers may be used.

ART. 38. WHARVES AND SEA WALLS

153. Wharves.—A wharf is a landing place for vessels and their cargoes and may be of timber or masonry construction. When projecting from the shore it is called a pier and when parallel with the shore it is termed a quay. Wharves of timber have been and still are used extensively. They are comparatively economical, but there is a tendency toward the use of reinforced concrete piles for wharves, especially in those localities where the teredo or marine borer has proved a menace to timber. Figure 82 (p. 327) shows several types of reinforced concrete piles.

Quays are built of block-stone or concrete masonry in those situations where stableized shipping will warrant the expenditure. Such walls usually must provide for vessels of considerable draft and must be designed to withstand water pressure on the face, fluid mud pressure on the back, hydrostatic uplift, or any combination of these, as well as impact of loaded vessels in rough weather. The Municipal Quay Wall at Oakland, Cal.,[4] may be cited as one type of such wall.

154. Sea Walls.—Sea walls may be said to be those which are built for the purpose of preventing the encroachment of the sea, or for breaking the force of the waves. Sea walls may be of the heavy rock type like the one at Manhattan Beach, L. I.,[5] or of specially designed concrete masonry like that at Galveston, Tex.[6]

155. Moles, Jetties, Breakwaters and Sills.—A *mole* is similar to a pier in that it projects outward from the shore. Though its primary purpose is to serve as a harbor protection, a mole may also serve as a landing place for vessels. The term is derived from the Latin word meaning mass. A mole is composed largely of heavy stones brought up above the water line, and it is sometimes topped off with block-stone or concrete masonry.

A jetty is a structure built in water to control or divert a current. Jetties may be built entirely of stone or of stone in connection with piles and mattresses of logs or woven brush. The stones used vary

[4] Engineering News, Vol. 69, p. 304.

[5] *Ibid.*, Vol. 71, p. 337.

[6] *Ibid.*, Vol. 49, p. 55 and Vol. 74, p. 424.

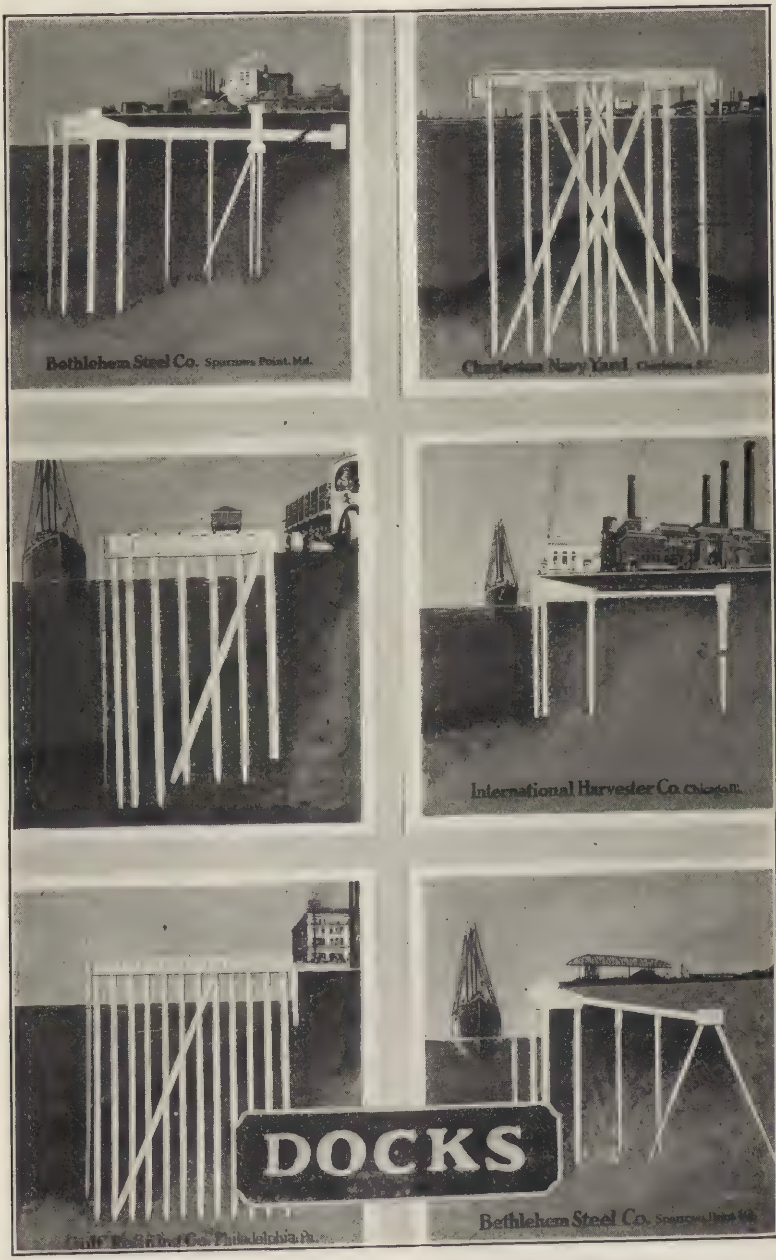


Fig. 82.—Types of Reinforced Concrete Pile Wharves.
(Courtesy of the Raymond Concrete Pile Company.)

in size. Spalls and chips may be handled with a shovel; small rip-rap weighs from 10 to 100 pounds; large rip-rap weighs from 100 to 1500 pounds; and large blocks, more or less regular in shape, weigh from 1 to 10 tons. The jetties at the mouth of the Mississippi river were built converging down stream and then continued parallel for a considerable distance until a depth of water of 40 feet was reached. The velocity of water thus obtained carries the suspended silt far enough out to be taken care of by the littoral current.

A breakwater is constructed for the purpose of supplying a safe harbor for vessels riding at anchor. The United States Government has built many of these on the great lakes, using timber cribs filled with broken stone, sunk below the water line, and topped off with concrete blocks of great weight.

A sill is a mound of rip-rap, the top being below water level. Sills are built for the purpose of creating still water in places where it is desired to promote the deposition of suspended matter.

ART. 39. CONSTRUCTION OF RETAINING WALL

156. Foundations.—As stated in Section 147, the most common cause of failure of retaining walls is defective foundations. Careful attention must always be given to the sufficiency of the foundation, footings being arranged so that excessive pressure does not come upon the soil upon which the structure rests.

On compressible soils it is important to equalize the pressures so that settlement under the toe of the wall may not cause the wall to tip forward. In constructing gravity walls this is accomplished by using a footing under the main wall which extends sufficiently beyond the base of the wall to cause the pressures to be equalized over the foundation soil, and bring the resultant near the middle of the foundation. Reinforced concrete walls must be given sufficient base to prevent excessive pressures on the foundation soil.

The extension of the front base cantilever may often be used as a means of securing good distribution of pressures upon the foundation; when this is not feasible, widening the base at the back of the wall may answer the same purpose.

When the soil is compressible, there is always some settlement, and this is greatest where the load is greatest. In many instances, therefore, it may be advisable to extend the footing sufficiently to bring the center of pressure back of the middle of the foundation so as to make the pressure greater at the heel than at the toe of the wall, and produce a tendency to tilt backward.

When soft materials are encountered, or when the pressures cannot be safely distributed over the foundation soil, a pile foundation or some other means of securing firm support for the wall must be employed. Methods of constructing such foundations, and the loads which may be borne by soils are discussed in Chapter XII.

The depth of foundations should be sufficient to prevent freezing in the soil under the footing of the walls, or of the earth in front of the wall at the depth of the bottom of the footing. This usually requires that the footing extend from 3 to 5 feet below the surface of the ground, depending upon local and climatic conditions.

157. Drainage and Back-Filling.—Failures of retaining walls have frequently occurred because of the lack of proper drainage, hence provision should always be made for the ready escape of water from the earth behind the wall. If the water is held in and the back-filling becomes saturated, the weight of the material is increased and the angle of friction decreased, thus producing a much heavier pressure against the wall. Freezing of wet material behind the wall may also produce dangerous pressures against the back of it.

To provide for drainage, weep-holes are commonly left through the base of the wall at intervals of 10 or 15 feet. In concrete walls, these are usually made by the use of drain tile about 3 inches in diameter. In stone masonry walls, the stones are set so as to leave an opening 2 or 3 inches wide through the course of masonry at the base of the wall.

When the back-filling is of retentive material through which water will not readily pass, a layer of cinders, gravel, or some other porous material should be placed against the back of the wall to permit the water to reach the drains without difficulty. It is always important that water be not held in the back-filling.

The manner of placing the back-filling may sometimes have an important effect upon the pressures against the wall. The layers in which the filling is placed should slope away from the wall. With some materials, there is a tendency for the earth to slide along the surfaces between the layers in compacting and settling into place, which may materially increase the pressure if inclined toward the wall.

158. Gravity Walls.—In constructing gravity walls, certain practices have developed as a result of experience.

It is quite common to make the face of the wall plumb where the retaining wall carries a roadway bridge over a railway cut, thus reducing the span. At other points in such cuts a batter of from $\frac{1}{2}$-inch to 1-inch to the foot is sufficient to compensate for any slight

settlement of the toe and to overcome the optical illusion of the wall leaning forward. A face batter of 2 inches to the foot throws the center of gravity of the wall back slightly and conveys the impression of great stability. It was frequently used in heavy block stone masonry construction.

A width of base of 0.45 of the height will be found ample for the great majority of retaining walls, but a thorough knowledge of local conditions must always be the guide in structures of importance.

A footing with a projection of at least 6 inches at both toe and heel is favored by most designers.

In stone masonry the back face was stepped and early concrete construction followed this practice. The present tendency is toward the use of the plane inclined surface for the back face.

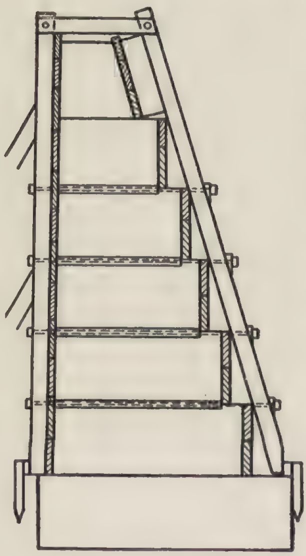


FIG. 83.—Gravity Wall of Concrete.

A coping width of 30 inches is common, though in some cases a lesser width may be admissible. A coping projection of 3 inches is, perhaps, used more frequently than any other.

A heavy frost batter is favored, as much as four horizontal to three vertical, in latitudes where such precaution is necessary.

Figure 83 shows a typical section of a concrete retaining wall with stepped back, such as might be used for carrying a railway embankment.

Concrete is quite largely replacing stone masonry in the construction of retaining walls. For high walls, reinforced concrete is economical and usually employed, while for walls less than 20 or 25 feet high, gravity walls may often be less expensive than reinforced walls. A larger quantity of concrete is required for the gravity wall, but concrete of less rich character may be employed and no steel is needed. For reinforced walls, about 1 to 2 to 4 concrete is usually used for the body of the work, while 1 to 3 to 6 concrete may commonly be used for gravity walls. The cost of forms does not vary greatly for the two types of wall.

CHAPTER VIII

MASONRY DAMS

ART. 40. GRAVITY DAMS

159. Stability of Dams.—A gravity dam, like a retaining wall, depends upon the weight of the mass of masonry to resist the thrust of the water against it. As the dam carries water pressure instead of earth pressure, the loads to which the dam is subjected are definitely known, and the thrusts are everywhere normal to the surfaces of contact.

Let $ABCD$, Fig. 84, represent a slice, 1 foot thick, of a gravity dam sustaining a head of water as shown.

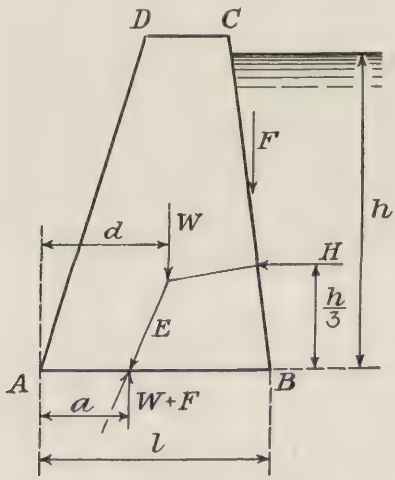


FIG. 84.—Gravity Dam.

h = height of water above section AB;
H = horizontal pressure of water against the dam;
F = vertical pressure of water on back of dam;
W = weight of dam above section AB;
E = resultant pressure upon section AB;
k = horizontal distance from inner edge of base to line of action of F;

l = width of base AB;
d = distance from outer edge of base to line of action of W;
a = distance from outer edge of base to point of application of resultant E.

The conditions of stability for the dam are the same as for the retaining wall:

It must not slide or shear on a horizontal section.

It must not overturn about outer edge of section.

The masonry must not be crushed by pressure upon the section.

Stability against Sliding.—Taking the weight of water as 62.5 lb./ft.,[3] the horizontal thrust against the dam above AB is $H = 31.25h^2$. This is the shear upon the section AB. If AB is a joint in the dam, or the base of the dam, H must be resisted by the friction of the masonry upon the masonry below, or upon the foundation under the dam, and the value of $H/(W+F)$ must not exceed the coefficient of friction for the material. If AB is a section in a concrete dam, H is resisted by the shearing strength of the concrete as well as by the friction.

Continuous joints are not usually employed in construction of masonry dams, and the interlocking of stones eliminates the tendency to slide without shearing blocks of stone. The possibility of sliding need usually only be considered at the foundation.

Stability against Overturning.—The overturning moment about the outer edge of the section at A, due to pressure of water, is

$$M_0 = H \times \frac{h}{3} - F(l-k).$$

The resisting moment of the weight of wall is $M_r = Wd$, and the distance from the outer edge A to the point of application of the resultant pressure on the base is

$$a = \frac{M_r - M_0}{W+F} = \frac{Wd - Hh/3 + F(l-k)}{W+F}. \qquad (1)$$

If the water face of the dam is vertical, $F = O$ and

$$a = \frac{Wd - 10.42h^3}{W}. \qquad (2)$$

Assuming that pressures upon AB are distributed with uniform variation from A to B, a should be greater than $l/3$ in order that

no tension may be developed in the section, as in the gravity retaining wall.

Stability against Crushing.—The total pressure normal to the section AB is $W+F$, distributed over the section with center of pressure distant a from A. The maximum normal unit pressure is therefore (see Section 54, p. 85)

$$f_c=\frac{(W+F)(4l-6a)}{l^2}. \qquad (3)$$

This is approximately the crushing stress in the masonry at the outer edge of the section, or the maximum pressure upon the foundation if AB is the base of the dam.

When the reservoir is empty and the water pressure is removed, the pressure upon the section AB will be W, with center of pressure distant d from the outer edge. The unit pressure at the outer edge of the section will be

$$f_c=\frac{W(4l-6d)}{l^2},$$

and at the inner edge,

$$f_c''=\frac{W(6d-2l)}{l^2}. \qquad (4)$$

In dams of unsymmetrical cross-sections it is necessary to consider the pressures coming upon the bases of sections when the water pressures are removed, as when the reservoir is empty. In this case, the weight of dam will be the only load, and the centers of pressure due to this weight must always come within the middle third of the base, and the crushing stress be within proper limits, so that removal of the water pressure may produce no harmful effects upon the dam.

160. Graphical Method of Determining the Profiles of Dams.—For low dams carrying low heads of water, trapezoidal cross-sections may be used, and designs made in a manner similar to that employed for retaining walls, using water pressure instead of earth pressure upon the back face of the dam. As the depth increases such a section becomes more and more uneconomical and the form of cross-section should be modified so as to make the thickness only that required to carry the load above, and the profile such as to distribute the material to the best advantage.

Figure 85 shows the method of determining the line of action of the resultant pressure for successive sections by the graphical method.

The section o'–o, k–k' is that of a dam 100 feet high with a top width of 10 feet. A slice of the dam 1 foot thick is assumed and the section is divided by horizontal planes a'–a, b'–b, etc., into layers 10 feet thick. The weight of water is taken at 62.5 lb./ft.^{3} and that of masonry at 150 lb./ft.^{3} The horizontal distances from some datum plane such as o–k'' to the various centers of gravity are perhaps

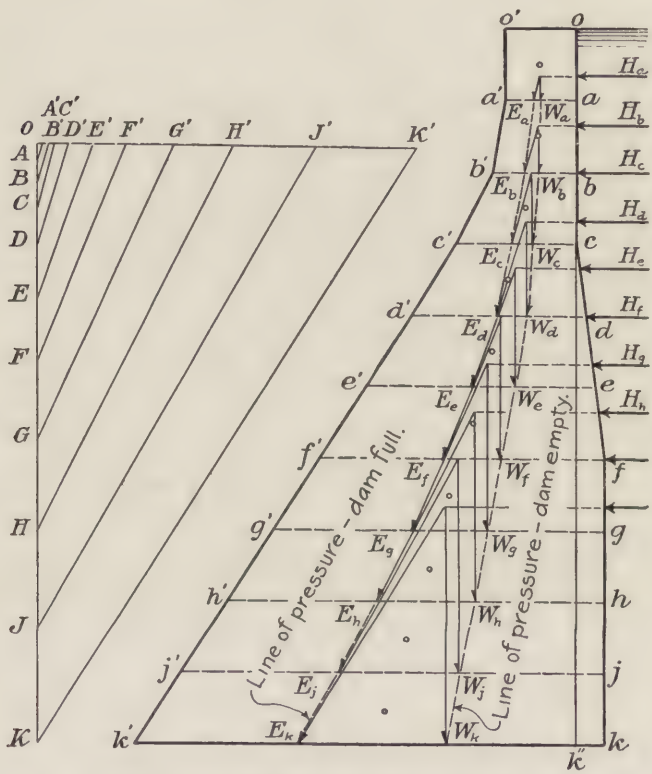


FIG. 85.—Graphical Analysis of Gravity Dam.

best found by moments, though one of several graphical methods may be employed, if preferred.

Starting at the top the widths of the various horizontal planes are determined by trial, and they must be such that the resultant, E, of the weight of masonry above any plane and the water pressure affecting that plane shall fall within the middle third.

The water pressures for the various planes are found by using the formula $H = \frac{1}{2} wh^2$. For example, the water pressure for the section o'–o, a–a' is $H_a = \frac{1}{2} \times 62.5 \times 10 \times 10 = 3125$ pounds, and for the section

o'–o, b–b' is $H_b = \frac{1}{2} \times 62.5 \times 20 \times 20 = 12{,}500$ pounds. These pressures are applied at a distance of $h/3$ above the plane under consideration.

Compute the water pressures for all sections and lay off the values to a suitable scale at convenient place on the sheet, as, O–A', O–B', O–C', etc.

The width a'-a is 10 feet, the same as the top, and the weight of the masonry is $10 \times 10 \times 150 = 15{,}000$ pounds. The distance of the center of gravity of this section is 5 feet from the datum plane o-k.″ From O lay off vertically downward the weight of the masonry $OA = 15{,}000$ pounds, to the same scale used for water pressure, and draw A-A'. Prolong H_a until it intersects a vertical through the center of gravity of o'-o, a-a', and through this point of intersection draw a line parallel to A-A'. This is the resultant, E, for the top section and it will cut the plane a'-a at a distance of 4.3 feet from the toe, or well within the middle third.

Assume the width b'-b as 12 feet. The water pressure is 12,500 pounds applied 6.66 feet above b'-b. The weight of the masonry o'-o, b-b' is $210 \times 150 = 31{,}500$ pounds, and its center of gravity is 5.27 feet from the datum plane o-k''. Lay off O-B equal to 31,500 pounds and draw B-B'. Through the point of intersection of H_b produced and a vertical through the center of gravity draw a line parallel to B-B'. This resultant intersects the plane b'-b at a distance of 4.08 feet from the toe, or just within the middle third.

In a similar manner, determine the widths, c'-c, d'-d, etc.

It must be kept in mind that the center of pressure must be within the middle third when the dam is empty as well as when it is full. It will be found that below the plane c'-c, it will be necessary to extend the base plane toward the water face as well as away from it.

161. Design of Profile.—In designing a profile for a dam, commence at the top with the assumed thickness and find by trial the required base thickness for each horizontal layer, making each thickness such that the line of pressure remains everywhere within the middle third of the section. This may be done by the use of the formulas given in Section 159 or by the graphical method of Section 160.

The crushing stress upon the masonry must also be kept within safe limits.

Let l = the width of the section;
a = the distance from the outer edge to the point of application of E;
y = the distance from the inner edge to the point of application of W.

The maximum crushing stress at the outer edge of the section is given by the formula,

$$f_c = \frac{W(4l-6a)}{l^2}. \qquad (5)$$

The maximum crushing stress at the inner edge is

$$f_c'' = \frac{W(l-6y)}{l^2}. \qquad (6)$$

The allowable crushing stress depends upon the quality of masonry used, and the conditions under which the dam is to be constructed. In high dams, where the front face of the dam has considerable batter, the pressure allowed at the outer face is often made less than that at the inner face. The maximum pressure at the inner edge of a section occurs when the dam carries no water pressure, and the resultant pressure on the base is vertical. The maximum pressure at the outer edge occurs when full water pressure is on the dam, the batter of the outer face is greater than that of the inner face, and the resultant pressure is inclined, only its vertical component being considered in determining the stress. For these reasons Rankine's recommendation that the allowable unit crushing stress at the outer edge be made less than that at the inner edge has been followed by some designers. Pressures of from 8 to 15 tons per square foot have been allowed in a number of large dams of massive rubble or cyclopean concrete.

The profile resulting from this method of design is somewhat irregular and may be modified by fitting it with more uniform batters and smooth curves, thus giving a more pleasing appearance and better profiles for construction purposes, without appreciably affecting its stability.

Vertical Water Pressure.—As the water face of the dam is nearly vertical, it is usual to disregard the vertical component of the water pressure, which is of small consequence in dams of less than about 180 to 200 feet in height. This component has the effect of diminishing the stress upon the outer edge of the section while somewhat increasing the total pressure. Its neglect is therefore a small error on the safe side until a depth is reached at which the slope of the inner face may make it of more importance.

The shape of the profile depends upon the top width given to the dam, and the weight of the masonry used.

The top width must be sufficient to resist any probable wave action and ice pressure, and should usually be made greater for high dams than for low ones. This is a matter of judgment in each case, about one-tenth of the height of dam being frequently used, with a minimum of about 5 feet and a maximum of 20 feet where no roadway is carried on top of the dam.

The dam should always extend to a sufficient height above the normal water surface to prevent water passing over the dam due to waves of floods for which wasteways might not be quite sufficient. This may require the dam to be raised 5 or 10 feet above the elevation of the expected water surface. In designing the dam, water should be assumed level with the top.

The weight of masonry used in dam construction commonly varies from about 135 to 150 pounds per cubic foot. The heavier the masonry is assumed to be, the less the required width of section until a depth is reached at which the width is determined by the necessity of providing sufficient area to carry the weight of masonry above. Below this point, usually about 200 feet below the water surface, the width required is greater for the heavier masonry if the same unit compression be allowed.

Uplift and Ice Pressure.—If water under hydrostatic pressure has access to the interior of the dam, the upward pressure will tend to lift the masonry and diminish its effective weight in the moment which prevents overturning. In this discussion it has been assumed that the dam is constructed water-tight, but as this is not altogether possible, in many instances it may be necessary to allow for upward pressure in designing the profile, or make special provision for drainage—a topic discussed in Section 164.

If ice forms on the surface when the reservoir is full, a considerable pressure may be brought against the top of the dam, which should be considered in its design. This will be a concentrated horizontal thrust at the surface of the water equal to the crushing strength of the ice, and has been assumed in a number of important dams at from 2500 to 4500 pounds per linear foot of dam. In storage reservoirs, generally, heavy ice is not likely to occur with full reservoir, and if water be low when freezing occurs no special allowance for ice pressure is necessary. Local conditions must determine the necessity of allowing for ice pressure in each instance.

162. Diagonal Compressions.—The common method of analysis, already described, considers only the stresses upon horizontal sections and resolves the diagonal thrusts into normal compressions and parallel shears upon these sections. This method does not give the

actual maximum compressions, but by using proper unit stresses has seemed to give satisfactory results in use. Several methods have been proposed for computing more accurately the maximum unit compressions.

Diagonal Compression upon Horizontal Section.—In 1874 Bouvier [1] used the actual diagonal pressure (E, Fig. 86) in computing the maximum unit compression upon a horizontal section, claiming that the unit compressive stresses produced by E parallel to its line of action are greater than those normal to the section. He considered E to be distributed along $A-B$, so as to act upon successive small sections normal to its direction as shown in Fig. 86. If β is the angle made by E with the normal to section $A-B$, and l is the width of section, the area upon which E acts is $AC=l\cdot\cos\cdot\beta$, and the maximum intensity of the compressive stresses is

$$f_{cd}=\frac{E(4l-6a)}{l^2\cos\cdot\beta}=\frac{W(4l-6a)}{l^2\cos^2\beta}=\frac{f_c}{\cos^2\beta}, \quad . \quad . \quad . \quad . \quad (7)$$

in which f_{cd} is the unit compression at tne outer edge of the section parallel to E, and f_c is that normal to the section at the same point.

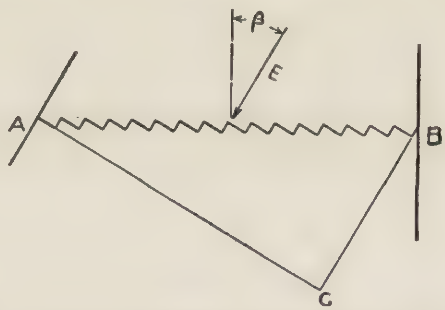


FIG. 86.—Diagonal Compression upon Horizontal Section.

Professor Unwin [2] has shown that the maximum unit compression at the face of the dam occurs on a section normal to the face, and that the maximum value of this compression at the outer edge of a horizontal section through the dam is $f_{cm}=\frac{f_c}{\cos^2\theta}$, in which f_c is the value of unit vertical compression and θ is the angle made by the batter of the face of the dam with the vertical.

[1] Annales des Ponts et Chaussées, 1875.

[2] Proceedings, Institution of Civil Engineers, Vol. CLXXII, Part II.

A method of finding the maximum diagonal compression and its direction at any point of a horizontal section of the profile of a dam is given by Professor Cain,[3] which agrees practically with Unwin's results for the stress at the edge.

Compression upon Inclined Sections.—The distribution of pressure upon an inclined section is sometimes investigated and the maximum unit stress at the outer face of the dam found to be greater than that for a horizontal section. In Fig. 87 using the same profile employed in Fig. 85, the pressure upon the inclined section k–n is that due to the water pressure (H) upon the inner face O–K of the dam combined with the weight of masonry (W) above the section k–n. The unit compression at n is obtained in the same manner as for the horizontal section. The stress for this profile is greater than for the same point when obtained by using the horizontal section through n, and about the same as that at the outer edge of the base section k–k.

Lateral Distribution of Stress.—In the trapezoidal distribution of stress, which considers the stresses to vary uniformly from the inner to the outer edge of the section, it is assumed that the whole width of the dam acts together as a single homogeneous body. It is not probable that this is the case in a wide section. The middle portion of the section carries more and the edges less stress than the assumed distribution shows, and for this reason many designers have considered that the ordinary method, with low allowable stress upon the outer edge, as proposed by Rankine, to be sufficiently exact. The ordinary method, taking successive horizontal sections, provides an easy way of determining an approximate profile. Careful study should, however, be given to the possible diagonal stresses in a high dam, and if such stresses exceed the allowable unit compression, the profile should be widened so as sufficiently to reduce them.

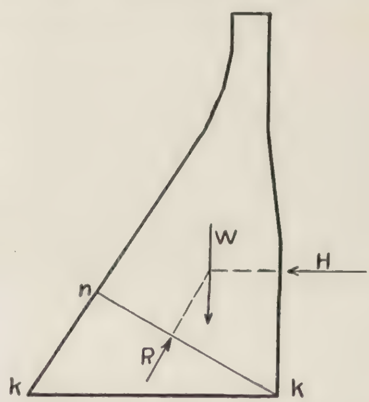


Fig. 87.—Pressure upon Inclined Section.

163. Horizontal Tension.—Experiments have been made by Sir J. W. Ottley and Mr. A. W. Brightmore [4] upon models of dams made

[3] Transactions, Am. Soc. C. E., September, 1909.

[4] Proceedings, Institution of Civil Engineers, Vol. CLXXII, p. 89.

of plasticine (a kind of modeling clay), and by Messrs. J. W. Wilson and W. Gore [5] on models made of india rubber.

The distribution of stresses through the profile was determined in each case by observing the horizontal and vertical displacement of points in the section. These experiments seemed to confirm, in general, the ordinary theory of the trapezoidal distribution of stresses, and to justify the methods of design in common use.

At the base of the dam, where the profile section joins the foundation, it was found that a different distribution of stress occurs, tension being developed at the inner edge of the base by the immovability of the foundation. Thus, in Fig. 88 the shear on A–B, due to the horizontal water pressure, causes horizontal or diagonal tension (T) in the foundation at the inner edge (A) of the base. In the plasticine models diagonal cracks (A–C) occurred at this point in the foundation.

Various methods have been suggested for meeting or reducing this tension by modifying the shape of the profile at the base or reinforcing the foundation. This does not seem necessary for dams as usually constructed. A high masonry dam is usually on solid rock foundation, and the strength of the rock is such that no break in the foundation is to be anticipated from this cause. In most dams the foundation is in rock at considerable depth below the bed of the stream, and the lower part of the dam is enclosed on both sides by gravel or other soil which usually may be considered to strengthen the dam, although the full depth of water pressure should be assumed to act upon it. If, however, this filling is soft material, which flows when saturated, it may increase the pressure against the dam and may be considered as a fluid heavier than water.

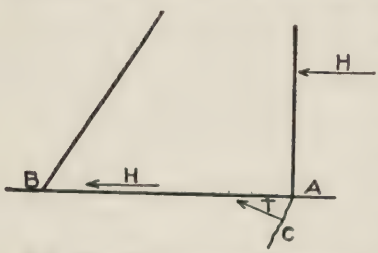


FIG. 88.—Horizontal or Diagonal Tension in Foundation.

164. Uplift.—If a dam be so constructed that water under pressure may penetrate into the interior of the dam or under its base, the effect of such pressure must be considered in its design. There is considerable difference of opinion among engineers concerning the necessity for providing for uplift in designing the profiles for dams. Some allow for it in all cases; while others claim that properly con-

[5] Proceedings, Institution of Civil Engineers, Vol. CLXXII, p. 107.

structed masonry or concrete will be so nearly water-tight that the effect of uplift may be neglected.

Interior Pressure.—It is always possible that some water may be forced into imperfect joints in the masonry and, if it be prevented from escaping at the lower side of the dam, have the full hydrostatic pressure of the head in the reservoir. For this reason it is important that the water face of the dam be made as nearly impervious as possible, and that the interior of the dam be drained so that any water passing into the masonry may escape without damage. It is evident that uplift of the interior of the masonry can exist only where continuous joints for considerable distances are filled with water under pressure. If concrete be porous and its voids filled with water under hydrostatic pressure, no uplift occurs until the pressure becomes sufficient to overcome the cohesive strength of the concrete. In properly constructed masonry dams, it is usually unnecessary to consider the effect of uplift on sections above the base of the dam.

Upward Pressure on Base.—The probability of uplift under the base of a dam depends upon the character of the foundation. Careful attention should always be given to the determination of the character of the foundation material to considerable depths below the base of the dam. The kind of material of which the foundation is composed, and the existence of seams in the rock, or of strata of permeable material must be accurately investigated.

When the foundation is of solid rock without seams, if care be used in joining the base to the foundation and cut-off wall be used under the inner edge of the base, there is little chance of appreciable uplift under the base.

When the foundation is permeable and there is water against both sides, as is frequently the case in dock walls the full hydrostatic head is usually considered to act under the whole base. This is somewhat excessive, as it implies that the dam is floating upon a continuous surface of water. Probably two-thirds of this pressure would represent about the maximum which could be reasonably be expected in any case.

When the foundation is stratified horizontally, so that water may be expected to pass under the dam and escape below, a uniformly diminishing upward pressure from the inner to the outer edge of the base may be assumed; the pressure at the inner edge being taken at about two-thirds the hydrostatic head above the dam, and that at the lower edge at zero.

The probability of upward pressure on the foundation should always be carefully investigated, and the section, where necessary,

increased sufficiently to provide weight of masonry to overcome the overturning moment of this water pressure. This subject is very fully treated in the discussion of a paper by the late C. L. Harrison in the Transactions of the American Society of Civil Engineers for December, 1912. Mr. Harrison's conclusions are:

1. For any stable dam, the uplift in the foundation cannot act

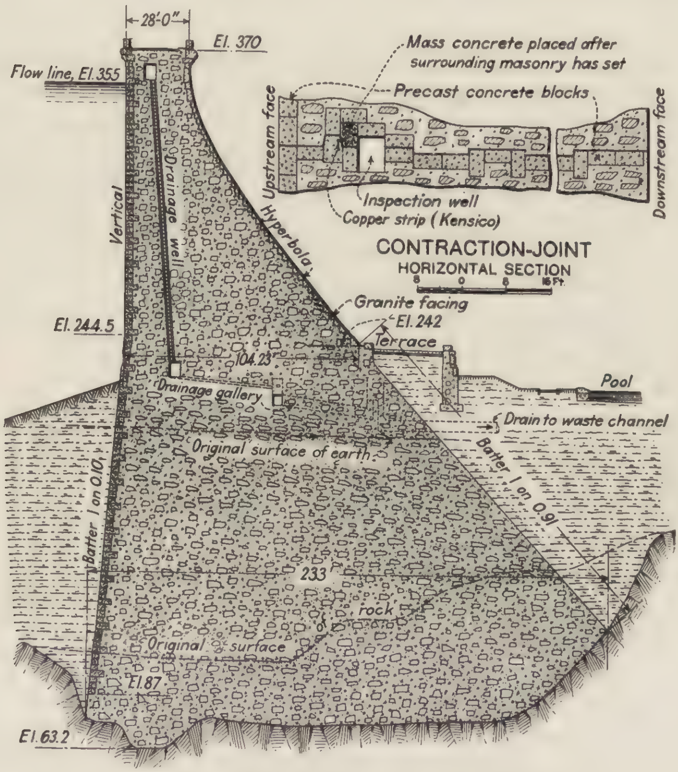


FIG. 89.—Kensico Dam—Maximum Section.
(Courtesy of the New York City Board of Water Supply.)

over the entire area of any horizontal seam, and in the masonry it cannot act over the entire area of any horizontal joint.

2. The intensity of uplift at the heel of the dam can never be more, and is generally less, than that due to the static head. Also, this uplift decreases in intensity from the heel to the toe of the dam, where it will be zero if the water escapes freely, and will be that due to the static head if the water is trapped.

3. The uplift in the foundation should be minimized by a cut-off

wall, under-drainage, and grouting when applicable; and in the dam itself by using good materials and workmanship, and by drainage when advisable.

4. The design should be based on the conditions found to exist at each site after a thorough investigation by borings, test-pits, and otherwise, and modified if found necessary after bed-rock is uncovered.

165. Kensico Dam.[6]—Figure 89 shows the maximum section of the Kensico Dam, which was begun in 1910 and finished in 1916. The crest length is 1843 feet; the top width, 28 feet; the maximum bottom width, 235 feet; and the maximum height, 307 feet. The dam contains 900,000 cubic yards of masonry, the interior being cyclopean concrete; the upstream face, precast concrete blocks; and the exposed parts of the downstream face, cutstone masonry. The contractors' plant was exceptionally complete, and cost about $1,000,000. Figure 90 is a photographic view of the finished structure.

The dam was built under the direction of the Board of Water Supply of New York City. Mr. J. Waldo Smith was Chief Engineer, and Mr. John R. Freeman, Professor William H. Burr, Mr. Frederick P. Stearns, and Mr. Alfred Noble, were Consulting Engineers.

ART. 41. DAMS CURVED IN PLAN

166. Curved Gravity Dams.—In constructing dams across narrow valleys, it is often desirable to curve the dam in plan, so as to make it form a horizontal arch, convex upstream. When so arranged, a portion of the water pressure may be transmitted to the sides of the valley by arch action, thus diminishing the overturning moment which would exist in a straight dam of the same section.

In certain locations, the shape of the valley and depth of suitable foundations make the use of the curved form economical in saving materials, although the length of dam is increased by the curvature. The curved form for gravity dams has not usually been adopted for the purpose of securing the arch action, although the advantage of the curved form is recognized and the added security obtained by the possibility of the upper part of the dam acting as an arch is worth considering when it does not materially increase the cost.

In order to develop free arch action in any horizontal slice of a dam, it would be necessary that the section be free to move horizontally when the pressure comes against it. As each section is rigidly

[6] Edward Wegmann, Design and Construction of Dams, Seventh Edition, 1922, p. 435.

FIG. 90.—Kensico Dam.
(Courtesy of the Board of Water Supply, City of New York.)

connected with those above and below it and the base is attached to a practically immovable foundation, the arch action is very imperfect. Near the top of a gravity section, deflection of the section may be sufficient to permit a portion of the water pressure to be resisted by the arch, but in the lower half of the dam such resistance is inappreciable.

There is no satisfactory way of determining how much of the pressure is borne by the arch in a curved gravity dam. In an analysis of the stresses in the Cheeseman dam, Mr. Silas H. Woodward estimated [7] roughly the amount of water pressure carried by the arch action, by determining the deflection at various points in the mid-section of the dam, considering the resistance of horizontal slices of the dam by arch action, and the resistance of a vertical slice as a cantilever beam, fixed at the bottom to the foundation. He concluded that in the Cheeseman dam, the arch carried about half the water pressure at the top and about 6 per cent at the mid-height of the middle section.

Mr. Woodward's analysis seemed to indicate that, while added security might be obtained through arch action at the top of the dam, the lines of pressure of the gravity section were only slightly modified by considering part of the load carried by the arch. His conclusion was that no diminution of the gravity section would be justified because of dependence upon arch action.

The use of curved plans for gravity dams may be of advantage in affording a possibility of motion when expansion and contraction take place, without cracking the masonry. The advantages to be gained by using curved plans, however, do not seem sufficient to make them worth while when they involve increase in cost. In constructing gravity dams across narrow valleys where arch action might be developed, the sides of the valley may also offer considerable support to a straight dam, causing horizontal slices of the dam to act as beams supported at the ends. In any such dam the actual stresses are probably considerably less than those obtained by considering the gravity resistance only.

167. Arch Dams.—Dams are sometimes constructed which depend for stability mainly upon arch action, and are designed as horizontal arches. A number of dams of this type have been constructed across narrow valleys, with sections much lighter than could be used for gravity dams. In some, the lines of pressure fall quite outside the bases when considered as gravity sections.

[7] Transactions, Am. Soc. C. E., Vol. LIII, p. 108.

Let A–B, Fig. 91 represent a horizontal slice, 1 foot thick, through a circular dam.

R = radius of water section;
t = thickness of section;
P = water pressure per foot of length;
f_c = unit compression on the masonry;
h = height of water surface above section;
w = weight of water per cubic foot.

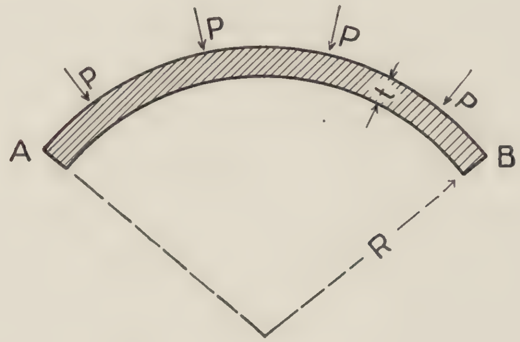


Fig. 91.—Arch Dam.

If the slice be supposed to act freely as an arch and carry the water pressure to the abutments,

$$f_c = \frac{PR}{t} = \frac{whR}{t}. \qquad (8)$$

If a limiting value of f_c be assumed, the thickness of section required at any depth will be

$$t = \frac{PR}{f_c} = \frac{whR}{f_c}.$$

For a dam of constant radius (R) the required thickness varies uniformly with h, or the vertical section of the dam is triangular.

As the ends of the arch at A and B are built into the sides of the valley and not free to move toward the center O when subjected to the water pressure, the lines of thrust of the arch will not be exactly axial as assumed in Formula (8), and bending stresses will develop in the arch, giving a maximum compression somewhat greater than the average value. This effect will usually be small as compared with

the stress due to arch action, although French authorities recommend that the line of thrust be assumed at the outer edge of the middle third at the crown, thus making the maximum compression double the average. The use of vertical expansion joints through the dam, dividing it into voussoirs, has the effect of largely eliminating the bending stresses. In practice the bending stress is commonly neglected, very conservative values for f_c being used.

When the length of the arch is small as compared with its thickness, it becomes a curved wedge which acts as a beam between the abutments supporting its ends, and should be considered as a curved beam—a condition frequently occurring near the bottom of a curved dam, where the valley is narrow and the thickness of the dam considerable. The thickness obtained by considering such a section as an arch is always sufficient.

A masonry structure cannot be considered to act as an arch when the thickness of the arch ring is more than from one-quarter to one-third of the radius of its outer surface. The exact limitations within which such action may take place are not definitely known and are seldom of importance in a dam.

Resistance of Vertical Cantilever.—As a dam is rigidly fastened to the foundation, it is evident that complete arch action cannot take place, and that in the lower part of the dam, the arch can carry very little of the load. A vertical section of the dam may be considered as a cantilever fixed at the bottom as in a gravity dam, and the resistance of the cantilever to deflection will limit the extent to which arch action may occur.

Attempts have been made by estimating the relative deflections of the horizontal arch and the vertical cantilever at various heights upon the mid-section of the dam, to determine what portion of the load is resisted by each. Such studies have been made by Mr. Silas H. Woodward [8] for the Lake Cheeseman dam, which is a curved dam of gravity section (see Section 166) and by Mr. Edgar T. Wheeler [9] for the Pathfinder dam, which was designed as an arch, and has a section considerably lighter than could have been employed in a gravity dam. The section of the dam has a width of 10 feet at the top, a batter of .25 on the downstream and .15 on the upstream face.

These analyses, with accompanying discussions, are interesting as throwing light upon the probable action of such dams when subjected to water pressure, but afford no means of determining the actual stresses occurring. The vertical cantilever has the effect of

[8] Transactions, Am. Soc. C. E., Vol. LIII, p. 89.

[9] Engineering News, August 10, 1905.

reducing the stresses in the arches, but it is not proposed to consider the combined actions in designing dams, or to attempt to use the actual stresses, as limited by the cantilever resistance in proportioning the arches. In practice, the arches are given sections which would enable them to carry the whole water pressure, and the vertical resistance is considered as a source of additional security.

Horizontal Shear.—As the dam is fixed at the bottom to the foundation and the various horizontal slices are not free to act independently of each other, the thickness at any point should be sufficient to carry the total water pressure above as horizontal shear. If S be the safe unit shear per square foot, the thickness should not be less than $t=\frac{wh^2}{2S}$. Such shearing stresses can exist only near the bottom of the dam, where it is rigidly attached to the foundation, and can never reach the assumed value if the water pressures toward the top of the dam are carried by arch action.

Weight of Masonry.—Each horizontal slice of an arch dam must carry the weight of the portion of the dam above as a vertical compression. This compression is computed as in the gravity section when the dam is empty, and must not exceed a safe unit stress on any part of the section. The weight of masonry above also produces a distortion of the horizontal section. The value of Poisson's ratio for concrete may be taken as approximately one-fifth of the unit horizontal compression produced through the mass of masonry, if prevented from expanding laterally is approximately one-fifth of the unit vertical compression which causes it. The effect of this horizontal compression is to cause an expansion of the horizontal section, increasing the length of the arch ring, and deflecting the crown of the arch upstream. When water pressure is brought against the dam, a portion of the pressure, sufficient to produce compression in the arch equal to the unit horizontal pressure due to the vertical load, will be used to bring the arch back to its initial position, and no deflection due to arch action will occur until this pressure has been passed.

When the crown of the arch has been deflected upstream by the weight of masonry, stress is brought upon the vertical cantilever by its resistance to bending in that direction. If water pressure be now brought against the dam, the vertical cantilever action will offer no resistance to downstream motion until the pressure upon the arches becomes sufficient to bring the dam back to its original unloaded position.

The existence of this initial distortion due to the weight of masonry

may depend upon the manner in which the dam is constructed. In order to produce this effect it is necessary that the horizontal layers be completed and hardened in position before the load above is applied. If portions of the work be carried up in vertical sections, or if vertical contraction joints be left, to be afterward grouted, the deflection due to weight of masonry may take place only to a very limited extent.

Constant-angle Arches.—Arch dams are usually constructed across narrow gorges which can readily be spanned by an arch of moderate radius. The gorges vary in cross-section, being usually much narrower at bottom than near the top of the arch. The arch at bottom will therefore be much shorter than at the top and if the same radius be used at top and bottom, or the centers lie in the same vertical line, the central angle included by the dam will be greater at top than at bottom. It has been shown by Mr. Lars R. Jorgensen [10] that a dam with a constant central angle of 133° 34′ requires theoretically, the minimum amount of masonry in its construction, and that angles from 110° to 150° vary but little from the minimum. It has therefore been proposed to vary the radius from the top to the bottom, so as to keep within these ranges of central angles. This makes the radius of the dam vary with the width of the gorge at different elevations. Several dams have been constructed in which this principle has been approximately applied. The topography of the site must be carefully studied in every instance and the dam fitted to its location, keeping in mind the general principles involved.

Temperature Stresses.—Comparatively little is known concerning the changes of temperature to be expected in a mass of masonry like a dam, but it is evident that distortions produced by such changes may sometimes be of importance, and careful attention should be given to their probable effect. Temperature above the normal at which the masonry was placed cause deflection upstream through expansion, which may bring bending stresses upon the vertical section when the water is low behind the dam. Contractions due to temperatures below the normal, causing tensile stresses which the masonry or concrete is not calculated to bear may cause cracks, or prevent the arch action through shortening the arch near the top. It is desirable that masonry which may be injuriously affected by low temperature, be placed when the temperature is low, thus giving a low normal and probable small range below. Mr. Wisner [11] urges that reinforcement be used on the faces of the upper portion of arched dams to prevent

[10] Transactions, Am. Soc. C. E., Vol. LXXVIII, p. 685.

[11] Engineering News, Aug. 10, 1905.

cracks; vertical rods on the downstream face to take up the possible vertical tensions due to expansion, and horizontal rods on the upstream side to prevent contraction cracks.

168. Roosevelt Dam.[12]—Figure 92 shows the cross-section of the Roosevelt Dam of the Salt River Irrigation Project in Arizona.

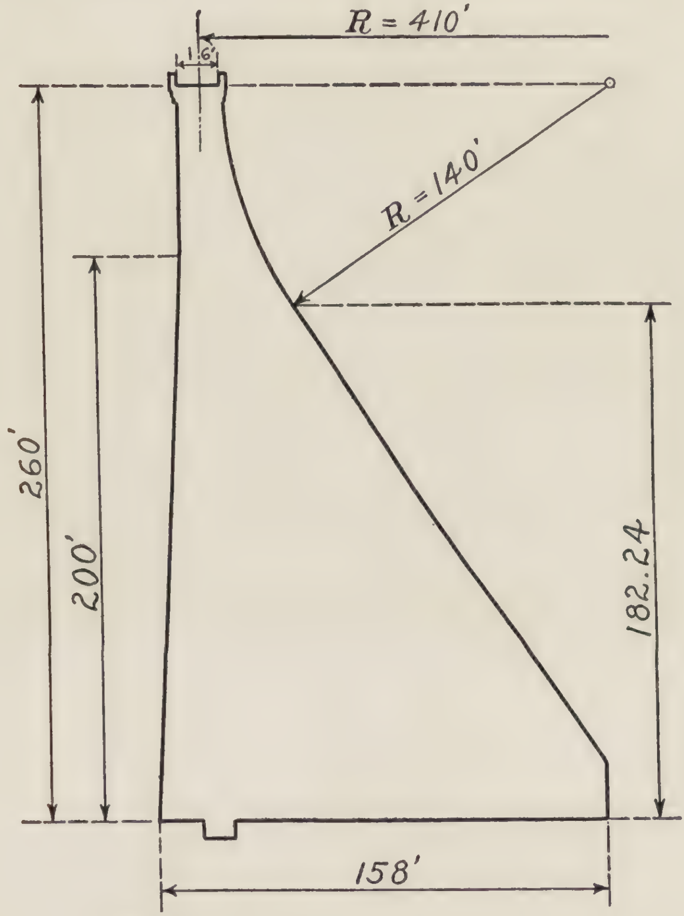


FIG. 92.—Roosevelt Dam—Cross-Section.

This dam impounds 420,000,000,000 gallons of water. The contract for the structure was signed April 8, 1905, and the dam was formally opened by President Roosevelt, March 18, 1911. The length of the

[12] Edward Wegmann, Design and Construction of Dams, Seventh Edition, 1922, p. 423.

Fig. 93.—Roosevelt Dam.
(Courtesy of the United States Department of the Interior—Bureau of Reclamation.)

crest is 1080 feet; width of roadway across the top, 16 feet; width at base, about 170 feet; and maximum height, 284 feet. The plan of the dam is curved to a radius of about 400 feet. The structure contains about 340,000 cubic yards of masonry. The United States Government constructed a cement plant at the site and furnished all the cement used in the construction, at a saving of $400,000. Figure 93 is a photographic view of the completed work.

The dam and reservoir were under the direction of Mr. F. H. Newell until March, 1907, when he was appointed Director of the Reclamation Service. He was succeeded as Chief Engineer by Mr. A. P. Davis. Mr. Loomis C. Hill was Supervising Engineer, and Mr. Chester W. Smith was Resident Engineer.

169. Experimental Arch Dam.[13]—Valuable information is expected from the results of tests which are to be made on the experimental dam which is being constructed on Stevenson Creek, sixty miles east of Fresno, Cal. This work is being carried on under the direction of the Committee on Arch Dam Investigation of the Engineering Foundation with a fund of $100,000 at its disposal. The upstream curvature of the dam has a constant radius of 100 feet. The final height of the dam will be 100 feet. It will be of the extreme thin arch type with a thickness of 2 feet from the crest to a level 30 feet above the base, from which level there will be a gradual curved batter on the down-stream face to the base which will be $7\frac{1}{2}$ feet thick. The construction will be carried on until a height of 60 feet is reached, after which a complete series of tests will be made. The work will then be continued to the crest and the final series of measurements made.

The general purpose of the test is to obtain "precise information, on working scale, concerning the stresses, movements, and changes in volume of thin arch dams, the theory of which is not in completely satisfactory condition."

To quote from the article referred to in the Engineering News-Record: "The experiment is expected to supply partial answers to some or all of the following points of question: (1) As to stresses: Division of the water pressure between the different resisting elements; arching in inclined planes, and secondary arching in interior of thick arches; horizontal stress variation between crown and abutment of arch elements; applicability of common theory of flexure to triangular cantilevers; shear deformation in cantilever beams. (2) As to temperature and distortion effects; variation in temperature

[13] Engineering News-Record, Sept. 24, 1925, p. 510.

change from upstream to downstream face; relation between temperatures of air, water, and concrete; shrinkage due to setting; swelling due to moisture; flow of concrete under sustained load; effect of lateral deformation; yielding of foundation and abutments. (3) As to construction influences: Effect of vertical construction joints or cracks; effect of horizontal cracks on cantilever action; effect of water pressure in horizontal cracks or construction joints; importance of uniformity of concrete, arrangement and preparation of construction joints, and difference of age between successive horizontal sections.

"Laboratory tests bearing on a number of these points will be carried on at the same time as the measurements on the dam."

Mr. Alfred D. Flinn is Director of the Engineering Foundation, Prof. Charles Derleth of Berkeley, Cal., is Chairman, and Mr. F. A. Noetzli of Los Angeles, Cal., is Secretary of the Committee of Investigation, and Mr. Harry Hawgood, Consulting Engineer of Los Angeles, is Chairman of the Sub-committee on the Test of the Dam.

170. Multiple-Arch Dams.—Dams consisting of a series of concrete arches supported by buttresses are sometimes used for moderate heights where suitable foundations are available and the cost of gravity dams would be greater. The amount of concrete required is much less than for gravity dams, and where concrete materials are expensive considerable savings in cost may result from their use. The form work required and the thin sections of concrete, make the unit costs much more than for gravity dams, and under favorable conditions for cheap concrete work gravity sections may be cheaper to construct. As the buttresses must carry the thrust of the water pressure, it is essential that they be established upon very substantial and unyielding foundations. Usually this is solid rock, although some dams of this type have been built upon gravel or fissured rock. Where the foundation is stable but of character which may permit water to penetrate it, this type of dam has advantages over a gravity dam on account of the less importance of possible uplift.

Two types of multiple-arch dams are in use: (1) those in which the axes of the arches are vertical, the water pressures coming horizontally against the faces and being transmitted as horizontal thrusts against the buttresses; (2) those with inclined axes, the water pressures acting normal to the sloping axes and bringing vertical as well as horizontal thrusts upon the buttresses.

Let Fig. 94 represent an inclined arch dam. A slice of the arch ring normal to the axis carries a water pressure which varies from

the crown to the springing line, and also carries a portion of its own weight to the buttress. If a slice of the arch ring be divided into voussoirs as shown, the water pressures upon each voussoir (P_1–P_5) varies with the depth (h_1–h_5) below the surface of the water. The weights of the voussoirs (G_1–G_5) may be considered as divided into components, ($N = G \sin \theta$) normal to the section and ($W = G \cos \theta$) parallel to the section. The normal components are carried as longitudinal thrusts to the foundation, while the parallel components (W_1–W_5) are carried by the arch ring to the buttress. Having determined these loads, an approximate line of thrust may be drawn by the method used for voussoir arches (see Section 189), from which stresses may be determined.

In designing such an arch, the required thickness at various depths may be approximately determined by finding the thickness for a

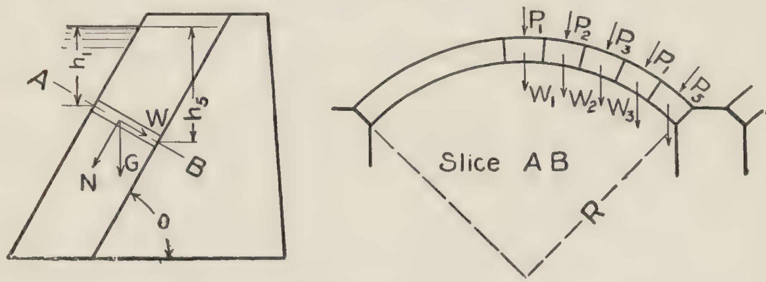


Fig. 94.—Inclined Multiple-Arch Dam.

horizontal arch at the same depth, then using this thickness in the analysis, modifying it as required. Practically an assumed thickness is given the ring at the top and tapered to the required thickness at some point below.

When the arch axis is vertical, the arch carries only the water pressure, which is uniformly distributed over the face. The weight of the arch, in this case, is normal to the arch section and is carried vertically to the foundation. The thickness required for the arch ring may be found from Formula (8), (Sec. 167, p. 345).

The stresses upon the buttresses of a multiple-arch dam may be found by the methods used for gravity dams. In Fig. 95 G–F is a section through the crown of an inclined arch; $ABCD$ being the side projection of the buttress. The form of the buttress must be such that the resultant thrust upon any horizontal section A–B will act approximately at the middle of the section. The loads acting are:

(1) The horizontal water pressure ($H=\frac{1}{2}wh^2L$) due to the depth of water above the plane $A-B$, upon a length of dam (L) equal to the distance between the middle points of adjacent arches.

(2) The vertical water pressure ($V=H\cdot\cot\cdot\theta$). The center of pressure for the vertical water pressure is at the center of gravity of a horizontal section of the water face of the arch at two-thirds the depth below the surface of the water.

(3) The weight (W_1) of the two half arches upon each side of the buttress. The center of gravity for the weight of the arch is at the center of gravity of that horizontal section of the arch ring which passes through the center of gravity of the vertical section ($G-F$) of the crown of the arch. If the centers of gravity be determined for

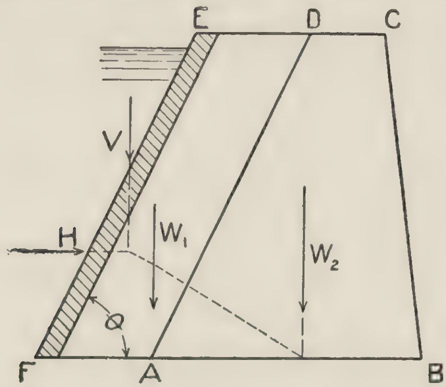


FIG. 95.—Buttress and Multiple-Arch Dam.

horizontal sections of the arch ring at the top and bottom of the arch, all intermediate centers will lie upon the line joining these points.

(4) The weight of the buttress itself, acting through its center of gravity.

The resultant (R) of these loads should cut the base $A-B$ near its middle point, in order to secure uniform distribution of pressure over the section.

Buttresses, for dams of this type, are usually made very thin in comparison with their widths, and are therefore stiffened laterally by the use of horizontal struts from buttress to buttress, or by the use of cross walls. The design of these struts is purely a matter of judgment on the part of the designer.

In the design of multiple-arch dams, the general lay-out is a matter which must depend upon local topography. Each dam is a problem by itself, and must be made to fit its location. It has been found that

in some instances, where the conditions are favorable to the construction of masonry dams of moderate height, multiple-arch dams may be built at much less cost than gravity structures. Forty to 60 or 70 feet between centers of buttresses are commonly found economical distances. Arches with axes making angles of 30° or 40° with the vertical are apt to show some saving of material as compared with vertical axes, but this is not always the case. The unit cost of construction is usually somewhat greater for inclined arches.

In constructing gravity dams, a cheaper grade of masonry may be employed, and the form work costs less than for multiple-arch dams. Careful studies of local conditions, and tentative trial designs are necessary in each case for best results.

Temperature Stresses, due to temperatures lower than those at which the arches are constructed, are to be expected in all structures, These produce shortening of the arch and give tensile stresses which may result in cracks when the dam is empty. Horizontal reinforcement near the downstream face at the crown and near the upstream face at the springing line is desirable to resist this tendency to crack.

ART. 42. REINFORCED CONCRETE DAMS

171. Reinforcement in Arch Dams.—Several designers have used steel reinforcement in arch dams, where it has seemed desirable to prevent the possible development of cracks, or to give additional security where the uncertainty concerning stresses made tensions seem possible under certain conditions. Cracks which may result from changes of temperature when dams are empty are frequently guarded against by using reinforcement, as has been mentioned in the previous articles. The stresses in most of these cases are practically indeterminate, and the reinforcement is placed according to the judgment of the designer.

These structures are sometimes called reinforced concrete dams in published reports, but are not designed as reinforced structures and are not properly so classed. No fully reinforced arch dams have as yet been constructed, and no dams have been designed in which the stresses have been determined by the use of the theory of the elastic arch.

172. Flat Slab and Buttress Dams.—For dams of moderate height reinforced flat slab and buttress construction has frequently proven economical. In this type of construction the buttresses are

usually placed from 12 to 18 feet apart, and the slabs extending between buttresses are inclined at an angle of 40° to 45° with the vertical so that the resultant of the normal water pressures passes near the middle of the base of the buttress. Most of the dams of this type in use have been constructed under the patents of the Ambursen Hydraulic Construction Company.

Figure 96 shows a dam of this type in section through the inclined slab. The loads carried by the slab consist of the normal water pressure and the normal component of its own weight. The slab may be designed by the ordinary method for reinforced concrete beams, but the values used for allowable stresses should be very conservative.

The buttress should be made of sufficient width to cause the

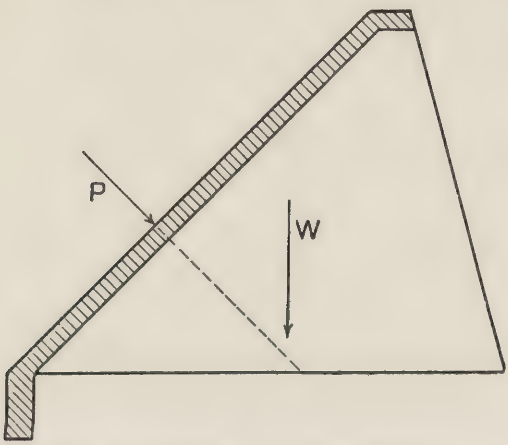


FIG. 96.—Flat Slab and Buttress Dam.

resultant thrust upon its base to pass approximately through its middle point when fully loaded, and must have sufficient base area to keep the pressure upon the foundation within reasonable limits. Lateral stiffness of the buttresses may be provided by giving them sufficient thickness, and using reinforcement on the sides, or it may be obtained by ties and struts between buttresses.

In many dams of this type, cellular construction is adopted in which the spaces between buttresses are divided into cells by horizontal floors, openings through the buttresses providing opportunity to pass under the dam throughout its length. Sometimes vertical walls provide rooms which may be utilized for power house or other purposes.

Slab and buttress dams, like any other masonry dams, require

firm foundations. For locations where substantial foundations may be obtained for buttresses on porous material, they possess an advantage over gravity dams which would be subjected to upward pressure. In these cases it is necessary to provide cut-off walls at the heel of the dam to prevent water passing under and washing out the foundation.

ART. 43. CONSTRUCTION OF MASONRY DAMS

173. Foundations.—Masonry dams are ordinarily applicable only to situations where foundations of solid rock may be obtained. Careful examinations of the character of the rock should always be made to considerable depths below the foundation in order to make sure that no seams or strata of porous materials exist, which might cause slipping of the foundation when subjected to pressure of water behind the dam.

Nearly all of the failures of masonry dams which have been recorded have been due to defective foundations, causing settlements through washing out the foundation materials, or sliding of base of dam, and foundation on seams or soft strata through which water under pressure found its way.

Where the depth to solid rock is considerable and the rock or gravel near the surface is of a character to give substantial support to the structure, masonry dams may sometimes be used without carrying the base of the dam into the solid rock. In such cases, curtain walls at the heel of the dam should be carried down to the rock to shut off leakage and possible washing of the foundation.

When the rock is seamed or fissured, it may frequently be made tight by grouting, which is done by drilling into it and forcing grout (usually of neat cement and water) under pressure into the fissures until the cracks become sufficiently filled to force grout to the surface through adjacent drill holes.

When a high dam is to be constructed, the geological structure of the valley should be studied, and core drill borings made over the site of the dam so as to determine fully the stability of the foundation and the probability of leakage around or under the dam.

The placing of the foundations of a dam is usually the most difficult part of the work of construction. Commonly it is necessary to divert the water of the stream to be dammed, and seepage water must be handled in making the excavations and placing the masonry. The methods used in such work are described in Mr. Chester W. Smith's "Construction of Masonry Dams," New York, 1915.

174. Masonry for Dams.—Several types of masonry are sometimes used in dams of massive construction.

Heavy rubble masonry has commonly been employed, in which the large stones are set in mortar beds, and the vertical joints filled with mortar and small stones carefully placed by masons. The stones are put into place by derricks and must be held and lowered so as to seat evenly upon the mortar bed, being set with careful attention to bond, so that no continuous joints exist in any direction. The complete filling of all joints in important.

Cyclopean masonry, in which stones weighing more than 100 pounds are individually embedded in concrete and the joints filled with soft concrete, has recently been used to considerable extent. The joints are made thicker than mortar joints, and the labor required in setting the stones and making good joints is much less than in the ordinary rubble.

Rubble concrete is masonry in which stones weighing less than 100 pounds are individually embedded in a mass of concrete. This differs from cyclopean masonry mainly in the size of stones used and the larger quantity of concrete. It uses more cement, but is more rapidly constructed and requires less hand labor.

Plain concrete is now frequently used, without large stone, for massive work, as well as for the dams of thin sections. The plant required is less, as derricks are needed for handling the heavy stone, and usually more rapid progress is possible in placing the concrete. The nature and location of materials and character of labor available determine the relative costs of the different methods of construction. The rapidity of construction is usually greater as the quantity of large stone becomes less.

175. Overflow Dams.—When water is to flow over the top of a dam or spillway, the section must be modified to provide for the passing of the water with the least disturbance possible, and to take into account the additional head of water above the dam.

If the water falls freely over the dam, its crest should be given such form as to eliminate the possibility of causing a vacuum behind the sheet of falling water. The effect of the impact of the falling water must also be taken into account, and provision made for protecting the toe of the dam against erosion, which is frequently done by providing a water cushion into which the stream may fall.

In overflow weirs of considerable height, the downstream face of the dam is given approximately the form of the curve that the water would take in falling freely over the weir under maximum head. The water may then follow the surface of the dam, and, by reversing the

curve in the lower part of the section, be turned to horizontal direction at the toe of the dam. In designing such a section, the weight of the water on the downstream face is neglected, the pressure on the upstream face being taken as that due to the full head at greatest expected flood. Special attention should also be given to the possibility of uplift or of scouring at the toe.

CHAPTER IX

SLAB AND GIRDER BRIDGES

ART. 44. LOADINGS FOR SHORT BRIDGES

176. Highway Bridges.—*Sidewalks* of bridges in towns may be considered as carrying a live load of 100 pounds per square foot of sidewalk area. In the more crowded districts of cities, larger loads are sometimes employed, but in general this is ample for all probable occurrences.

Roadways of highway bridges should be able to carry the heaviest motor trucks which may reasonably be expected to come upon them. In the development of truck transportation there is a tendency to increase the weights carried by a single truck, and careful attention should be given to this possibility in designing bridges intended to last a long time. A motor truck weighing 20 tons, with 6 tons on one axle and 14 tons on the other, the distance between wheels being 6 feet and between axles 12 feet, may reasonably be assumed as a maximum load for a bridge upon an important country highway or street of a town. This load is a very exceptional one for ordinary highways and probably in most cases a truck weighing 7 to 10 tons is as large as is likely to be met under present conditions, and possibly a road roller may be a more probable maximum load. The use of maximum loads not likely to be exceeded in the near future is always desirable in such work.

For country bridges under moderate or light traffic, a truck weighing 8000 pounds on each of two axles, 10 feet apart, may be used as a probable maximum load under present conditions, or a 15-ton road roller, 6 tons on the front wheel, which is 4 feet wide, and 4.5 tons on each of the rear wheels, each 20 inches wide.

Street Railway Track.—When the bridge is to carry a street railway, the load of a car weighing 50 tons on four axles spaced 5, 14, and 5 feet apart may be assumed as a probable maximum load. This load may be considered as distributed over an area of bridge floor about 35 feet in length and 10 feet in width, giving a maximum uniform load of about 300 pounds per square foot.

For light traffic roads, a car weighing 35 tons on the same wheel distribution may be used, giving a uniform loading of about 200 pounds per square foot.

177. Distribution of Concentrated Loads.—Investigations of the distribution of concentrated loads upon slabs have been made by Mr. Goldbeck for the U. S. Office of Public Roads. These tests [1] seemed to indicate that for a slab whose width is greater than its span, the effective width of distribution of a concentrated load might be taken at about eight-tenths of the span.

From a series of tests at the University of Illinois, Mr. Slater concluded [2] that for a slab whose width is greater than twice the span, the effective width (e) might be assumed as $e=\frac{4}{3}x+d$, where x is the distance from the concentrated load to the nearest support and d is the width over which the load is applied. As the ratio of width to span decreases, the effective width becomes less, the coefficient in the formula becoming about 1.2 when the span equals the width.

From tests for the Highway Department of the State of Ohio [3] Professor Morris recommends for a concentrated load applied to the concrete floor of a highway bridge that $e=0.6S+1.7$, where e is the effective width in feet for a slab whose width is greater than its span, and S is the clear span in feet. This agrees well with the results of Mr. Slater if the load be placed at the middle of the span ($x=S/2$).

When the load comes upon the floor of the bridge through a pavement or fill, it may also be considered as distributed lengthwise over a certain area. For earth fill, the length of distribution may be taken as twice the depth of fill. For gravel or macadam road surface, three or four times the depth of surface may be used.

In T-beam construction, when a slab is continuous over several girders and a load comes upon the slab immediately over one of the girders, the whole of the load will not be borne by the girder under the load, but a portion of it will be transferred by the slab to adjacent girders. In the Ohio tests mentioned above, this distribution was investigated and the following conclusions reached:

(1) The percentage of reinforcement has little or no effect upon the load distribution to the joists, so long as safe loads on the slab are not exceeded.

(2) The amount of load distributed by the slab to other joists than

[1] Proceedings, American Society for Testing Materials, 1915, p. 858.

[2] Proceedings, American Society for Testing Materials, 1913, p. 874.

[3] Bulletin No. 28, Ohio State Highway Department, 1915.

the one immediately under the load, increases with the thickness of the slab.

(3) The outside joists should be designed for the same live load as the intermediate joists.

(4) The axle load of a truck may be considered as distributed uniformly over 12 feet of roadway.

178. Railway Bridges.—For short spans, railway moving loads may be considered as uniformly distributed by the track and ballast. If the heaviest locomotive load per foot of length be distributed over a width of about 10 feet, the result will be well on the safe side. When the bridge is covered by a fill under the tracks, the width of distribution may be increased by twice the depth of fill.

The weights for maximum locomotive loads may vary from about 8000 to 10,000 pounds per linear foot of track, or from 800 to 1000 pounds per square foot when distributed over a width of 10 feet. For bridges longer than about 35 feet, it may be preferable to use actual locomotive wheel loads, or to somewhat reduce the load per square foot.

ART. 45. DESIGN OF BEAM BRIDGES

179. Slab Bridges.—When the span of a bridge is not more than 12 to 15 feet, the simple slab spanning the opening and resting upon the abutments at its ends is usually the most economical form to use. Under heavy loading, the economic limit of length may be only 10 to 12 feet, while for lighter loads, slabs 16 to 20 feet in length may be desirable. The design of a bridge slab will be illustrated by a numerical example.

Example 1.—Design a highway slab of 11 feet clear span, and width of 18 feet to carry a macadam road with the loading given in Section 176.

Solution.—

Assume weight of road material	= 80 pounds per square foot.
Weight of Slab	=150 pounds per square foot.
Total dead load	=230 pounds per square foot.

Live load is auto truck with 14,000 pounds on each of two wheels 6 feet apart. From Section 177, effective width, $e=.6S+1.7$. As $.6S$ is more than the distance apart of wheels, the loads would overlap, and we consider both loads distributed over $e=.6S+1.7+6=14.3$ feet. The live load per foot of width is $28000/14.3=1950$ pounds. This load may be considered as applied over a length of

1.7 feet = 20 inches. The effective length of the beam is distance between centers of bearings, or 1 foot more than the clear span. (11+1 = 12 feet).

Bending moments,

M (live) = 1950/2 (72−5)	= 65325 in.-lb.
M (impact) = 25 per cent of live	= 16330 in.-lb.
M (dead) = 230×12×12×12/8	= 49680 in.-lb.
Total moment, M	= 131335 in.-lb.

Table XXVIII (p. 205) gives $d = 10\frac{1}{4}$ inches.

Maximum shear occurs when center of live load is $1\frac{3}{4}$ feet from support, in which case,

V (live) = 1950×10.65/12	= 1722 pounds.
V (impact) = 25 per cent of live	= 430 pounds.
V (dead) = 230×11/2	= 1265 pounds.
Total shear V	= 3417 pounds.

Diagram IX (p. 220) gives $d = 8\frac{1}{4}$ inches.

Make $d = 10\frac{1}{4}$ inches, then allowing concrete to extend 1.75 inches below steel, weight of beam is 12×121×150/144 = 150 pounds per square foot, as assumed.

Reinforcement.—Table XXVIII gives $A_s = 0.95$ inch ² and Table XXVII (p. 204) shows that $\frac{3}{4}$-inch round bars spaced 5.5 inches center to center will answer. For these the maximum unit bend stress is

$$u = \frac{V}{jd\Sigma_0} = \frac{3417}{.875 \times 10.25 \times 5.12} = 74 \text{ lb./in.}^2.$$

Figure 97 shows the slab in longitudinal section. For lateral reinforcement $\frac{1}{2}$-inch round bars, 12 inches apart, are used. To prevent cracking due to negative moment where the slab joins the abutments, $\frac{1}{2}$-inch round bars 12 inches apart are placed in the ends of the slab at the top. Expansion joints, usually tar paper, are often placed on the top of the abutment under the slab, thus preventing the development of negative moment and allowing for temperature changes.

180. T-Beam Bridges.—When the length of the bridge is too great for a simple slab, it is found economical to use girders to support the slab. If the head room is sufficient and the span not too great, T-beam construction may be used. This consists of a series of T-beams extending from abutment to abutment, girders being placed under the

slab to form the stems of the T-beams, and the slab being continuous over the girders for the width of the bridge.

Example 2.—Design a T-beam highway bridge with clear span of 24 feet, to carry a roadway 18 feet wide, using loadings as in Example 1.

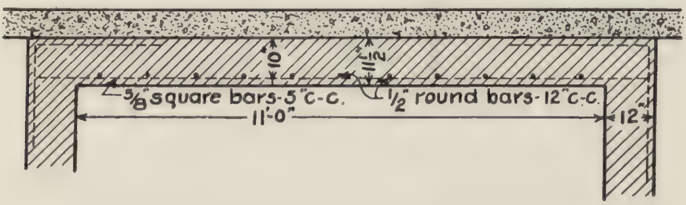


Fig. 97.—Slab Bridge.

Solution.—Allowing 12 inches for width of base of guard rail, the full width is 20 feet. Use five girders, spaced 4 feet on centers, the outside girders being 2 feet from end of beam (see Fig. 98).

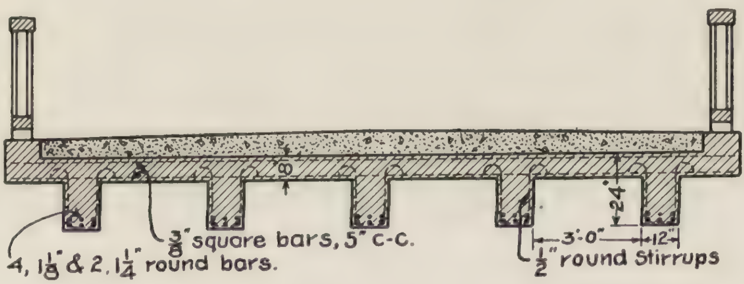


Fig. 98.—T-beam Girder Highway Bridge.

Slab.—Weight of road material = 80 pounds per square foot.
Assume weight of slab = 100 pounds per square foot.

Total dead load = 180 pounds per square foot.

The live load is a single wheel load of 14,000 pounds distributed over a width $.6 \times 4 + 1.7 = 4.1$ feet. The live load per foot of width is $14000/4.1 = 3415$ pounds. This may be considered as distributed over 2 feet of length. The slab is continuous and taking the moment

of the concentrated load as four-fifths of the moment for a simply supported beam, we have

$$\begin{aligned} M\text{ (live)} &= (3415/2)(24-6)\tfrac{4}{5} &&= 24588 \text{ in.-lb.}\\ M\text{ (impact)} &= 25 \text{ per cent of } 24588 &&= 6147 \text{ in.-lb.}\\ M\text{ (dead)} &= 180\times4\times4\times12/12 &&= 2880 \text{ in.-lb.}\\ \hline \text{Maximum moment, } M & &&= 33615 \text{ in.-lb} \end{aligned}$$

Table XXVIII (p. 205) gives $d=5.25$ inches.

The shear is a maximum when the load is placed next to the support, and assuming width of girder at 12 inches,

$$\begin{aligned} V\text{ (live)} &= 3415\times2.5/4 &&= 2134 \text{ pounds.}\\ V\text{ (impact)} &= 25 \text{ per cent of } 2134 &&= 533 \text{ pounds.}\\ V\text{ (dead)} &= 180\times3/2 &&= 270 \text{ pounds.}\\ \hline \text{Total shear, } V & &&= 2937 \text{ pounds.} \end{aligned}$$

Diagram IX (p. 220) gives $d=7$ inches.

Using $d=7$ inches,

$$A_s=\frac{M}{f_sjd}=\frac{33615}{16000\times.875\times7}=0.34 \text{ in.}^2$$

From Table XXVII (p. 204), $\frac{1}{2}$-inch round bars spaced 7 inches apart will answer.

When the concentrated load is at the middle of a span, adjacent unloaded spans will be under negative moment throughout their lengths. Maximum negative moment is approximately the same as positive moment, and $\frac{3}{8}$-inch square bars 5 inches apart will therefore be put through the top as well as the bottom of the slab.

With concrete extending 1 inch below the steel, the total depth of slab is 8 inches and the weight of slab $=8\times150/12=100$ pounds per square foot as assumed.

Girders.—The maximum stresses in the girder occur when two wheels are directly over the girder. A portion of this load is distributed by the slab to adjacent girders. This rolling load consists of one wheel carrying 14,000 pounds and one carrying 6000 pounds, 12 feet apart. Assuming this distributed over a width of 6 feet (see Section 177), the load carried by one girder covers 4 feet of width and the loads are $14000\times4/6=9333$ and $6000\times4/6=4000$ pounds.

Assuming the stem of girder to weigh 250 pounds per foot, the dead load is $180\times4+250=970$ pounds per linear foor of girder.

The position of moving load for maximum moment is that in which the heavier wheel is as far to one side of the middle of the beam as the center of gravity of the two loads is to the other, and the moment (taking length of beam as 25 feet) is:

$$M\text{ (live)} = \frac{13333(12.5-1.8)^2}{25}\times 12 \quad = 732736 \text{ in.-lb.}$$

$$M\text{ (impact)} = 25 \text{ per cent of } 732736 = 183184 \text{ in.-lb.}$$

$$\text{M (dead)} = 970\times 25\times 25\times 12/8 \quad = 909375 \text{ in.-lb}$$

$$\text{Total moment, } M \quad = 1825295 \text{ in.-lb.}$$

Maximum shear occurs when the heavier load is adjacent to the support, and the center of gravity of the loads (considering the loads distributed over 2 feet of length) is 5.1 feet from the center of support

$$V\text{ (live)} = 13333(25-5.1)/25 \quad = 10610 \text{ pounds.}$$

$$V\text{ (impact)} = 25 \text{ per cent of } 10610 \quad = 2650 \text{ pounds.}$$

$$V\text{ (dead)} = 970\times 25/2 \quad = 12125 \text{ pounds.}$$

$$\text{Total } V \quad = 25385 \text{ pounds.}$$

The depth required for shear with 12-inch width of stem is

$$d = \frac{V}{b'jv} = \frac{25385}{12\times .874\times 120} = 20.1 \text{ inches.}$$

The girder is a T-beam with flange 48 inches wide and 8 inches thick, and stem 12 inches wide and 20 inches deep.

$$d/t = 20/8 = 2.5, \quad \text{and} \quad Q = \frac{M}{btd} = \frac{1825295}{48\times 8\times 20} = 238.$$

From Diagram IX (p. 220), we see that the neutral axis is in the flange.

$$R = \frac{M}{bd^2} = \frac{1825295}{48\times 20\times 20} = 95.$$

From Table XXII (p. 199), we see that for $f_s = 16000$ and $R = 95^2$, $f_c = 600$ and $p = .0068$. Then $A_s = .0068\times 48\times 20 = 6.53$ in.^{2} From Table XXVI (p. 203), it is found that four $1\frac{1}{8}$- and two $1\frac{1}{4}$-inch round bars will answer. These are placed in two rows, two $1\frac{1}{8}$- and one $1\frac{1}{4}$-inch bars in each row, making the total depth of the beam 24 inches. The weight of stem is then $= 16\times 12\times 150/144 = 200$ pounds per foot, which is less than the assumed weight.

Diagonal Tension.—The maximum shear at the middle of the

girder occurs when the moving load is at one side of the middle of the beam, or V (middle) $= 9333 \times 11.5/25 = 4293$ pounds; with impact this becomes 5366 pounds and v (middle) $= \frac{5366}{12 \times .875 \times 20}$ $= 25.5$ pounds. The maximum unit shear varies from 25.5 lb./in.^{2} at the middle to 120 lb./in.^{2} at the supports. Stirrups are necessary from the support to the point where the shear is 40 lb./in.^{2} Using Formula 13 of Section 120, if U-shaped stirrups of $\frac{1}{2}$-inch round steel be used, the spacing at the ends should be

$$s = \frac{2A_v f_s}{vb'} = \frac{2 \times .39 \times 16000}{120 \times 12} = 8 \text{ inches.}$$

Use this spacing for eight stirrups, then change to 12 inches spacing and continue to middle of girder. Two of the horizontal bars may also be turned up near the abutment.

181. Through Girder Bridges. For spans of considerable length, or where the head room under the roadway is too contracted to permit

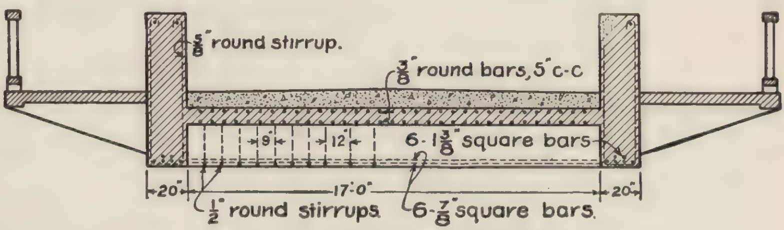


FIG. 99.—Through Girder Bridge.

the use of T-beam construction, through girders may be used at the sides of the roadway, the slab floor being hung from the bottoms of the side girders. The floors in such bridges may be simple slabs, extending from one girder to the other, or the floor slab may be carried by T-beams across the bridge from girder to girder.

Figure 99 shows a bridge of this type. The method of design is the same as for the other types. If the loading of Example 2 be used, the T-beam cross-girder would carry the two loads of 14,000 pounds each, 6 feet apart, or if the width of the bridge and importance of traffic are sufficient, two passing trucks might give a loading of four such wheels spaced 6, 2, and 6 feet apart. In a bridge for heavy traffic, where passing loads might come upon it, each girder should be able to carry the whole weight of a truck as a rolling load in addition to the dead weight of one-half the bridge. On a country highway, designing for the passing of a single truck is usually sufficient, as the meeting

of two unusually heavy loads on the bridge is a very remote contingency.

When sidewalks are to be carried at the side of the roadway, the through girder may be placed between the roadway and sidewalk, and the sidewalk carried by cantilever beams attached to the girders. These cantilevers should be continuations of the cross-girders, the tension steel extending through the main girder and being anchored into the cross-girders.

Example 3.—Design the principal members for a bridge of 35 feet clear span, 17 feet wide between girders, to carry roadway and loads as in Example 2. Also to carry sidewalks 5 feet wide, loaded with 100 pounds per square foot.

Solution.—Assume the spacing of cross-girders at 4 feet c. to c. and the road slab as in Example 2; slab 8 inches thick, $d=7$ inches, $\frac{1}{2}$-inch round steel 7 inches c. to c. top and bottom.

Cross-beams.—The dead load upon the T-beams, assuming weight of stem at 150 pounds per linear foot will be $180\times4+150=870$ pounds per linear foot. The live load is composed of two 14,000-pound wheel loads, 6 feet apart. As these are distributed over 6 feet of width, $14000\times4/6=9333$ will be carried by the 4-foot width of beam.

The effective length of beam is 18 feet, and

$$M \text{ (live)} = \frac{18666(9-1.5)^2}{18} = 700000 \text{ in.-lb.}$$

$$M \text{ (impact)} = 25 \text{ per cent of } 700000 = 175000 \text{ in.-lb.}$$

$$M \text{ (dead)} = \frac{870\times18\times18\times12}{8} = 422820 \text{ in.-lb.}$$

$$\text{Total moment, } M = 1297820 \text{ in.-lb.}$$

Assuming that the nearest wheel load may pass 18 inches from the side girder, maximum shear in the cross-beam is

$$V \text{ (live)} = \frac{18666(18-5)}{18} = 13480 \text{ pounds.}$$

$$V \text{ (impact)} = 25 \text{ per cent} = 3370 \text{ pounds.}$$

$$V \text{ (dead)} = 870\times9 = 7830 \text{ pounds.}$$

$$\text{Total shear, } V = 24680 \text{ pounds.}$$

Assuming width of stem of T-beam to be 12 inches, Diagram IX (p. 220) gives the required depth, 19.5 inches.

By Table XXII (p. 199), for $f_s = 16000$ and

$$R = \frac{1297820}{48 \times 19.5 \times 19.5} = 71,$$

we find $f_c = 500$ and $p = .005$. Then

$$A_s pbd = .005 \times 48 \times 19.5 = 4.68 \text{ in.}^2$$

Use six $\frac{7}{8}$-inch square bars, four in lower row, two in upper.

The total depth of beam is 22 inches, and the weight of the stem is $12 \times 14 \times 150/144 = 175$ pounds per linear foot, 25 pounds more than assumed.

Using U-shaped stirrups of $\frac{1}{2}$-inch round bars, the spacing at end of beam is, from Diagram XII (p. 223), $0.34\,d = .34 \times 19.5 = 6.5$ inches. At the center of the span the spacing will be $0.45\,d = .45 \times 19.5 = 8.5$ inches. Make one space of $3\frac{1}{4}$ inches at end, six spaces of $6\frac{1}{2}$ inches, 6 spaces of $7\frac{1}{2}$ inches, and the balance of about $8\frac{1}{2}$ inches toward the center of the span.

The sidewalk slab carries 100 pounds per square foot moving load, on 4-foot continuous spans. We will make it 3 inches thick, reinforced with $\frac{1}{2}$-inch round bars $7\frac{1}{2}$ inches apart. The sidewalk supports are cantilever beams carrying 4 feet of sidewalk with its load and 4 feet of handrail at the end.

Side Girders.—The sidewalk with its load weighs about 800 per pounds per foot of girder. One-half the weight of bridge floor and T-beams is 1900 pounds per foot. Assume weight of girder as 1600 pounds per foot, and the total dead load is 4300 pounds per linear foot.

The maximum moving load is the weight of a truck whose nearest wheels are 18 inches from the girder. These loads are

$$\frac{28000 \times (18-5)}{18} = 20200 \text{ and } \frac{12000 \times (18-5)}{18} = 8700 \text{ lb., 12 feet apart.}$$

Take effective length of girder as 36 feet and we have

$$M \text{ (live)} = \frac{28900(18-1.8)^2}{18} \times 12 \qquad = \ 2528200 \text{ in.-lb.}$$

$$M \text{ (impact)} - 25 \text{ per cent of } 2528200 = \ 632050 \text{ in.-lb.}$$

$$M \text{ (dead)} = \frac{4300 \times 36 \times 36}{8} \times 12 \qquad = \ 8359200 \text{ in.-lb.}$$

$$\text{Total} \qquad M = 11519450 \text{ in.-lb.}$$

$$bd^2 = \frac{11519450}{108} = 106666.$$

Assuming $b=20$ inches, we find $d=73$ inches.

$$A_s=pbd=.0078\times 20\times 73=11.38 \text{ in.}^2$$

Table XXVI (p. 203) shows that nine $1\frac{1}{8}$-inch square bars may be used, or six $1\frac{3}{8}$-inch square bars will answer. The latter can be spaced four in the lower and two in upper row. The maximum bond stress for the latter is

$$u=\frac{V}{jd\Sigma_0}=\frac{106250}{.875\times 73\times 33}=50 \text{ lb./in.}^2$$

Shear.—Considering the live loads to be applied over a length of 2 feet,

$$\begin{aligned} V \text{ (live)} &= 28900(36-5.1)/36 &&= 24800 \text{ pounds.}\\ V \text{ (impact)} &= 25 \text{ per cent of } 24800 &&= 6200 \text{ pounds.}\\ V \text{ (dead)} &.\ 4300\times 17.5 &&= 75250 \text{ pounds.}\\ \text{Total } V & &&= 106250 \text{ pounds.}\end{aligned}$$

The maximum shear at the middle of the beam occurs when the heavier load is just past the middle point, or

$$V=28900\ (18-4.6)/36=10760 \text{ lb.}$$

but, in accordance with the recommendation of the Joint Committee, it will be taken at $\frac{106250}{4}=26560$ lb.

and

$$v \text{ (middle)}=\frac{26560}{20\times .875\times 73}=21 \text{ lb./in.}^2$$

The maximum shear varies from 83 lb./in.^{2} at the support to 21 lb./in.^{2} at the middle of the girder. Reinforcement for diagonal tension is needed where v is more than 40 lb./in. From Diagram XIII (p. 224), use $\frac{1}{2}$-inch square double U-stirrups spaced 14.5 inches c. to c. next to the abutment and gradually increase to 32 inches at the middle of the span. The upper ends of these bars must be hooked to secure ample bond.

Hangers.—To prevent the T-beams breaking loose from the girders, bars passing under the steel in the stem of the T-beam, and extending up into the girder are used to carry the reactions at the ends of the T-beams. These reactions equal the maximum shear upon the T-beams, and the area of steel required is $A_h=24680/16000 =1.54$ in.^{2} By Table XXVI we find 1-inch round bars to be needed. These should extend upward a distance sufficient to develop a bond strength equal to the tensile strength of the bars, or at least 50 diameters.

CHAPTER X

MASONRY ARCHES

ART. 46. VOUSSOIR ARCHES

182. Definitions.—A masonry arch is a structure of masonry spanning an opening and carrying its loads as longitudinal thrusts, which exert outward as well as vertical pressures upon the abutments. The masonry arch combines utility, beauty, and permanence. Its utility dates back to "the dawn of civilization," and its beauty and permanence to the time of the Etruscans and Romans.

FIG. 100.—Stone Arch, Echo Bridge, Newton, Mass.
(From "Artistic Bridge Design" by courtesy of Mr. H. Grattan Tyrrell.)

We know that the Egyptians were acquainted with the arch, but they did not use it to any great extent because it did not harmonize with the style of architecture developed by them. We know that the Asiatics were familiar with the arch because they have a saying that,

"The arch never sleeps." Improperly designed abutments with gradual settlements followed by final failures would naturally give rise to such an expression.

A *voussoir arch* is one in which the arch ring is composed of a number of independent blocks of stone, brick, or concrete masonry. The great variety of textural effects obtainable, from rough boulders to the highest grade of cut-stone masonry, will continue to make this type a favorite with engineers and architects for those situations where esthetic effect is a prime consideration. Fig. 11 (p. 7) shows a semicircular stone arch at Troy constructed by the Romans during the second century, A.D. Fig. 100 (p. 372) shows one of the handsomest modern stone arches at Newton, Mass.

Parts of an Arch.—The principal parts of an arch are as follows:

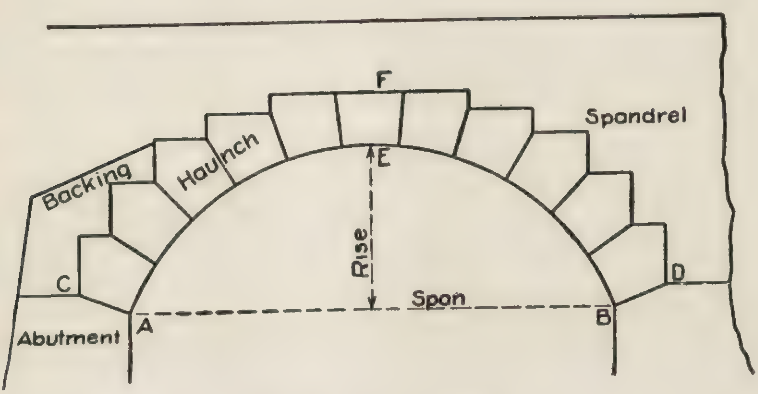


FIG. 101.—Voussoir Arch.

The under or concave surface of an arch is called the *soffit.* The outer or convex surface is the *back.*

The *crown* is the highest part of the arch ring (*E–F*, Fig. 101).

The *skewbacks* are the joints at the ends of the arch where it rests upon the abutments (*C–A*, *B–D*, Fig. 101).

The *intrados* is the intersection of the soffit with a vertical plane perpendicular to the axis of the arch (*A–E–B*, Fig. 101).

The *extrados* is the intersection of the outer surface with a vertical plane perpendicular to the axis (*C–F–D*, Fig. 101).

The *springing lines* are the intersections of the skewbacks with the soffit.

The *span* is the distance between springing lines.

The *rise* is the perpendicular distance from the highest point of the intrados to the plane of the springing lines.

The *voussoirs* are the wedge-shaped stones of which an arch is composed.

The *keystone* is the voussoir at the crown of the arch (E–F).

The *springers* are the voussoirs next the skewbacks.

The *haunch* is the portion of the arch between the keystone and springers.

The *arch ring* is the whole set of voussoirs from skewback to skewback.

The *ring stones* are voussoirs showing on the face of the arch.

The *arch sheeting* is the portion of the arch ring not showing at the ends.

Backing is masonry above and outside the arch ring.

The *spandrel* is the space between the back of the arch and the roadway above. The walls above the ring stones at the ends of the arch are spandrel walls and the filling between these walls is spandrel filling.

Kinds of Arches.—A full-centered arch is one whose intrados is a semicircle. A *segmental arch* is a circular arch whose intrados is less than a semicircle. A *pointed arch* has an intrados composed of two circular arcs which intersect at the crown. A three-centered arch composed of arcs tangent to each other is sometimes called a *basket-handled* arch.

A *right arch* is one whose ends are perpendicular to its axis. An arch whose ends are oblique to its axis is called a *skew arch.*

Hinged arches are those in which hinged joints are used at crown and skewback. Those without hinges are called *solid arches.*

183. Theory of Stability.—A voussoir arch is supposed to be composed of a number of independent blocks in contact with each other and held in place by the pressures between them. In Fig. 102, let $ABCD$ represent a voussoir at any part of an arch ring. If P is the pressure received from the voussoir above and W the external load carried by the voussoir, the resultant, R, of these forces will be the pressure transmitted to the voussoir below. If the line of action of this resultant should pass outside of the joint A–D, the arch will fail by the voussoir rotating about the edge of the joint.

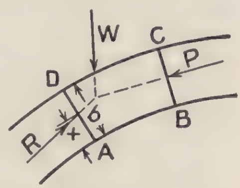


Fig. 102.—A Voussoir.

If the point of application of R is outside the middle third of

A–D, there will be a tendency for the joint to open on the opposite side, and the area of contact between the voussoirs will be reduced. If the line of action of R makes an angle with the normal to the joint A–D greater than the angle of friction for the surfaces upon each other, the voussoirs may slide upon each other, causing failure of the arch.

For stability of the arch:

(1) The resultant pressures between voussoirs should act within the middle third of the joints.

(2) The components of the resultant pressures parallel to the joints ($R \sin \alpha$) should be less than the frictional resistance of the voussoirs to sliding upon each other.

(3) The unit pressures at the surfaces of contact should be less than the safe compressive strength of the material of the voussoirs.

If b represents the width of the joint AD, x the distance of the point of application of R from the nearest edge and α the angle made by R with the normal to the joint, the maximum unit compression will be represented by

$$f_c = \frac{R(4l - 6a)}{l^2} \cos \alpha. \quad \text{(See Section 147.)}$$

Usually the angle α is so small that $\cos \alpha$ may be taken as 1 without sensible error, or R may be considered as equal to its normal component.

Line of Pressure.—If an arch ring be divided into a number of voussoirs, and the points of application of the resultant pressures upon the joints between these voussoirs be determined, the broken or curved line joining these points of application is known as the line of pressure for the arch. In Fig. 103 the line *abcdef* is called the line of pressure for the half arch, when H is the crown thrust and P_1, P_2, etc., are the external loads coming upon the several divisions. The true line of pressure, or of resistance, is a curve circumscribing the polygon *abcdef*. The larger the number of divisions of the arch ring, the more nearly will the polygon approach this curve.

In determining the line of pressure, the arch ring is divided into a convenient number of parts, usually six to sixteen on each side of the crown, and the external loads (P_1–P_5, Fig. 103) coming upon the various divisions are found. It is now necessary to know certain points through which the line of pressure must pass in order to draw it. If the arch be hinged, the line of pressure must pass through the centers of the hinges and may be drawn without difficulty. In a solid arch, the points of application of the pressures upon the various joints are

not definitely known, and certain assumptions must be made concerning them. Any number of different lines may be drawn as these assumptions are varied.

Hypotheses for Line of Pressure.—If Fig. 103 represents half of a symmetrically loaded arch, the crown pressure H will be horizontal. Assuming its point of application, a, and that its line of resistance passes through a definite point on one of the other joints as f, the amount of H may be found by taking a center of moments at f and writing the moment equation for all the loads upon the half arch equal to zero. H is then known in amount, direction and point of application and the line of pressure may be drawn, as shown.

Several hypotheses have been proposed for the purpose of fixing the position of the line of thrust. Professor Durand-Claye assumed

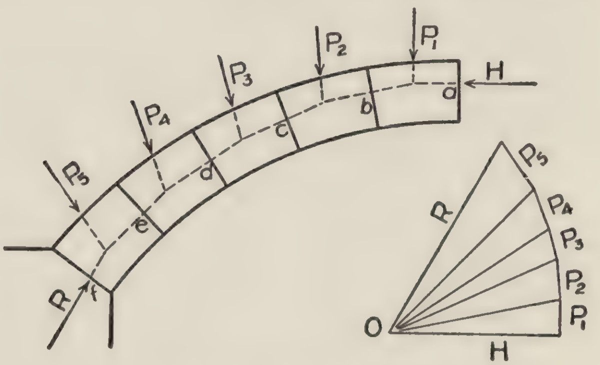


FIG. 103.—Line of Pressure.

that the true line of resistance is that which gives the smallest absolute pressure upon any joint. This method is outlined in Van Nostrand's Engineering Magazine, Vol. XV, p. 33. Professor Winkler suggested that "for an arch ring of constant cross-section, that line of resistance is approximately the true one which lies nearest to the axis of the arch ring, as determined by the method of least squares." No practicable method of applying this principle to ordinary cases of voussoir arches has been devised. Moseley's hypothesis was that the true line of resistance is that for which the thrust at the crown is the least, consistent with stability. This occurs (Fig. 103) when H is at the highest and R at the lowest point it can occupy on the joint. This hypothesis is the basis of Scheffler's method of drawing the line of resistance.

Scheffler's theory assumes that H is applied at the upper edge of the middle third of the crown joint, and that the value of H is such as

to cause the line of pressure to touch the lower edge of the middle third at one of the joints (as *d*, *e*, or *f*) nearer the abutment. The joint at which the line of pressure is tangent to the lower edge of the middle third is known as the joint of rupture. The joint of rupture may be found by taking moments about the lower edge of the middle third of each of several joints and solving for H. All loads acting between the joint considered and the crown should be used in obtaining the moment, and the one giving the largest value of H is the joint of rupture. The value of H so determined is the least consistent with stability, as a less value causes the line of pressure to pass outside the middle third at the joint of rupture.

Should it be found that the line of pressure passes outside the middle third on the upper side of any of the joints between the joint of rupture and the crown, the point of application of H may be lowered without violating the hypothesis. This leads to the usual statement that "if any line of pressure can be drawn within the middle third of the arch ring the arch will be stable." This is justified by common experience.

When the loading upon the arch is not symmetrical, this method of finding the crown thrust cannot be used, and in this case it is usual to select three points through which to pass the line of pressure, one at the crown and one near each abutment. A line of pressure is then passed through these three points, and if the line so found does not remain within the middle third of the arch ring the positions of the points may be changed and new lines constructed. This may be repeated until it is determined whether any line of pressure can be drawn within the middle third.

ART. 47. LOADS FOR MASONRY ARCHES

184. Live Loads for Highway Bridges.—For the floors of open spandrel arch bridges, live loads should be considered in the same manner as for slab bridges (see Art. 43). In investigations of arch rings, live loads are usually taken as uniformly distributed. The loading which should be used in any design depends upon the location of the bridge, the character of traffic, and the length of span.

A heavy (20-ton) motor truck may bring a load of about 140 pounds per square foot upon a bridge of short span (about 40 feet). Bridges 60 to 100 feet span subjected to traffic of motor trucks and heavily loaded wagons may be considered to carry about 100 pounds per square foot. For longer bridges this load may be lessened, bridges over 200 feet being designed for about 75 pounds per square foot.

For bridges less than 100 feet in length carrying street railways, a load of 1800 pounds per foot of length for each track may be taken. For spans of 200 feet or more, this may be reduced to 1200 pounds per foot of track. These loads are considered as distributed over a width of about 9 feet, giving loads of 200 and 133 pounds per square foot respectively. For spans between 100 and 200 feet, the loads may vary according to the length of span.

For light traffic lines on country roads, a load of 1200 pounds per foot of track may be used for arches less than 100 feet in length and 1000 pounds per foot for those 200 feet or more in length. Frequently bridges must be built for special service, or where the traffic conditions are unusual and should be designed for any loads that may reasonably be expected to come upon them. Traffic conditions are constantly undergoing important changes, and in determining the loading to be used in any particular instance, it is desirable to consider the possible effect upon future traffic of the rapid increase in the use of heavy auto-trucks and traction engines. As masonry arches are structures of permanent character, the probable future development of traffic should be considered and liberal loadings used in design.

185. Live Loads for Railway Arches.—Standard locomotive loadings are used in the design of floor systems for open spandrel arches, as in beam bridges, and are also sometimes employed in investigations of arch rings. Equivalent uniform loadings may, however, commonly be used in arch-ring design.

Loadings should correspond with the heaviest locomotive and train loads to be expected. For spans less than about 60 feet, a load of 8000 pounds per foot of track, or 1000 pounds per square foot of road surface is frequently used. When the span is 80 feet or more a load of 5600 pounds per foot of track, or about 700 pounds per square foot, is used, which are approximately the same as Cooper's E 40 loading. Impact is not taken into account in the arch-ring investigation.

A concentrated load upon a fill may be considered as distributed downward through the fill at an angle of 45° with the vertical, the top of the distributing slope being taken from the ends of the ties. Wheel loads are taken as distributed over three ties and then transmitted to the filling.

186. Dead Loads.—In arch bridges, the dead weights of the arch ring and of the filling or structure above constitute the principal loads upon the arch rings. The live loads are much less in amount, and are important mainly as producing unsymmetrical loading when

the load does not extend over the whole arch. In computing the dead load upon an arch ring, the actual weights of the materials to be used should be taken when they are accurately known. It is common to assume the weight of earth filling as 100 pounds per cubic foot, and that of concrete or other masonry as 150 pounds per cubic foot.

In open-spandrel arches the dead weights act vertically through the columns or walls supporting the floor of the roadway, and may be readily computed. When the spandrels are filled with earth, each section of the arch ring is assumed to carry the weight of the filling and roadway vertically above it.

The earth pressures upon the inclined back of the arch ring are not actually vertical, but may have certain horizontal components. For arches of small rise, these horizontal pressures are small and may be neglected, but when the rise of the arch is large, the horizontal earth thrusts may be considerable, and should be taken into account, although their omission is usually an error on the safe side. While the amount of horizontal earth pressure cannot be exactly determined, it is usual to use Rankine's minimum value for unit horizontal earth pressure in terms of the unit vertical pressure, which is

$$H=V\frac{1-\sin\phi}{1+\sin\phi},$$

in which H is the horizontal and V the vertical unit pressure, and ϕ the angle of friction for the earth. For ordinary earth filling, this would make the unit horizontal pressure at any point approximately one-fourth of the unit vertical pressure at the same point, the probability being that a horizontal pressure of at least this amount may always be developed.

The methods used for determining pressures upon retaining walls evidently are not applicable to this case. The actual horizontal earth pressure may vary within rather wide limits, and cannot be accurately determined. In retaining-wall design, the maximum earth thrust which may come against the wall is computed, while for the arch we need to know the minimum horizontal pressure which may be relied upon to help sustain the arch. That the actual pressure may sometimes be considerably more than the computed minimum is quite probable.

When an arch carries a continuous masonry wall, as in an opening through the wall of a building, or the spandrel wall at the end of an arch bridge, the wall itself would arch over the opening and be capable of self-support if the arch were removed. The load upon the

arch would therefore be only that due to a triangular piece of wall immediately above the arch as in the case of a stone lintel. (See Section 55).

ART. 48. DESIGN OF VOUSSOIR ARCHES

187. Methods of Design.—In designing masonry arches, the form and dimensions of the arch ring are first assumed and the stability of the arch, as assumed, is then investigated. The graphical method of the investigation is commonly employed, a line of pressure (see Section 183) being drawn and the maximum unit compression computed. Stability requires that the line of pressure remain within the middle third of the arch ring and that the unit compression does not exceed a safe value. If the first assumptions are not satisfactory the shape or dimensions of the arch ring may be modified and the new assumptions tested as before.

Arches subjected to the action of moving loads should be tested for conditions of partial loading, which may cause unsymmetrical distortion of arch ring, as well as for full load over the whole arch. For ordinary loadings and spans of moderate length, it is usually sufficient to draw the line of pressure for arch fully loaded and with live load extending over half the arch, but in large and important structures, or those with unusual loadings, it may be desirable to test the arch ring with live loads in other positions which seem likely to produce maximum distortions of the line of pressure.

188. Thickness of Arch Masonry.—The choice of dimensions for the trial arch ring is necessarily based upon judgment founded upon knowledge of the dimensions of existing arches, which are found to differ widely, and rules have been formulated by several authorities for the purpose of aiding in selecting the dimensions.

Crown Thickness.—Several different formulas have been proposed for determining the thickness at the crown. *Trautwine's formula* for the depth of keystone of first-class cut-stone arches, whether circular or elliptical, is

$$\text{Depth of key in feet} = \frac{\sqrt{\text{Radius} + \text{half span}}}{4} + .2 \text{ foot.}$$

For second-class work this depth may be increased about one-eighth part; or for brick or rubble about one-third.

Rankine's formula for the depth of keystone for a single arch is

$$\text{Depth in feet} = \sqrt{.12 \text{ radius.}}$$

This gives results which agree fairly well with Trautwine's formula. For an arch of a series, Rankine also recommends

$$\text{Depth in feet} = \sqrt{.17 \text{ radius.}}$$

These formulas make the thickness depend upon the span and rise of the arch without regard to the loading. They agree fairly well with many examples of existing arches, but make the thickness rather large for arches of moderate span.

Douglas Formulas:— In Merriman's American Civil Engineer's Pocket Book, Mr. Walter J. Douglas gives the following rules for thickness at crown:

TABLE LIII.—THICKNESS IN FEET AT CROWN FOR HIGHWAY ARCHES

Kind of Masonry.	Span in Feet $=L$.			
	Under 20.	20 to 50.	50 to 150.	Over 150.
First-class ashlar....	$0.04(6+L)$	$0.020(30+L)$	$0.00012(11000+L^2)$	$0.018(75+L)$
Second-class ashlar or brick..........	$0.06(6+L)$	$0.025(30+L)$	$0.00016(11000+L^2)$	$0.025(75+L)$
Plain concrete.......	$0.04(6+L)$	$0.020(30+L)$	$0.00014(11000+L^2)$	$0.020(75+L)$
Reinforced concrete..	$0.03(6+L)$	$0.015(30+L)$	$0.00010(11000+L^2)$	$0.016(75+L)$

For railroad arches, add 25 per cent for arches 20- to 50-feet span, 20 per cent for 50 to 150 feet, and 15 per cent for those over 150 feet.

These formulas give smaller thickness for highway arches of short span than Trautwine's and do not vary the thickness with the rise of the arch.

Thickness at Skewback.—If the arch ring be made of uniform thickness, the unit pressure at the ends will be greater than at the crown. The pressure may often be made fairly uniform by making the thickness at any radial joint equal to the crown thickness times the secant of the angle made by the joint with the vertical.

In the American Civil Engineer's Pocket Book, Mr. Douglas recommends that the thickness at the springing line of a masonry arch be obtained by adding the following percentages to the crown thickness:

(1) Add 50 per cent for circular, parabolic, and catenarian arches having a ratio of rise to span less than one-quarter.

(2) Add 100 per cent for circular, parabolic, catenarian, and three-centered arches having a ratio of rise to span greater than one-quarter.

(3) Add 150 per cent for elliptical, five-centered and seven-centered arches.

Mr. Douglas recommends that the top thickness of abutments be assumed at five times the crown thickness. For a pier between

arches in a series he suggests a thickness at top of three and one-half times the crown thickness, but places an abutment at every third or fifth span.

Trautwine gives a method for design of abutment, approximately as follows (see Fig. 104):

Thickness at springing line in feet

$$a-b=\frac{\text{Radius}}{5}+\frac{\text{Rise}}{10}+2 \text{ feet.}$$

Lay off $a-c=$ rise, and $c-d=\text{ab}+\frac{\text{span}}{24}$.

Continue bd downward to bottom of abutment, and upward a distance $be=\text{rise}/2$. From e draw a tangent $e-f$ to the extrados.

It is also required that the thickness at bottom of abutment gh, shall not be less than two-thirds of the height ag.

If the abutment is of rough rubble, 6 inches is added to the thickness to insure full thickness in every part.

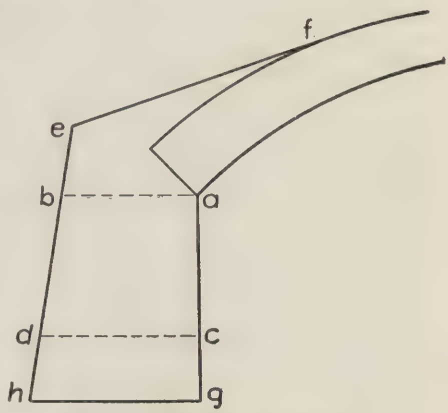


Fig. 104.—Arch Abutment.

These rules usually give arches which are amply strong for heavy railway service and heavier than necessary for highway bridges. For structures of small span, however, when voussoir or plain concrete arches are used, the saving effected by paring them down is small, and rather heavy work is common practice.

189. Investigation of Stability.—After assuming dimensions for the arch ring and abutments, the stability of the arch is investigated by the methods outlined in Section 156. The stability of the abutment is tested by continuing the line of thrust and determining whether it cuts the base of the abutment within the middle third. The sufficiency of the foundation for the abutment must also be examined and footings provided which will properly distribute the pressure over the soil upon which it is to be placed. The following example will outline the method of procedure.

Example.—A highway arch is to have a span of 40 feet and a rise of 10 feet. It is to carry a moving load of 200 pounds per square foot.

The depth of fill at crown is 2 feet. The weight of earth fill is 100 and of masonry 150 pounds per cubic foot.

We will try a segmental arch. By the Douglas rule, the thickness at crown would be 1.4 feet. By Trautwine's formula, it would be 1.95 feet. Make the crown thickness 18 inches. By the Douglas rule the thickness at springing would be between 1.5 and 2 times the crown thickness. We will try 30 inches. Draw the arch ring as shown in Fig. 105, and divide it into equal parts by radial lines. The line *z–t* represents the roadway and verticals from the points where the radial divisions cut the extrados divide the earth fill into parts resting upon the sections of the arch ring. These loads, including the weights of the sections of arch ring, are now computed, and their vertical lines of action determined.

In finding the loads, it is often convenient to draw the *reduced load contour*, which is obtained by reducing the height of the sections so that the volume contained by them may be considered to weigh the same per unit as the arch ring. Thus if the earth fill weighs 100 pounds and masonry 150 pounds per cubic foot, the height *ax* is made two-thirds of *az*, and the other verticals are reduced proportionately, giving the volume *a–x–u–g*, which has the same weight at 150 pounds as the earth fill at 100 pounds. In the same way *x–y–v–u* represents the live load which would come upon half the arch ring reduced to 150 pounds per cubic foot. In the example, the loadings given represent live load extending over the left half of the arch, dead load only upon the right half.

The horizontal thrusts against the arch ring are sometimes computed by assuming that the unit horizontal thrust bears a definite proportion (usually about one-quarter) to the unit vertical thrust. Thus in Fig. 105, if the vertical load upon the section *a–b* is 5085 pounds the horizontal component of the load on the section is

$$\frac{5085}{4} = 1200 \text{ pounds, approximately}$$

In the example, the horizontal components upon the two lower divisions on each side are used, those upon the upper divisions being too small to affect the results appreciably. The horizontal components of the loads are not usually considered in a problem of this kind unless the rise of the arch is large as compared with the span.

Having computed the loads, a line of pressure may now be drawn through any three points in the arch ring. Assume that it is to pass through the lower third point of the joint *a* on the loaded side, the

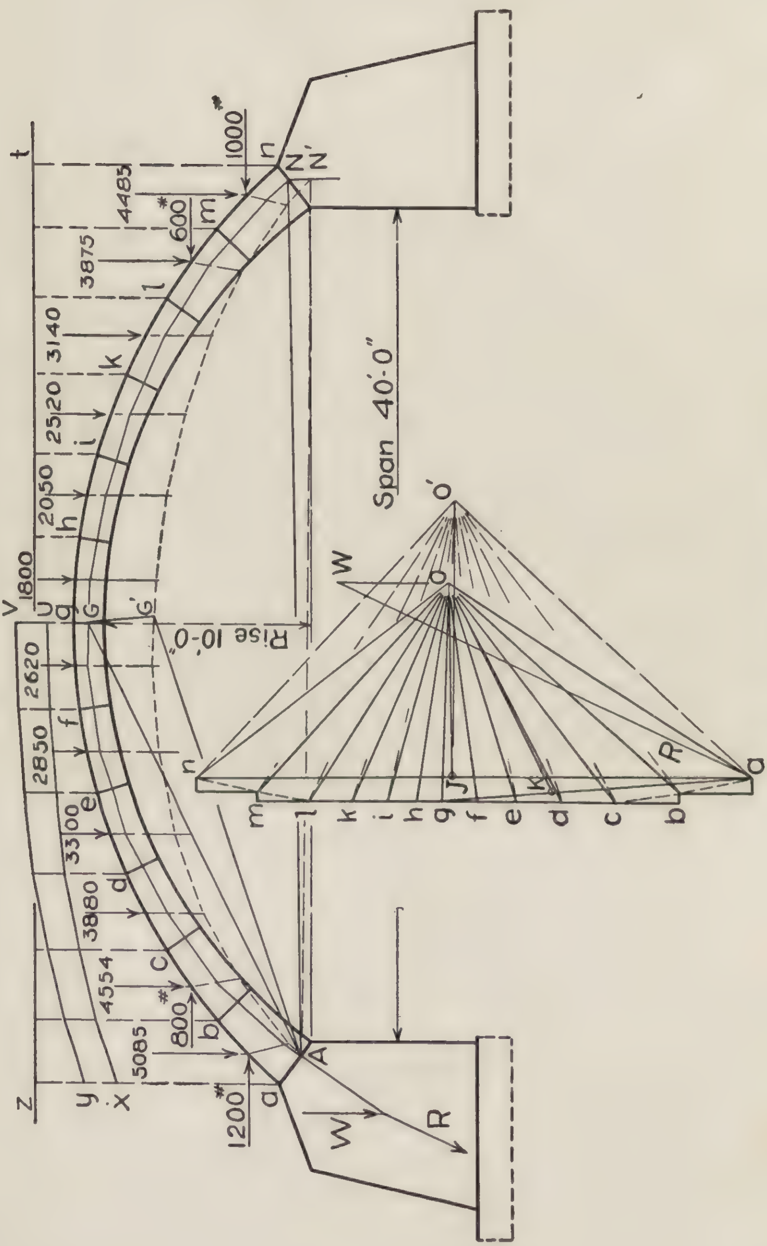


Fig. 105.—Voussoir Arch Design.

middle point at the crown, and the upper third point at the joint n on the unloaded side.

The load line is first plotted on a convenient scale by laying off the loads which come upon the various sections in succession, $n–m$, $m–l$, etc.; $n–a$ is now the resultant of all the loads upon the arch ring. A pole O' is assumed and the strings $O'a$, $O'b$, etc., drawn.

The equilibrium polygon, shown in broken lines, may now be drawn. Starting from A, the lower third point on joint a, with a line parallel to the string $O'a$ to an intersection with the line of action of the load upon the section $a–b$. From this intersection, draw a line parallel to $O'b$ to intersection with the line of action of the load on $b–c$, and continue it until a parallel to $O'n$ is intersected in N' upon a line through N parallel to the resultant $n–a$.

Connect N' with A, and from O' draw a line parallel to $N'–A$ to intersection J with the resultant $n–a$ of the loads, thus dividing the resultant into two reactions, $n–J$ and $J–a$, which would exist at the ends of the span if the horizontal thrust of the arch be neglected. Join the points A and N and from J draw a line parallel to $A–N$. A pole lying upon this line will give an equilibrium polygon passing through A and N.

The distance of the pole from J must now be determined to cause the equilibrium polygon to pass through the middle of the crown joint. The line $g–a$ in the force polygon, is the resultant of the loads upon the left half of the arch. From the middle of the crown section, draw $G–G'$, parallel to $g–a$, to intersection with the trial equilibrium polygon. Connect $A–G'$ and $A–G$. From O' draw $O'k$ parallel to $G'A$ to intersection with $g–a$ in k, and from k draw $k–O$ parallel to AG. The point O where KO intersects JO is the new pole.

From A, the new line of thrust may now be drawn with sides parallel to the strings, Oa, Ob, etc. This passes through the points G and N.

By inspection we see that the line of thrust, as thus drawn, is everywhere within the middle of the arch ring. The thrust upon the joint at a is represented by the length of the line $O–a = 27000$ pounds, and the maximum unit compression is

$$f_c = \frac{27000}{30 \times 12} \times 2 = 150 \text{ lb./in.}^2$$

The unit compression upon any other joint may be found in the same manner.

The resultant pressure R upon the base of the abutment is found by combining the weight W of the abutment with the thrust $O–a$ of

the arch against the abutment. The footing under the base of the abutment should be so designed as properly to distribute the load over the foundation soil.

ART. 49. THE ELASTIC ARCH

190. Analysis of Fixed Arch.—Reinforced concrete arches are commonly constructed as solid curved beams firmly fixed to the abutments. In analyzing them, it is assumed that the abutments are immovable and the ends of the arch firmly held in their original positions. Let Fig. 106 represent the left half of an arch, fixed in position at the end G–H, and carrying loads which produce thrusts and bending moments throughout the arch ring.

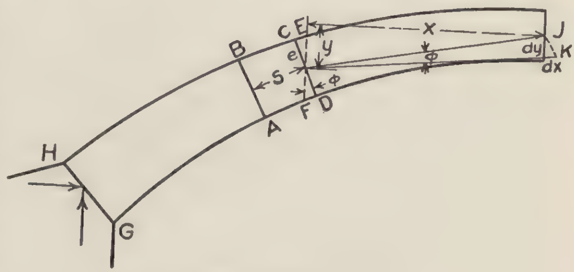


Fig. 106.—Elastic Arch.

The arch may be considered as made up of a number of small divisions. Suppose $ABCD$ to be one of these divisions, small enough so that its ends are practically parallel and its section area constant. The loads upon the arch bring a bending moment upon the division $ABCD$, which causes the end CD to take the position EF.

Let M = bending moment on the division;
s = length of division, $AD=BC$;
e = distance from center of section to outside fiber;
ds = elongation of fiber distant e from neutral axis;
f_m = Unit stress upon fiber distant e from neutral axis;
k = angle of distortion, COE;
E = modulus of elasticity of material;
I = moment of inertia of section;
x and y = horizontal and vertical coordinates of center of section, O, with reference to center of crown section, J.

The unit fiber elongation in the division $ABCD$ is $\frac{ds}{s}=\frac{e\cdot k}{s}$.

$$\text{Unit stress, } f_m=\frac{Me}{I} \quad \text{also} \quad f_m=\frac{ds}{s}E=\frac{e\cdot k}{s}\cdot E.$$

Equating these and solving,

$$k=\frac{Ms}{EI}. \qquad (1)$$

If the crown of the arch be free to move, the deflection of $ABCD$ into its new form $ABEF$ will bring the middle point of the arch ring J, to the new position K. Let dx and dy be the horizontal and vertical coordinates of K with respect to J. Then from similar triangles, $JK/OJ=dy/x=k$, and

$$dy=xk=\frac{Mxs}{EI}. \qquad (2)$$

Similarly,

$$dx=yk=\frac{Mys}{EI}. \qquad (3)$$

As the end section GH is fixed in position, the summation of all the angular distortions k, for the left half of the arch gives the distortion at the crown section. The summation similarly of those for the right half must give the same result with opposite sign, or indicating the left and right sides of the arch by the subscripts L and R respectively, and indicating summation by the sign Σ, we have

$$\Sigma k_L=-\Sigma k_R, \text{ also } \Sigma dy_L=\Sigma dy_R \text{ and } \Sigma dx_L=-\Sigma dx_R.$$

Substituting for these distortions, their values as found above, we have for a symmetrical arch:

$$\Sigma\frac{M_Ls}{EI}=-\Sigma\frac{M_Rs}{EI}, \qquad (4)$$

$$\Sigma\frac{M_Lxs}{EI}=\Sigma\frac{M_Rxs}{EI}, \qquad (5)$$

and

$$\Sigma\frac{M_Lys}{EI}=-\Sigma\frac{M_Rys}{EI}. \qquad (6)$$

If the lengths of the divisions of the arch ring be made directly proportional to the corresponding values of the moment of inertia, $\frac{s}{I}=$constant, the terms $\frac{s}{EI}$ in Equations (4), (5), and (6) are constant and may be eliminated, and we have,

$$\Sigma M_L=-\Sigma M_R \qquad (7)$$

$$\Sigma M_Lx=\Sigma M_Rx \qquad (8)$$

$$\Sigma M_Ly=-\Sigma M_Ry \qquad (9)$$

Figure 107 represents a symmetrical arch divided into parts the lengths of which are directly proportioned to the moments of inertia of the cross-sections at their middle points. $s/I=\text{constant}$. Suppose the arch to be cut at the crown and the separate halves supported by introducing the stresses acting through the crown section as exterior forces. These may be resolved into a horizontal thrust, a vertical shear and a bending moment.

H_c = horizontal thrust at crown;
V_c = vertical shear at crown;
M_c = bending moment at crown.

V_c is considered to be positive when acting in the direction indicated by the arrows. Moments are taken as positive when they

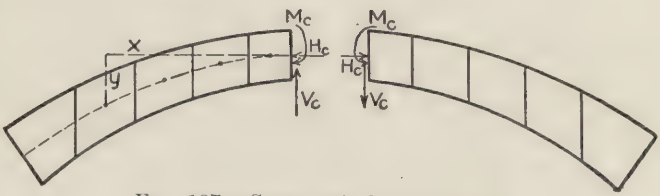


FIG. 107.—Symmetrical Elastic Arch.

produce compression on the upper and tension on the lower side of the section.

Let M_L = bending moment on mid-section of any division in left half of arch;
M_R = bending moment on mid-section of any division in right half of arch;
m_L = moment at middle of any division in left half, caused by external loads between that division and the crown section;
m_R = moment at mid-section of any division in right half, caused by external loads between that section and the crown section;
x and y = coordinates of middle point of any division with respect to middle of crown section.
n = number of divisions in the half span.

The bending moment at any section of a beam is equal to the moment at any other section plus the moments of the intermediate loads about the center of the section. Therefore,

$$M_L=M_c+V_cx+H_cy-m_L. \quad \quad (10)$$

$$M_R=M_c-V_cx+H_cy-m_R. \quad \quad (11)$$

Substituting these values in Equations (7), (8) and (9), we have,

$$2nM_c+2H_c\Sigma y-\Sigma m_L-\Sigma m_R=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$2V_c\Sigma x^2-\Sigma m_L x+\Sigma m_R x=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$2M_c\Sigma y+2H_c\Sigma y^2-\Sigma m_L y-\Sigma m_R y=0. \quad . \quad . \quad . \quad .(14)$$

Solving for the thrusts and moments at the crown

$$H_c=\frac{n\Sigma(m_L y+m_R y)-\Sigma(m_L+m_R)\Sigma y}{2n\Sigma y^2-2(\Sigma y)^2}, \quad . \quad . \quad . \quad (15)$$

$$V_c=\frac{\Sigma m_L x-\Sigma m_R x}{2\Sigma x^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$M_c=\frac{\Sigma m_L+\Sigma m_R-2H_c\Sigma y}{2n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

In analyzing an arch by this method, the arch is first divided into a number of parts in which s/I is a constant. The loads upon the divisions are then found and mL and mR computed for the several centers of division. The values of H_c, V_c and M_c may then be found from Formulas (15), (16) and (17), after which the line of thrust may be drawn beginning with the known values of H_c and V_c at the crown. The moment M_c is due to the eccentricity of the thrust at the crown, and the point of application for H_c may be found by dividing M_c by H_c. This gives the vertical distance of H_c from the center of gravity of the crown section. For M_c positive, H_c is above, and for M_c negative, H_c is below the center of section.

The thrust at any section of the arch may be obtained from the thrust diagram as in the voussoir arch. The bending moment at any section is the moment of the thrust upon the section about the center of gravity of the section. The bending moment at any section may also be obtained by the use of Formula (10) or (11).

In analyzing an arch bridge subject to moving loads, it is necessary to assume different conditions of loading and find the thrust and moments resulting from each. For a small arch, it is usually sufficient to make the analysis for arch fully loaded and for moving load over one-half the arch. The maximum stresses will be more accurately determined by dividing the moving load into thirds, and determining the stresses with span fully loaded, one-third loaded, two-thirds loaded, center third loaded, and with two end thirds loaded. If complete analysis be made for the arch under dead load alone, for live load over one end third, and live load over the middle third, the results of these three analyses may be combined to give the five conditions of loading above mentioned.

191. Effect of Changes of Temperature.—A rise in temperature tends to lengthen and a fall in temperature to shorten the span. If the ends of the arch ring are rigidly held in position, the tendency to change in length is resisted by moments and horizontal thrusts at the supports, which produce moments and thrusts throughout the arch ring.

If the arch ring were not restrained, a rise in temperature of t degrees would cause an increase in length $=CtL$; L being the length of span and C the coefficient of expansion of the material. The moments throughout the arch ring are therefore those which correspond to an actual change in length of span $=CtL$, or from Formula (3)

$$\Sigma dx=\Sigma\frac{Mys}{EI}=CtL.$$

From this, for a symmetrical arch ring

$$\Sigma\frac{M_Lys}{EI}=\Sigma\frac{M_Rys}{EI}=\frac{CtL}{2}, \quad \ldots \ldots \quad (18)$$

and

$$\Sigma M_L=-\Sigma M_R. \quad \ldots \ldots \quad (19)$$

As there are no exterior loads, m_L, and V_c are each equal to zero, and Formula (10) becomes $M_L=M_c+H_cy$. Substituting this in (18) and (19) and solving, we have

$$H_c=\frac{EI}{s}\cdot\frac{CtLn}{2n\Sigma y^2-2(\Sigma y)^2} \quad \ldots \ldots \quad (20)$$

$$M_c=-\frac{H_c\Sigma y}{n}. \quad \ldots \ldots \quad (21)$$

The line of thrust consists of a single force H_c, and is applied on a horizontal line at a distance, $e=\Sigma y/n$, below the middle of the crown section. The bending moment at any section due to H_c is

$$M_L=H_c\left(y-\frac{\Sigma y}{n}\right). \quad \ldots \ldots \quad (22)$$

The direct thrust upon any section of the arch ring is the component of H_c normal to the section.

For temperatures below the normal, H_c will be negative and may be found from Formula (20) by giving t the negative sign.

192. Effect of Direct Thrust.—Axial thrusts on the arch ring produce compressive stresses on the various sections and also tend to shorten the arch ring. As the span length does not change, this

tendency to become shorter causes stresses in the arch ring in the same manner as does lowering temperature. If f_c lb./in.^{2} be the average unit compression due to axial thrust, the arch ring if unrestrained would be shortened an amount $dx = f_c L/E$, from which,

$$H_c = -\frac{I}{s} \cdot \frac{f_c L n}{2n\Sigma y^2 - 2(\Sigma y)^2}$$

and

$$M_c = \frac{H_c \Sigma y}{n}. \qquad (23)$$

As the unit stress f_c is not uniform through the arch ring, a value obtained by finding the stresses at several points and averaging them may be used.

The stresses due to shortening of the arch ring are comparatively small and are often neglected in the analysis of ordinary arches; in some instances, however, they may be considerable.

ART. 50. DESIGN OF REINFORCED CONCRETE ARCH

193. Selection of Dimensions.—In designing an arch, it is necessary to first assume dimensions for the arch ring, and then investigate for the strength of the arch and the suitability of the assumed dimensions to the conditions of service. The methods of investigation usually employed are indicated in Art. 48. The investigation will show whether changes in form or thickness should be made in the arch ring. The shape of the arch should be such as to fit as closely as possible the lines of pressure, and the thickness should be such as to give allowable stresses under all conditions of loading.

Example.—As an illustration of the method of investigation, we will assume an arch of 60 feet clear span and 12 feet rise, to carry a live load of 100 pounds per square foot of road and a solid spandrel filling, 2 feet deep over the crown, weighing 100 pounds per cubic foot. For ordinary arches with solid spandrel filling, a three-centered intrados, with radii at the sides from three-fifths to three-fourths that at the crown, is apt to give better results than a segmental intrados. We will use an intrados composed of three arcs tangent to each other at the quarter points with radii of 52.5 feet and 31.25 feet respectively (see Fig. 109).

Weld's formula[1] for the crown thickness is $t = \sqrt{L} + \frac{L}{10} + \frac{W}{200} + \frac{W'}{400}$

in which

[1] Engineering Record, Nov. 4, 1905.

t = the crown thickness in inches;
L = clear span in feet;
W = live load in pounds per square foot;
W' = weight of fill at crown per square foot.

Assume a crown thickness of 18 inches. The thickness of the arch ring should increase from the crown to the springing line; the thickness at the quarter point may be made a little greater than that at the crown (about $1\frac{1}{4}$ to $1\frac{1}{3}$ times). Assume a thickness at the quarter point of 23 inches, and at the springing line of 42 inches. The extrados will now consist of three arcs tangent to each other at the quarter points and giving the desired thicknesses. Figure 109 shows the arch ring as assumed.

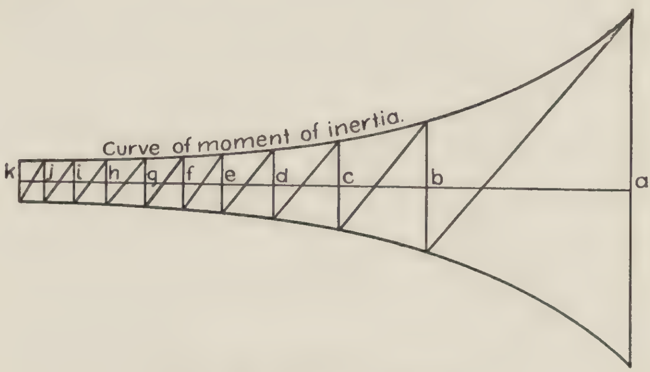


FIG. 108.—Division of Arch Ring.

The reinforcement may be assumed at from .4 to .7 per cent of the area of the section at the crown, to be placed at both extrados and intrados. We will use $\frac{1}{2}$-inch round bars spaced 7 inches apart.

194. Division of Arch Ring.—Having chosen the form and dimensions of the arch ring, it is necessary to divide the ring so that the lengths of the divisions shall be directly proportional to the moments of inertia of their mid-sections, s/I = constant. This may be done by trial, assuming a division next the crown, determining the value of s/I for the assumed division, and finding the corresponding lengths of other sections toward the abutment. Then changing the first assumption as may seem necessary to make the division come out properly at the abutment.

More easily, the division may be made graphically as shown in Fig. 108. The line a–k is laid off equal in length to half the arch axis (34.52 feet). The moments of inertia are then computed at several points

along the arch axis, and their amounts laid off normally to the line a–k, and the curves of moment of inertia drawn through the points so located.

A trial diagonal is then drawn from A to intersection with the curve in the point B. A vertical from B is drawn to intersection with the upper curve, and a second diagonal parallel to A–B, cutting the lower curve in C. Continue successive diagonals and verticals until the end k is reached. If these do not come out accurately at the end k the inclination of the diagonals may be varied until the division of a–k is made into the correct number of parts. This divides a–k into lengths which are proportional to the average of the moments of inertia at the ends of the divisions.

The lengths of the divisions, a–b, b–c, etc., are now transferred to the arch axis. The axis of the arch in Fig. 109 is thus laid off into ten divisions on each side of the crown section. The constant ratio s/I is found to be 5.1, all measurements being taken in feet.

The middle point of the arch axis in each division is now located, and the values of x and y are determined with reference to the middle of the crown section. These values and their squares are tabulated in Table L for use in the computations.

195. Analysis.—If vertical lines be drawn through the points of division of the arch axis, the weight of the portion of masonry and spandrel filling included between each pair of lines may be considered as the dead load resting upon the included division. The live load is similarly divided for the portion of the arch over which it is considered as acting. In Fig. 109 the live load is taken as extending over the left half of the arch, and the loads are as indicated. The values of m_L and m_R are now computed and placed in Table LIV, and include in each instance the moments of all loads between the division considered and the crown section about the center of division. The quantities m_Lx, m_Rx, m_Ly, and m_Ry may now be computed and placed in the table, and the summations of the various columns obtained. These substituted in Formulas (15), (16) and (17) give,

$$H_c=\frac{10(2375207+1988744)-(512291+425771)17.09}{2\times 10\times 80.85-2\times 17.09\times 17.09}=+26715.$$

$$V_c=\frac{10408051-8686133}{2\times 1769.5}=+487 \text{ lb.},$$

$$M_c=\frac{512291+425771-2\times 26715\times 17.09}{2\times 10}=+1247 \text{ ft.-lb.}$$

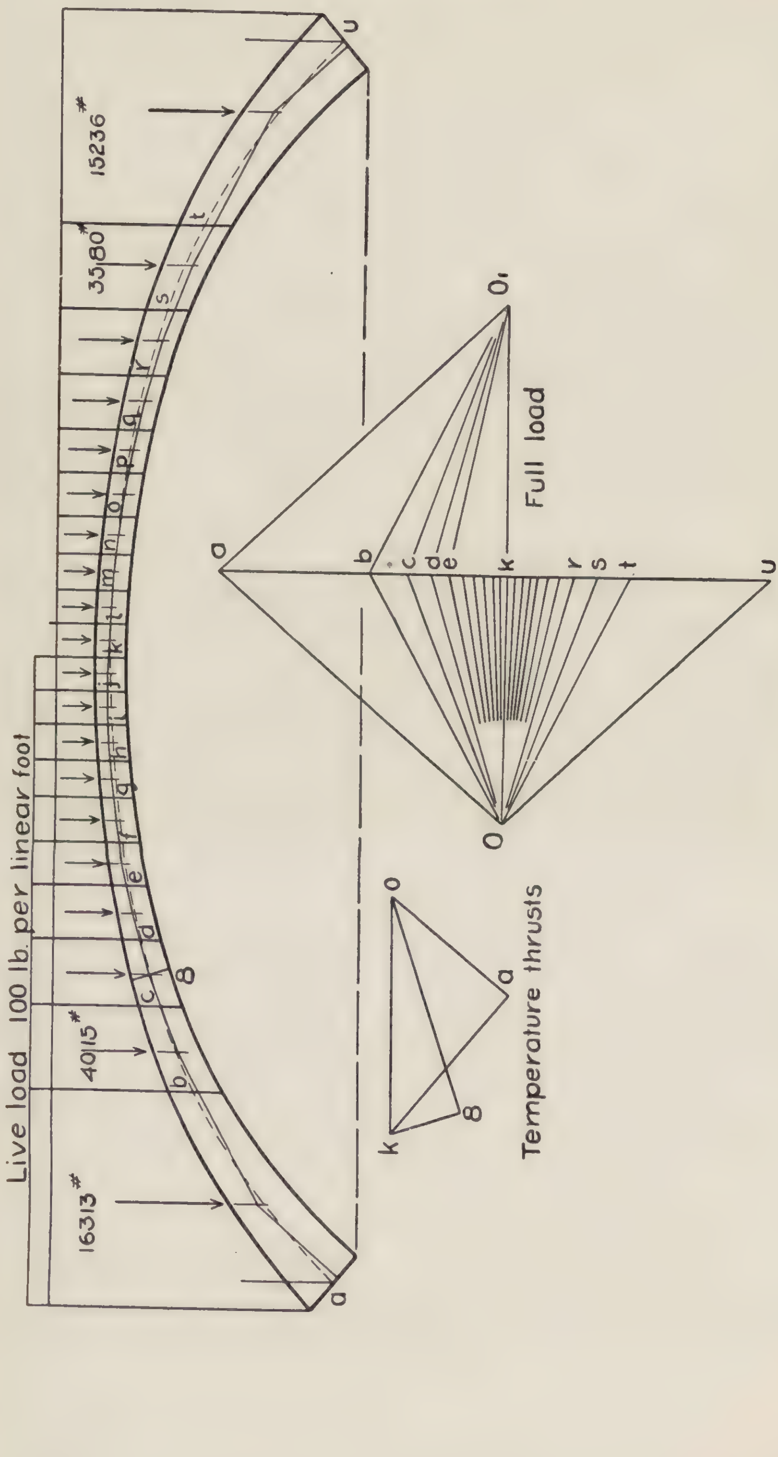


FIG. 109.—Elastic Arch Design.

TABLE LIV.—ORDINATES AND MOMENTS FOR COMPUTATIONS

Points.	x	y	x^2	y^2	m_R	m_L	m_Rx	m_Lx	m_Ry	m_Ly
1	0.84	0.01	0.7	0.00	00	00	00	00	00	00
2	2.56	0.06	6.5	0.00	1,230	1,519	3,149	3,889	74	91
3	4.32	0.16	18.7	0.03	3,812	4,703	16,468	20,317	610	752
4	6.16	0.34	38.0	0.11	7,990	9,841	49,218	60,620	2,717	3,346
5	8.14	0.58	66.4	0.34	14,237	17,502	115,889	142,467	8,257	10,151
6	10.33	0.95	106.9	0.90	23,389	28,670	241,609	296,151	22,210	27,236
7	12.85	1.45	165.4	2.12	37,012	45,195	475,594	580,756	53,667	65,534
8	15.88	2.25	252.5	5.06	58,125	70,608	923,025	1,120,255	130,781	158,868
9	19.75	3.58	390.4	12.85	93,992	113,278	1,856,342	2,237,240	336,491	405,535
10	26.91	7.71	724.0	59.44	185,984	220,972	5,004,839	5,946,356	1,433,937	1,703,694
Σ	107.74	17.09	1769.5	80.85	425,771	512,291	8,686,133	10,408,051	1,988,744	2,375,207

Having obtained the thrusts and moment at the crown, we may now proceed to find the thrusts and moments at any other section desired. The thrusts are obtained graphically by drawing the line of pressure. The load line is first constructed, as shown by the vertical line a–u. V_c and H_c are laid off from the mid-point k of this line, thus locating the pole O. The force diagram is then completed by drawing connections from O to the extremities of the various loads.

The equilibrium polygon is now drawn, beginning with the crown thrust $(O–K)$, the point of application of which is at a point $e=\frac{+1247}{+26715}=+0.05$ foot above the center of the crown section. The thrust upon each section is now shown in amount by the length of the line from O to the division in the load line, and its line of action by the corresponding line in the equilibrium polygon (or line of pressure).

The bending moment at any section may be found by multiplying the thrust upon the section by its perpendicular distance from the center of section, or it may be computed by the use of Formula (10) or (11). Usually the formula is employed and the measurement of the eccentricity used as a check.

196. Computation of Stresses.—*At the crown*, the stress due to direct thrust is

$$f_c=\frac{\text{thrust}}{\text{area}}=\frac{26715}{18\times 12}=+124 \text{ lb./in.}^2;$$

for the moment,

$$f_c=\frac{M_c}{I}=\frac{1247\times 12\times 9}{6388}=21 \text{ lb./in.}^2$$

This gives

$$124+21=+145 \text{ lb./in.}^2 \text{ at the extrados}$$

and

$$124-21=+103 \text{ lb./in.}^2 \text{ at the intrados.}$$

At the left support, by Formula (10),

$$M_L=1247+26715\times 11.7+487\times 31.25-349930=-20899 \text{ ft.-lb.}$$

and the measured thrust $=40300$ pounds, giving an eccentricity of $-20899/40300=-.5$ foot. This may be checked by measurement. Then for thrust,

$$f_c=\frac{40300}{42\times 12}=+80 \text{ lb./in.}^2$$

and for moment,

$$f_c=\frac{20899\times 12\times 21}{79098}=-67 \text{ lb/in.}^2,$$

giving at the extrados

$$f_c=80-67=+13 \text{ lb./in.}^2$$

and at the intrados

$$f_c = 80 + 67 = +147 \text{ lb./in.}^2$$

At the right support, by Formula (11),

$$M_R = 1247 + 26715 \times 11.7 - 487 \times 3125 - 303226 = -4632 \text{ ft.-lb.}$$

The measured thrust is 39,200, and the eccentricity

$$= -\frac{4632}{39200} = -.12 \text{ foot.}$$

For thrust, $f_c = \dfrac{39200}{42 \times 12} = 78$ lb./in.^{2}, and for moment, $f_c = -15$ lb./in.^{2} This gives at extrados $f_c = 63$ lb./in.^{2}, and at intrados $f_c = +93$ lb./in.^{2} *At point* 8_L,

$$M_L = 1247 + 26715 \times 2.25 - 487 \times 15.88 - 70608 = -1520 \text{ ft./lb.}$$

Thrust $= 27600$ pounds

$$f_c = \frac{27600}{21 \times 12} = 110 \text{ lb./in.}^2$$

For moment, $f_c = \dfrac{-1520 \times 12 \times 10.5}{10964} = 17$ lb/in.^{2};

this gives at extrados $f_c = 93$ lb./in.^{2}, and at intrados $f_c = 127$ lb./in.^{2}

At point 8_R, in the same manner, we have at extrados, $f_c = 58$ lb./in.^{2} and at intrados $f_c = 150$ lb./in.^{2}

Full Load.—When the live load extends across the whole span of the arch, the loading is symmetrical and the values given in Table LIV for m_R become equal to those for m_L. We then have

$$V_c = 0.$$

$$H_c = \frac{2 \times 10 \times 2375207 - 2 \times 512291 \times 17.09}{2 \times 10 \times 80.85 - 2 \times 17.09 \times 17.09} = 29030 \text{ pounds}$$

$$M_c = \frac{2 \times 512291 - 2 \times 29030 \times 17.09}{2 \times 10} = +1662 \text{ ft.-lb.}$$

The force diagram is now drawn for one-half of the arch, and the equilibrium polygon may be drawn as in the case of partial loading. To avoid confusion it is not drawn in Fig. 109. The stresses in the crown section due to this loading are

$$f_c = \frac{29030}{18 \times 12} = 131 \text{ lb./in.}^2 \quad \text{and} \quad f_c = \frac{1662 \times 12 \times 9}{6388} = +28 \text{ lb./in.}^2$$

This gives at extrados, total $f_c=131+28=+159$ lb./in.^{2}, and at intrados, total $f_c=131-28=103$ lb./in.^{2}

At section 8, $M=1662+29030\times2.25-70608=-3629$ ft.-lb. The thrust is 30,350 pounds, and the resulting unit stresses at extrados $f_c=120-36=84$ lb./in.^{2} and at intrados $f_c=156$ lb./in.^{2}

At the support in the same manner, the thrust is 42,600 pounds, and the moment, $M=1662+29030\times11.7-349930=-8617$ ft./lb. from which at extrados, $f_c=84-27=57$ lb./in.^{2}, and at intrados $f_c=111$ lb./in.^{2}

Temperature Stresses.—If we assume that a rise in temperature of 20° above the normal may take place, Formula (20) gives

$$H_c=\frac{288000000}{5.1}\times\frac{.0000055\times20\times62.5\times10}{2\times10\times80.85-2\times17.09\times17.09}=+3770 \quad \text{pounds}$$

and (21),

$$M_c=\frac{-3770\times17.09}{10}=-6448 \text{ ft.-lb.} \qquad e=-\frac{6448}{3770}=-1.71 \text{ feet.}$$

The equilibrium polygon is a horizontal line 1.71 feet below the center of the crown section, and the bending moment at any section of the arch ring is equal to 3770 times the vertical distance from the center of section to the line of thrust. At point 8 the moment due to change of temperature is

$$3770\times(2.25-1.71)=+2037 \text{ ft.-lb.}$$

and at point a,

$$M=3770\times(11.7-1.71)=37{,}696 \text{ ft.-lb.}$$

The stresses at the crown section are, at extrados

$$f_c=17-112=-95 \text{ lb./in.}^2,$$

and at intrados

$$f_c=17+112=+129 \text{ lb./in.}^2$$

The normal thrust on section at point 8 is the component of H_c normal to the section, given in diagram in Fig. 109, $=3345$ pounds. At section a, thrust $=2510$. These thrusts and moments give at 8, for extrados $f_c=14+24=38$ lb./in.^{2}; for intrados, $f_c=14-24=-10$ lb./in.^{2} at support; extrados $f_c=+127$ lb./in.^{2}; intrados, $f_c=-113$ lb./in.^{2}

For a fall in temperature the stresses are equal and opposite to those for rising temperature.

Arch Shortening.—The effect of direct thrust in shortening the span of the arch, taking average unit compression as 100 lb./in.^{2}

average of stresses at crown, point 8 and support under one-half live load by Formula (23),

$$H_c = \frac{1}{5.1} \cdot \frac{100 \times 144 \times 62.5 \times 10}{1033} = -1710 \text{ lb}$$

This is applied on the same line as the temperature thrust and the stresses are therefore equal to 1710/3770 = .45 of those for falling temperature.

Table LV shows the computed stresses upon the sections at crown, at point 8 and at supports. Examination of this table shows that the unit compression is nowhere excessive. Tension of 34 lb./in.^{2} occurs at the intrados in the crown section at low temperature. This is too small to cause cracking in the reinforced section. The tension of 150 lb./in.^{2} at the extrados of the support section would possibly crack the concrete. The compression at the intrados under the same conditions would be 331 lb./in.^{2}, and the reinforced section would be capable of bearing the load if the steel be assumed to carry all the tension. It might be desirable, however, to introduce additional reinforcement at this point to lessen the unit tension in the steel and prevent cracking, and these negative stresses might also be eliminated by slightly modifying the form of the arch, increasing the radius at the crown and decreasing those at the ends, although the form as shown agrees fairly well with the lines of thrust.

TABLE LV.—STRESSES IN ARCH SECTIONS, LB./IN.^{2}

Character of Load.	Crown.		Point 8_R.		Point 8_L.		Support R.		Support L.	
	Extr.	Intr.	Extr.	Intr.	Extr.	Intr.	Extr.	Intr.	Extr.	Intr.
Dead and one-half live load.......	+145	+103	+58	+150	+93	+127	+ 63	+ 93	+13	+147
Dead and full live load..........	+159	+103	+84	+156	+84	+156	+ 57	+111	+57	+111
High temperature	− 95	+129	+38	− 10	+38	− 10	+127	−113	+127	−113
Low temperature.	+129	− 95	−10	+ 38	−10	+ 38	−113	+127	−113	+127
Arch shortening..	+ 58	− 42	− 5	+ 17	− 5	+ 17	− 50	+ 57	− 50	+ 57
Maximum stresses	+346	+180	+117	+211	+126	+211	+140	+295	+184	+337
Minimum stresses	+108	− 34	+ 43	+157	+ 69	+134	−106	+ 37	−150	+ 55

ART. 51. TYPES OF CONCRETE ARCHES

197. Arrangement of Spandrels.—In ordinary bridges of short span, solid arches with filled spandrels are commonly employed, as shown in the example of the last article. In such arches, spandrel walls are used to retain the filling above the arch ring. These are usually light reinforced walls and must be designed to resist the side pressure of the filling with its live load. When the depth of filling is considerable, a thin wall with counterforts is often employed.

Figure 110 shows a concrete arch with filled spandrels.

For larger bridges, and where heavy filling would be required, open spandrels are often used. In these, the floor is usually carried by slabs and the loads are brought vertically upon the arch ring by cross walls. In such arches, the dead loads with their lines of action are definitely known, and the use of influence lines gives an accurate method of determining the effect of moving loads at any point of the road surface.

Figure 111 shows a concrete arch bridge with open spandrels.

In a large open spandrel arch, the arch ring, instead of being solid, is frequently composed of two or more longitudinal ribs. The bridge floor is supported by beams and slabs, and the load transferred to the ribs by a series of columns. The determination of stresses is made in the same manner as for the solid arch, the whole section of the arch rib being used to carry loads brought by the columns, instead of determining loads and sections for a 1-foot slice of the arch ring. The loads must be brought upon the ribs axially, so as to produce on horizontal bending moment, and the width of the rib must be sufficient to enable it to act as a column between points of support. The width may increase from the crown to the support so as to maintain a proper relation between width and depth. Figure 112 is a photographic view of a concrete arch bridge showing the use of longitudinal ribs.

198. Methods of Reinforcement.—There are several methods of arranging the reinforcement in concrete arches. Numerous patented systems are more or less in use, while many designers place reinforcing bars in any way that seem to best meet their needs without following any particular system.

The Monier system was the earliest type invented, and consists in placing wire netting near the surfaces of the arch at both intrados and extrados. This system has been quite largely used in Europe.

Fig. 110.—Concrete Arch Bridge with Filled Spandrels. Columbus, Ohio.
(Courtesy of Prof. Clyde T. Morris, Designing Engineer.)

Fig. 111.—Concrete Arch Bridge with Open Spandrels. Rocky River, Cleveland, Ohio.

(From "Artistic Bridge Design," by courtesy of Mr. H. Grattan Tyrrell.)

The Melan arch has steel ribs, consisting of bent I-beams, or of built-up lattice girders, spaced 2 or 3 feet apart, extending from abutment to abutment, they are self-supporting, and may sometimes be used to carry the forms in placing the concrete for the arch ring. Many Melan arches have been constructed in the United States, most of those built previous to 1900 being of this type. Figure 113 is photographic view of a Melan Arch.

In the *Thacher system*, steel bars are used in pairs, one immediately above the other near the extrados and intrados, the bars being independent of each other. Several modifications of the Thacher

FIG. 112.—Concrete Arch Bridge with Longitudinal Ribs.
(Courtesy of W. J. Watson.)

system have been patented, in which the rods alternate in position or are connected in some way.

In other systems, attempts are made to use single tension bars, bent to pass from the extrados at certain points to the intrados at others as the occurrence of tensions may require.

When the stresses upon the concrete in an arch are kept within proper limits, the unit stresses upon the steel are very small, and more steel must be used than would be necessary to carry the tensions if reasonable unit stresses for the steel could be allowed. The steel is not therefore economically used in carrying stresses. It is rather intended to give added security against unforeseen contingencies,

preventing cracks in the concrete, and guarding against distortions due to slight settlement of foundations or structural defects.

199. Hinged Arches.—Hinges are frequently used in arches for the purpose of making the stresses more nearly determinate, they give definite points through which the line of pressure must pass.

Three-hinged Arches.—Three hinges are usually employed and have the advantage of fixing the line of pressure so as to make it statically determinate. It is assumed that the hinge acts without friction and the line of pressure passes through the center of the hinges. Making this assumption, the horizontal and vertical components of the thrusts at the supports may be determined by means

FIG. 113.—Melan Arch Bridge, Eden Park, Cincinnati, Ohio. Built 1894. (Courtesy of the Cincinnati Chamber of Commerce.)

of moments about the hinge centers. In large arches the hinges have the advantage of eliminating the temperature stresses. Slight settlement of the foundations may occur in hinged arches without sensibly changing the stresses, while the accuracy of the computed stresses in a solid arch is dependent upon the rigidity of the supports. Hinges are usually expensive to construct, and the form of the arch, if economically designed, is not so graceful as that of a solid arch.

Two-hinged Arches.—Two hinges are sometimes used at the supports without the crown hinge. Two points upon the line of pressure are thus fixed and the vertical components of the end thrusts may be found by moments about the hinges. As the span of the arch remains unchanged upon the application of the loads, Formula (14) of Section 190 applies to this case, or

$$2M_c\Sigma y + 2H_c\Sigma y^2 - \Sigma m_L y - \Sigma m_R y = 0. \quad . \quad . \quad . \quad (14)$$

Let R_L = vertical component of the thrust at left support;
m_W = moment at crown of all loads between crown and left support;
$L/2$ = half span of the arch axis;
h = rise of the arch axis.

Then using the same notation as for the solid arch,

$$M_c = R_L L/2 - H_c h - m_W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Substituting this in (14), we have for arch with vertical loading,

$$H_c = \frac{\Sigma m_L y + \Sigma m_R y + 2m_W - R_L L}{2\Sigma y^2 - 2h}. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The thrusts and moments may now be determined in the same manner as for the solid arch.

200. Unsymmetrical Arches.—The formulas of Art. 48 apply only

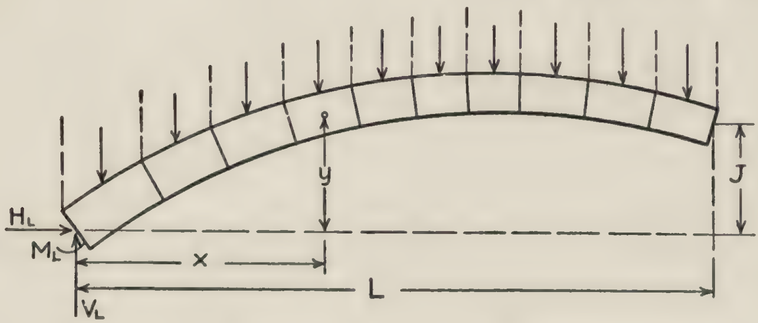


Fig. 114.—Unsymmetrical Arch.

to arch rings which are symmetrical with respect to the crown section. It is frequently necessary or desirable to construct arches which for topographical reasons are not alike upon the two sides of the crown. In these arches if s/I is made constant for the whole arch a division may not come at the crown section, the values of x and y will not be the same upon the two sides of the section nearest the crown, and the formulas produced as in Section 191 become quite complicated.

For this case, the origin of coordinates may be taken at the middle of the lower support, as in Fig. 114.

Let M = bending moment at mid-point of any division;
M_L = bending moment at left support;
V_L = vertical component of thrust at left support;
H_L = horizontal component of thrust at left support;

x and y = coordinates of mid-points of divisions from center of left support;

m = moment at any mid-point of division of all exterior loads between the division and the left support.

Then, using the method of Section 194, we have $\Sigma M=0$, $\Sigma Mx=0$, $\Sigma My=0$, and

$$M=M_L+V_Lx-H_Ly-m. \qquad (25)$$

From this by substitution we obtain

$$nM_L+V_L\Sigma x-H_L\Sigma y-\Sigma m \qquad =0;$$

$$M_L\Sigma y+V_L\Sigma xy-H_L\Sigma y^2-\Sigma my=0;$$

$$M_L\Sigma x+V_L\Sigma x^2-H_L\Sigma xy-\Sigma mx=0.$$

Combining these and solving, we find

$$H_L=\frac{[n\Sigma x^2-(\Sigma x)^2][\Sigma m\Sigma y-n\Sigma my]-[n\Sigma xy-\Sigma x\Sigma y][\Sigma m\Sigma y-n\Sigma mx]}{[n\Sigma x^2-(\Sigma x)^2][n\Sigma y^2-(\Sigma y)^2]-(n\Sigma xy-\Sigma x\Sigma y)^2}. \qquad (26)$$

$$V_L=\frac{[\Sigma m\Sigma y-n\Sigma my]-H_L[n\Sigma y^2-(\Sigma y)^2]}{\Sigma x\Sigma y-n\Sigma xy} \qquad (27)$$

$$M_L=\frac{\Sigma m+H_L\Sigma y-V_L\Sigma x}{n} \qquad (28)$$

Having found the values of H_L, V_L and M_L, the moment at any section may be calculated by Formula 25, and the line of pressure may be drawn, beginning at the left support.

The line of thrust due to change of temperature will be parallel to a line joining the ends of the arch axis.

If L = the horizontal span of the arch axis, and J = the height of its right end above its left end, using the notation of Section 168, we have

$$\Sigma M=0,\ \Sigma Mys/EI=ctL/2,$$

$$M=M_L+V_Lx-H_Ly,\ V_L=H_LJ/L,\ M_L=\frac{H_L\Sigma y-V_L\Sigma x}{n}, \qquad (29)$$

and

$$H_L=\frac{s}{EI}\cdot\frac{ctLn}{2\frac{J}{L}(n\Sigma xy-\Sigma x\Sigma y)-2[n\Sigma y^2-(\Sigma y)^2]}. \qquad (30)$$

201. Arches with Elastic Piers.—In the ordinary theory of the elastic arch, the supports are supposed to be rigid and unyielding. This can never be strictly true, but it is practically correct where good foundations are obtained and a sufficient weight of abutment is used.

In the construction of a series of arches, light piers are sometimes employed to carry the vertical loads, the arches being depended upon to carry the horizontal reactions. In such systems, the tops of the piers are subject to lateral motion which may materially affect the stresses in the arch rings.

The bases of the piers must always be designed so that the resultant of the loads fall within their middle thirds, so that the bases will remain in contact with the foundations throughout. When this is the case, the piers become cantilevers held firmly at their bases and fixed between the arches at their upper ends.

When the structure is composed of nearly equal spans, and the thrust against the pier does not differ greatly on the two sides under dead load, the effect of the flexibility of the pier may be investigated for moving loads by a method of approximation. In Fig. 115, T_L and T_R are the thrusts of the spans upon the left and right respectively and R is their resultant acting upon the pier. The maximum load is supposed upon the left span and dead load only upon the right. The difference of the horizontal components of T_L and T_R, $H_L - H_R$, is the horizontal component of R.

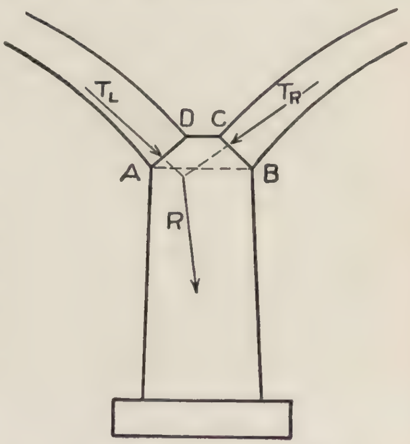


Fig. 115.—Elastic Pier.

This is applied at the top of the pier, causing the pier to act as a cantilever fixed at the bottom. If we assume that the top of the pier is firmly held by the ends of the arch, so that no rotation takes place, the top of the pier will have only a horizontal motion. The effect of this motion is to lengthen the span of the arch upon the left of the pier and decrease that of the arch upon the right, which will decrease the value of H_L and increase that of H_R.

Let Q = the horizontal motion at top of pier;

h = the height of pier;

I_p = average moment of inertia of horizontal sections of pier.

The crown thrust for the span on the left then becomes,

$$H_c = \frac{n(\Sigma m_L y + \Sigma m_R y) - (\Sigma m_L + \Sigma m_R)\Sigma y}{2n\Sigma y^2 - 2(\Sigma y)^2} - \frac{nQEI/s}{2n\Sigma y^2 - 2(\Sigma y)^2}, \quad (31)$$

and for the span upon the right,

$$H_c=\frac{n(\Sigma m_L y+\Sigma m_R y)-(\Sigma m_L+\Sigma m_R)\Sigma y}{2n\Sigma y^2-2(\Sigma y)^2}+\frac{nQEI/s}{2n\Sigma y^2-2(\Sigma y)^2}. \quad (32)$$

$$Q=\frac{(H_L-H_R)h^3}{12EI_p}.$$

The formulas for V_c and M_c are unchanged by the motion of the top of the pier, and are the same as for the arch with fixed ends.

If values of H_L and H_R be found by the formula for fixed supports, and the value of Q corresponding to their difference computed, the actual value of Q will be less than the computed value, and a trial value may be used in obtaining new values of H_L and H_R, until the values of the three quantities are in fair agreement.

The above is inaccurate in neglecting possible bending at the top of the pier. If the top of the pier in Fig. 115 be held against rotation, a bending moment will be produced in section $A-B$ equal to $M_p=(H_L-H_R)h/2$. The actual bending moment in the section $A-B$ is that produced by the eccentricity of the resultant of the thrusts of the arches against the pier, or $M_p=Re$. In order that no tendency to rotate exist, Re should not be less than $(H_L-H_R)h/2$. The error due to this cause may usually be made insignificant by careful design.

Methods of analyzing arches with elastic piers may be found in the works of Melan [2] and Hool.[3] In a paper by A. C. Janni in the Journal of the Western Society of Engineers, May, 1913, a graphical method of analysis is outlined, by the use of the ellipse of elasticity, which may be applied to a system of arches with elastic piers. These methods are complicated and cannot be discussed here; they all involve assumptions which make it necessary to exercise care in their application.

ART. 52. OTHER METHODS OF ANALYSIS

202. Analysis by Influence Lines.—In important structures, other conditions of loading than those mentioned in the preceding paragraphs may be desirable, and a more complete analysis may be obtained by determining the effect of individual loads at the various points of loading, which is accomplished by using influence lines to determine the effect of a unit load at each load point. In open spandrel arches, when the loads are brought upon the arch ring at defi-

[2] Plain and Reinforced Concrete Arches, by J. Melan, translated by D. B. Steinman, New York, 1915.

[3] Reinforced Concrete Construction, Vol. III, by George A. Hool, New York, 1915.

nite points, by vertical walls or columns, this method may be easily applied.

Figure 116 represents an arch 80 feet long, 16 feet rise; depth at crown, 2 feet; at springing line, 2.8 feet. It is reinforced with 1.6 in.^{2} of steel per foot of arch, placed 2.5 inches from both extrados and intrados. The loads are assumed to be applied through cross walls at points 10 feet apart.

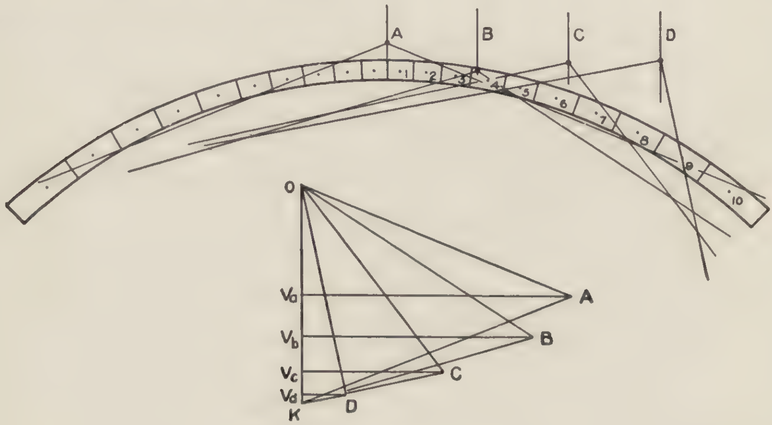


FIG. 116.—Influence Lines.

The arch ring is divided into ten parts on each side of the crown, so that the ratio s/I is constant; s being the length of division and I the moment of inertia at the middle of the division. Using the notation of Section 194, the values of x and y for centers of the various divisions arre as given in Table LVI.

TABLE LVI

Points.	x	y	x^2	y^2	
1	1.42	0.02	2.0	0.0	
2	4.39	0.23	19.3	0.1	
3	7.58	0.50	57.5	0.2	$2n\Sigma y^2-2(\Sigma y)^2=3928$
4	11.01	1.06	115.8	1.1	
5	14.70	1.89	216.1	3.6	
6	18.67	3.09	348.6	12.1	$2\Sigma x^2=8935.8$
7	22.94	4.73	502.5	22.4	
8	27.50	6.94	756.3	48.2	$2n=20.$
9	32.35	9.86	1046.5	97.2	
10	37.46	13.72	1403.3	188.2	
Σ	178.02	42.04	4467.9	373.1	

Values o. m_L, m_Lx and m_Ly are now computed for unit load at each load point and tabulated in Table LVII.

TABLE LVII

Points.	Load at *A*.			Load at *B*.		
	m_L	m_Lx	m_Ly	m_L	m_Lx	m_Ly
1	1.42	2.0	0.0			
2	4.39	19.3	1.0			
3	7.58	57.5	3.8			
4	11.01	115.8	11.1	1.01	11.1	1.0
5	14.70	216.1	27.8	4.70	69.1	8.9
6	18.67	348.6	57.7	8.67	161.9	26.8
7	22.94	526.2	108.5	12.94	296.8	61.2
8	27.50	756.3	190.9	17.50	481.2	121.5
9	32.35	1046.5	318.1	22.35	723.0	219.5
10	37.46	1403.3	514.0	27.46	1028.7	376.8
	178.02	4467.9	1232.9	94.63	2770.7	813.7
	Load at *C*.			Load at *D*.		
7	2.94	67.4	13.9			
8	7.50	206.2	52.1			
9	12.35	399.5	120.9	2.35	76.0	22.3
10	17.46	644.1	239.6	7.46	269.5	102.4
	40.25	1317.2	426.5	10.81	345.5	124.7

Substituting values from these tables in Formulas (15), (16), and (17), we have:

$$\text{Load at } A.\ \begin{cases} H_c=\dfrac{10\times1232.9-178\times42.0}{3928}=+1.235 \\[2ex] V_c=\dfrac{4467.9}{8935.8}=0.50 \\[2ex] M_c=\dfrac{178.0-2\times1.235\times42.0}{20}=+3.71 \end{cases}$$

$$\text{Load at } B.\ \begin{cases} H_c=\dfrac{10\times813.7-94.6\times42.0}{3928}=+1.063 \\[2ex] V_c=\dfrac{2770.7}{8935.8}=0.31 \\[2ex] M_c=\dfrac{94.6-2\times1.063\times42.0}{20}=+0.27 \end{cases}$$

Load at C.
$$\left\{\begin{array}{l} H_c=\dfrac{10\times426.5-40.2\times42.0}{3928}=0.656 \\[2ex] V_c=\dfrac{1317.2}{8935.8}=0.147 \\[2ex] M_c=\dfrac{40.2-2\times0.656\times42.0}{20}=-0.74 \end{array}\right.$$

Load at D.
$$\left\{\begin{array}{l} H_c=\dfrac{10\times124.7-10.8\times42.0}{3928}=0.20 \\[2ex] V_c=\dfrac{345.5}{8935.8}=0.04 \\[2ex] M_c=\dfrac{10.8-2\times0.20\times42.0}{20}=-0.30 \end{array}\right.$$

The thrusts and moments at any given section of the arch ring, due to each load, may now be found graphically (see Fig. 116). For this purpose, draw the force polygon, laying off O–$K=1.0$, the unit load. From K, the value of V_c is measured vertically, K–$v=V_c$, for each load, and H_c horizontally, v–A, v–B, etc. The distance M_c/H_c, measured vertically from the middle point of the crown section, gives the point of application of the crown thrust, k–A, k–B, etc. The equilibrium polygon in each case consists of two lines intersecting on the line of action of the loads and parallel to the corresponding lines in the force polygon.

The thrusts upon any section of the arch ring due to each unit load may now be taken from the force polygon, while the moment is found by multiplying the value of H_c for the given load by the vertical distance from the center of section to the equilibrium polygon.

Moments and thrusts at any section due to dead or live load at each load point may now be found by multiplying the values found for unit load by the amount of the load. If these be tabulated and combined, the maximum and minimum stresses may be obtained.

203. Analysis Using Arbitrary Divisions.—The method of analysis given in Art. 46 requires that the arch ring be so divided as to make s/I constant for all divisions. This simplifies the formulas used in obtaining values for H_c, V_c, and M_c, but makes the lengths of divisions vary greatly where the thickness of the arch ring increases from crown to springing line, and frequently gives very long divisions near the ends of the arch, which may sometimes introduce considerable error into the results.

A method of analysis based upon the principle of work in deflec-

tion is sometimes employed. This is demonstrated by Professor Hudson [4] and is applied to the analysis of the stresses in a conduit by Professor French [5] under the name of the method for indeterminate structures. Practically the same formulas may be produced by the method of Art. 48 by leaving the term s/I as a variable in the formulas.

If the constant T be eliminated from Formulas (4), (5), and (6) of Section 163, we have

$$\Sigma M_L\frac{s}{I} = -\Sigma M_R\frac{s}{I},\quad \Sigma M_L x\frac{s}{I} = \Sigma M_R x\frac{s}{I} \text{ and } \Sigma M_L y\frac{s}{I} = -\Sigma M_R\frac{sy}{I}.$$

Combining these with Equations (10) and (11) of the same section, and solving we find

$$H_c = \frac{\Sigma(m_L+m_R)y\frac{s}{I}\cdot\Sigma\frac{s}{I} - \Sigma(m_L+m_R)\frac{s}{I}\cdot\Sigma y\frac{s}{I}}{2\Sigma y^2\frac{s}{I}\Sigma\frac{s}{I} - 2\left(\Sigma y\frac{s}{I}\right)^2}. \quad . \quad . \quad (33)$$

$$V_c = \frac{\Sigma(m_L-m_R)x\frac{s}{I}}{2\Sigma x^2\frac{s}{I}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$M_c = \frac{\Sigma(m_L+m_R)\frac{s}{I} - 2H_c\Sigma y\frac{s}{I}}{2\Sigma\frac{s}{I}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

In the same manner for a rise in temperature, we have

$$H_c = \frac{ECtL\Sigma\frac{s}{I}}{2\Sigma\left(y\frac{s}{I}\right)^2 + \Sigma y^2\frac{s}{I}\cdot\Sigma\frac{s}{I}},$$

and

$$M_c = -\frac{H_c\Sigma y\frac{s}{I}}{\Sigma\frac{s}{I}}.$$

As an illustration of the use of this method of analysis we will

[4] Deflections and Statically Indeterminate Stresses, New York, 1911.

[5] American Sewage Practice, by Metcalf and Eddy, Vol. I, New York, 1914.

compute the values of H_c, V_c and M_c for the arch ring given in the example of Art. 49 with the loading employed in Section 195. Figure 117 shows the arch with divisions of equal length and the loads upon

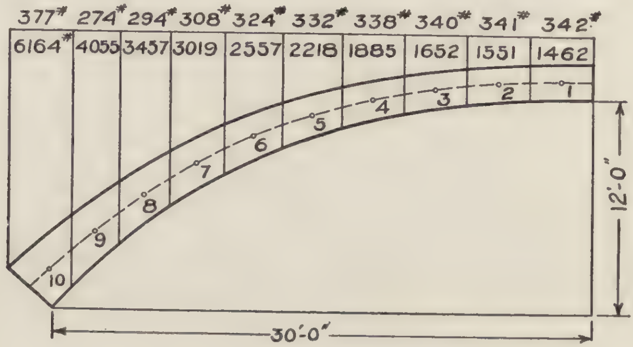


FIG. 117.—Arch with Equal Divisions.

each division. Table LVIII gives the coordinates of the centers of divisions, the value of s/I for the mid-section of each division, and

TABLE LVIII.—COORDINATES AND s/I FOR CENTERS OF DIVISION

Points.	x	y	s/I	xs/I	ys/I	x^2s/I	y^2s/I
1	1.71	0.03	9.17	16.26	0.28	27.6	0.00
2	5.12	0.22	7.90	40.45	1.76	207.0	0.39
3	8.53	0.62	6.62	56.47	4.10	481.9	2.52
4	11.92	1.02	5.63	67.10	5.64	800.0	5.85
5	15.27	2.02	4.83	73.75	9.76	1126.4	19.70
6	18.55	3.09	3.62	67.15	11.19	1245.6	34.57
7	21.27	4.47	2.62	57.98	11.71	1234.8	52.40
8	24.72	6.15	1.86	45.98	11.44	1136.6	70.35
9	27.56	8.12	1.30	35.83	10.56	987.7	85.71
10	30.19	10.35	0.83	25.06	8.59	756.5	88.91
Σ			44.72		75.03	8004.1	360.40

combinations of these quantities required in the computations. Table LIX gives the computations of the moments at centers of divisions, and of the terms in the formulas which include these moments. These computations might be somewhat shortened by expressing the loads in units of 1000 pounds and the moments as thousand foot-pounds.

TABLE LIX.—MOMENT COMPUTATIONS

Points	Total Loads		Lever Arms	m_R	m_L	$(m_L-m_R)x\frac{s}{I}$	$(m_L+m_R)\frac{s}{I}$	$(m_L+m_R)y\frac{s}{I}$
	Dead	Live						
1	0	0	0	0	0	0	0	0
2	1,462	342	3.41	4,985	6,151	47,223	87,974	19,354
3	3,013	683	3.41	15,256	18,751	197,362	225,126	139,573
4	4,665	1023	3.39	31,069	38,032	467,217	389,038	396,613
5	6,550	1361	3.35	52,012	62,534	777,523	553,256	1,117,577
6	8,768	1693	3.28	80,791	96,866	1,078,632	643,118	1,987,234
7	11,325	2017	3.16	116,578	139,027	1,302,042	669,785	2,993,939
8	14,344	2325	3.01	159,753	189,200	1,354,562	649,032	3,993,546
9	17,801	2619	2.84	210,308	247,193	1,320,483	549,751	4,829,378
10	21,856	2893	2.63	267,789	312,283	1,116,799	481,459	4,983,100
Σ	....	......	......	....	....	7,661,843	4,292,539	20,459,414

Substituting values from the tables in Formulas (33), (34), and (35),

$$H_c=\frac{20459414\times 44.7-4292539\times 75}{2\times 360\times 44.7-2(75)^2}=+28300 \text{ lb.}$$

$$V_c=\frac{7661843}{2\times 8004}=+478 \text{ lb.}$$

$$M_c=\frac{4292535-28300\times 75\times 2}{2\times 44.7}=+532 \text{ ft.-lb.}$$

These results are preferable to those obtained in Section 195 on account of the better division of the arch axis and the inclusion of a larger portion of the load in the moments. The labor required in the use of this method is not materially greater than that involved in the use of the ordinary method as given in Section 195.

NOTE.—For comprehensive discussion of elastic arches, see paper on "Design of Symmetrical Concrete Arches" by Mr. Charles S. Whitney in the *Proceedings* of the American Society of Civil Engineers, Nov., 1924, p. 1327.

For a description of research work carried on by Prof. George E. Beggs on elastic arches and piers, in which celluloid and heavy cardboard model sections were used, see discussion of paper entitled, "The Design of a Multiple-Arch System and Permissible Simplifications" in the *Proceedings* of the Am. Soc. C. E., Jan., 1925, p. 93.

CHAPTER XI

CULVERTS AND CONDUITS

ART. 53. CULVERTS

204. Types of Culverts.—The term culvert is usually applied to structures intended to provide small waterways through earth embankments. Such structures are usually constructed according to certain standard plans, depending upon the size of opening required. For the smaller openings, pipe culverts of vitrified clay, plain or reinforced concrete, cast iron or corrugated iron, are frequently used.

For openings larger than 24 or 30 inches in diameter, box culverts or arch culverts of stone or brick masonry or of concrete, either plain or reinforced are commonly employed. Concrete for this purpose has recently been gradually replacing the older types of construction, on account of its ease of application in most localities, and its low cost as compared with other types of equal strength and durability.

Wooden culverts have been largely used in the past upon highway work, but are now rapidly giving way to more permanent structures, for, while cheaper in first cost than the other types, they are very uneconomical on account of their rapid deterioration and high cost of maintenance.

All culverts require walls of masonry or concrete at the ends to prevent the possible penetration of water around the culvert, and to sustain the bank of earth and hold it from falling into and clogging the waterway. For small culverts, such walls are usually parallel to the roadway; they should be long enough to permit the earth to stand at a slope of about $1\frac{1}{2}$ to 1 without reaching the waterway of the culvert and sufficiently high to sustain the earth fill above the culvert.

205. Area of Waterway Required.—The waterway provided for a culvert must, for safety, be sufficiently large to pass the maximum flow of water that is likely to occur, while for economy it should be as small as possible. There are at long intervals, in most localities, records of storms of extraordinary character, to provide for which would need large increase of capacity in the culverts and add greatly

to their cost, and while these unusual storms can hardly be taken into account in the design of the structures, effort should be made to provide for any flow of water that may reasonably be anticipated. The maximum flow of a stream depends upon a number of local conditions, most of which are very difficult of accurate determination. Among these are the maximum rate of rainfall, the area drained by the stream, the shape and character of the surface drained, and the nature and slope of the culvert channel.

The maximum rate of rainfall varies widely in different localities, the heaviest occurring over very limited areas and short periods of time, and are therefore important only for small culverts. For larger areas, the maximum rainfall of sufficient duration to permit water from all parts of the tributary area to reach the culvert gives maximum results.

The amount of water reaching the culvert depends upon the permeability of the soil, its degree of saturation, and the amount of vegetation. The rapidity with which water reaches the culvert from the far portion of the watershed depends upon the slope and smoothness of the surface and whether it is covered with vegetation. The shape of the area to be drained is important in that it determines the distance the water must travel in reaching the culvert.

The quantity of water which will pass through a culvert in a given time depends upon the smoothness of its interior surface, the disturbance of flow at entrance to the culvert, and the freedom with which the water flows away after passing the culvert. If the culvert is so constructed that water may stand against its upper end, causing it to discharge under pressure, its capacity will be considerably increased.

The determination of the area of waterway required in any instance is a matter of judgment, and there is no way in which it may be accurately computed. A number of formulas have been proposed for the purpose of aiding in estimating the probable quantity of water from a given area or the size of opening required for a given area. The formula of Professor Talbot has been used to considerable extent in the Middle West with good results. This formula is: Area of waterway in feet $= C\sqrt[4]{(\text{drainage area in acres})^3}$, in which C is a coefficient depending upon local conditions. For rolling agricultural country subject to floods at time of melting snow, and with length of valley three or four times its width, $C = \frac{1}{3}$. When the valley is longer, decrease C. If not affected by snow and with greater lengths, C may be taken at $\frac{1}{5}$, $\frac{1}{6}$, or even less. For steep side slopes, C should be increased. Where the ground is steep and rocky, C

may vary from $\frac{2}{3}$ to 1. Table LX gives roughly the sizes of openings required for different areas, computed from the formula of Professor Talbot.

TABLE LX.—AREA IN SQUARE FEET OF WATERWAY REQUIRED

Area Drained, Acres.	Steep Slopes, Sq. Ft.	Rolling Agricultural Country, Sq. Ft.	Level Country, Sq. Ft.
10	6	2	1
25	11	4	2
50	19	7	4
75	25	9	5
100	32	11	6
200	54	18	10
300	72	24	15
500	106	35	21
1000	180	60	35

For most cases in practice the size of waterway may be determined from the knowledge which usually exists in the vicinity regarding the character of a stream, from the sizes of other openings upon the same stream, or from comparison with other streams of like character and extent in the same locality. Where data of this kind do not exist, careful examination of water marks on rocks, the presence of drift, etc., may be made to determine the height to which water has previously risen. The shape of the valley and the slope of the surface is of more importance than the area of country drained. The use of a formula like Talbot's assists the arrangement of the factors which enter into the determination, and is intended only as an aid to judgment in selecting the size of opening required.

206. Pipe Culverts.—Vitrified clay pipes make satisfactory as well as comparatively cheap culverts when small openings are required, and for openings from 12 to 24 inches in diameter, they may often be used economically. It is not usually desirable to build a culvert less than 12 inches in diameter. For those larger than 24 inches concrete will usually be found more suitable, although vitrified pipes 30 and 36 inches in diameter are sometimes used.

The best quality of double-strength, salt-glazed sewer pipe should be used for culverts. These pipes are made in lengths of 24 and 30 inches and diameter from 12 to 36 inches, with socket joints. They should be sound and well burned, giving a clear ring when lightly struck with a hammer.

The joints should be filled with Portland cement mortar a requirement particularly necessary where the pipe is likely to flow full, or under pressure, as it will prevent the water being forced out and the earth being washed from around the pipe.

Vitrified pipes cannot safely be used where they are directly exposed to the shocks of traffic, and many failures of such culverts have been due to this cause. In highway work they should be protected by at least 2 feet of filling, the roadway being graded so that a vehicle may pass smoothly and without shock over the culvert. In railway work a fill of about 5 feet over the culvert is usually necessary. The use of vitrified pipe for railway culverts is desirable only under favorable conditions, when danger from shocks of traffic can be avoided, and good foundations make breakage from settlement improbable.

The cost of vitrified pipe varies widely with tne conditions of trade, and with the expense for freight and haulage to the site of the work. The cost of laying the pipe depends upon local conditions and the way the work is handled. Table LXI gives areas, weights, and rough averages of costs in a number of localities in the middle West before the War.

TABLE LXI.—APPROXIMATE DIMENSIONS, WEIGHTS AND COSTS OF VITRIFIED PIPE CULVERTS

Inside Diameter, Inches	12	15	18	24
Area opening, square feet	0.78	1.26	1.76	3.14
Weight of pipe, pounds per foot	52	70	100	175
Cost of pipe, per foot	$0.40	$0.50	$0.75	$1.20
Cost of laying, per foot	0.40	0.60	0.75	1.25

The ends of pipe culverts should always be protected by a masonry or concrete wall. Figure 118 shows a vitrified pipe culvert with end wall as used in highway work. These walls should extend at least 2 feet below the bottom of the culvert to prevent water passing under the culvert and undermining it, and should also reach above the surface of the roadway, thus serving as a protection both to the culvert and to the road. When the culvert is under an embankment, the wall should rise high enough to catch the slope of the embankment and form a curb to retain the earth.

Table LXII gives dimensions that may be used for end walls for highway culverts under ordinary conditions.

Culverts of cast-iron pipe have been used to considerable extent in railway work for sizes from 1 to 4 feet in diameter. The present tendency, however, is to use concrete for the larger openings, on

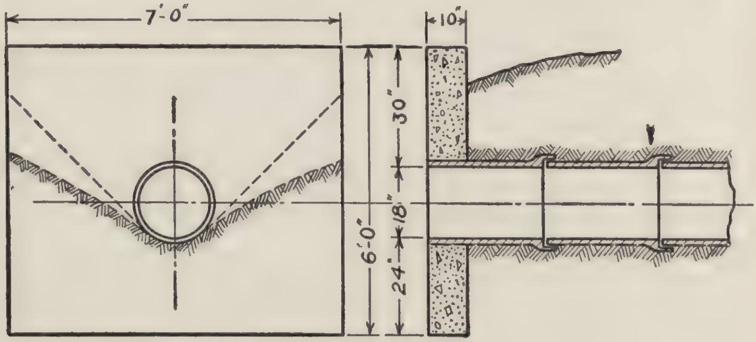


FIG. 118.—Vitrified Pipe Culvert.

account of its relative cheapness and the occasional cracking of the large iron pipes. Ordinary water pipe is sometimes used, but heavier pipe made for the purpose is more commonly employed.

TABLE LXII.—CONCRETE END WALLS FOR PIPE CULVERT

Diameter of Pipe, Inches	12	15	18	24
Thickness of walls, inches	10	10	10	10
Height of walls, feet, inches	5–6	5–9	6–0	6–6
Length of walls, feet, inches	5–0	6–0	7–0	9–0
Concrete in two walls, cubic yards	1.7	2.1	2.5	3.4

For highway work, cast-iron pipe has the advantage of resisting shocks better than vitrified pipe, and may be used for small openings where the service is severe. It is not extensively used on account of its cost. Special culvert pipes in lengths of 3 or 4 feet are now available, which are made lighter than ordinary water pipe, some of them being made with a thinner shell reinforced by ribs. They are also made in longitudinal sections to be bolted together.

Corrugated metal culvert pipe is made lighter than cast iron, and does not ordinarily differ greatly in price from clay pipe. It is rather easy to handle and is less likely to break under shocks than vitrified pipe. It should, however, be covered by a thickness of at least 1 foot of road material.

The life of a culvert of this kind depends upon the ability of the metal to resist rust. Wrought iron is much better than steel in this respect, but must be selected with special reference to its resisting qualities. Pipes made of nearly pure iron have given good results, although numerous failures have resulted from the use of improper material.

Concrete Pipe Culverts.—Reinforced concrete culvert pipes are sometimes made from 18 to 48 inches in diameter, and in lengths from 4 to 8 feet. They usually have a hoop reinforcement, as shown in Fig. 119, passing near the interior surface at top and bottom and

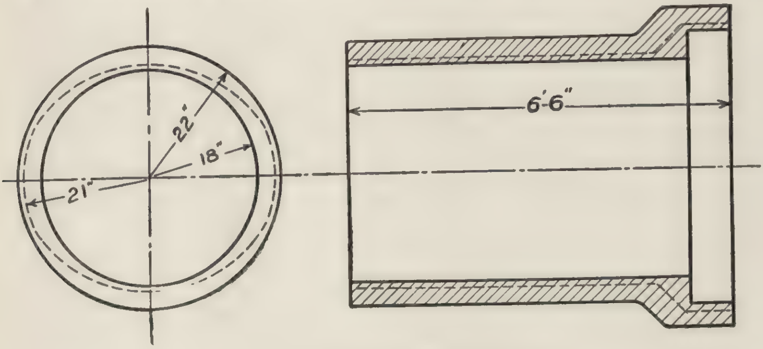


Fig. 119.—Concrete Pipe Culvert.

near the exterior surface at the sides, the reinforcement being bent to circular form and the pipe made in oval form with the larger diameter vertical. Concrete pipe is also sometimes made with a double reinforcement, one line near each surface. Table LXIII gives dimensions recommended by the Iowa State Highway Commission for circular pipe with double reinforcement.

TABLE LXIII.—CONCRETE CULVERT PIPE

Diameter, Inches.	Thickness of Shell, Inches.	Steel Area for Each Line, Per Foot of Pipe.
15	2.25	.058 in.^{2}
18	2.50	.077 in.^{2}
24	3.00	.102 in.^{2}
30	3.50	.151 in.^{2}
36	4.00	.170 in.^{2}
42	4.50	.225 in.^{2}

The load to be carried by a culvert under an embankment may usually be taken as equal to the weight of embankment immediately

above the culvert, and the live load carried by the roadway considered as distributed through the fill. For pipes in trenches the weight of filling is partly borne by the sides of the trenches. A study of pressures on pipes in trenches has been made by Professor Marston at the Iowa State College, and the very interesting results published in a bulletin of the Engineering Experiment Station of the College.

A uniform horizontal earth pressure over the whole width of a pipe produces positive bending moments at the top and bottom sections and negative moments at the ends of the horizontal diameter which are each equal to $M=Wd/16$, where W is the total load and d the diameter of the pipe. The pipe must be uniformly supported over its whole width in this case. If it is supported only at the middle, as when laid in a flat trench, the moments at top and bottom will be about doubled. In laying pipe the bottom of the trench should be rounded to fit it, being cut a little deeper under the middle, so that the bottom is free, not quite touching the soil, and letting the pipe rest upon the soil at the sides. Depressions should also be dug for the sockets to prevent the pipes being supported at the sockets and thus subjected to longitudinal bending.

Pipe should be laid from the down stream end with the sockets upstream. It is also desirable to give a slight crown to the grade of the culvert to provide for possible settlement.

207. Box Culverts.—Rectangular culverts are commonly used for sizes too large for pipes. These may be open boxes consisting of a slab top resting upon sidewalls, or closed boxes, in which a bottom slab connects the bases of the side walls and distributes the load over the foundation soil.

Stone box culverts have been extensively used in the past, but are now being superseded by reinforced concrete; but where suitable stone is available, they may often be found satisfactory and economical.

The side and end walls should be built of stone at least 6 inches thick, laid in cement mortar, and with frequent headers extending through the wall. The walls should extend downward sufficiently to obtain good foundations and to be safe from frost. The floor of the culvert between the side walls should be paved with stone, unless it is of material which will resist erosion.

The width of opening for stone box culverts is limited by the dimensions of the cover stones available and is never more than 4 or 5 feet. The cover stones should have a thickness at least one-quarter of the width of opening, and should have a bearing of about 1 foot upon each wall.

TABLE LXIV.—CONCRETE BOX CULVERTS

Size of opening:									
Span, feet	5	5	5	6	6	6	8	8	8
Height, feet	3	4	5	3	4	5	4	5	6
Top slab:									
Thickness of concrete, inches	7½	7½	7½	8	8	8	10	10	10
Reinforcement:									
Diameter of round steel rods, inches	⅝	⅝	⅝	⅝	⅝	⅝	⅝	⅝	⅝
Spacing of rods, c. to c., inches	7½	7½	7½	7	7	7	5½	5½	5½
Length of rods, feet, inches	6–6	6–6	6–6	7–8	7–8	7–8	9–8	9–8	9–8
Reinforcement lengthwise:									
Diameter of steel rods, inches	½	½	½	½	½	½	½	½	½
Spacing of rods, c. to c., inches	12	12	12	12	12	12	12	12	12
Quantities per foot of length:									
Concrete, cubic yards	0.16	0.16	0.16	0.20	0.20	0.20	0.31	0.31	0.31
Steel, pounds	13.5	13.5	13.5	17.1	17.1	17.1	26.7	26.7	26.7
Abutments:									
Thickness at top, inches	10	10	10	12	12	12	12	12	12
Thickness at bottom, inches	24	24	30	24	24	30	30	30	36
Height, feet and inches	4–6	5–6	6–6	4–6	5–6	6–6	5–6	6–6	7–6
Concrete per foot, cubic yards	0.47	0.58	0.80	0.50	0.61	0.84	0.71	0.84	1.10

Concrete box culverts are sometimes constructed with a reinforced slab top resting upon side walls which may or may not be reinforced. The design of short bridges of this type has been discussed in Chapter IX. Where many culverts are to be constructed, it is common to adopt specific loadings and work out standard forms and dimensions to be used. Such standards have been adopted by many railways and State highway departments. Table LXIV shows dimensions suitable for ordinary highway culverts 5 to 8 feet in span, to carry the loadings used in Section 177. The steel is to be placed $1\frac{1}{2}$ inches from the lower surface of the slab.

Closed box culverts of reinforced concrete are frequently used for small openings, as they require less headroom than arched openings and are easily applied when openings are too large for convenient use of pipes. The stresses in such a culvert cannot be accurately determined on account of the indeterminate character of the loads. A load applied upon the top of the culvert produces an equal upward thrust upon the bottom of the culvert, as shown in Fig. 120, which causes a moment tending to bend the top and bottom slabs inward and the sides outward.

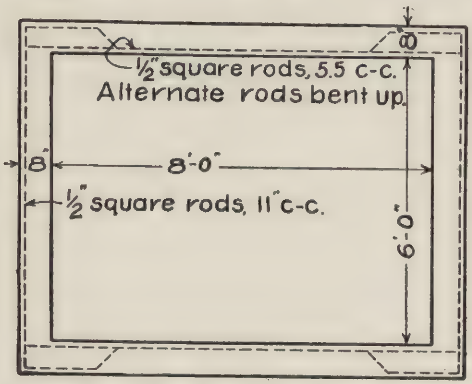


FIG. 120.—Closed Box Culvert.

Let b = width of culvert;

h = height of sides;

w = uniform load per foot;

M_1 = bending moment in top and bottom slabs;

M_2 = bending moment at middle of sides;

M_3 = bending moment at corners;

I_1 = moment of inertia of sections of top and bottom;

I_2 = moment of inertia of sections of sides.

If we assume the load to be uniformly distributed over the top, using the method of Section 203 (p. 410), as the loads are normal to the top of the culvert and symmetrically placed, $m_L = m_R = m$; $H_c = 0$ and $V_c = 0$. For any point of the top slab $m = +\frac{wx^2}{2}$; for the side $m = +\frac{wb^2}{8}$; and for the bottom

$$m=\frac{w}{2}\left(\frac{b}{2}-x\right)^2-\frac{wb}{2}\left(\frac{b}{2}-x\right),$$

and

$$\Sigma m=\frac{s}{I}\int_0^{\frac{b}{2}}\frac{wx^2dx}{2I_1}+\int_0^h\frac{wb^2}{8I_2}dy+\frac{w}{2I_1}\int_0^{\frac{b}{2}}\left(\frac{b}{2}-x\right)^2dx-\frac{wb}{2I_1}\int_0^{\frac{b}{2}}\left(\frac{b}{4}-x\right)dx$$

$$=\frac{wb^3}{24I_1}+\frac{wb^2h}{8I_2}.$$

Substituting these values in Formula 35, (p. 411) we find

$$M_1=\frac{wb^2}{8}\cdot\frac{b/3I_1+h/I_2}{b/I_1+h/I_2}$$

and

$$M_2=M_3=M_1-wb^2/8.$$

The pressure of earth against the sides of the culvert produces moments in the top, bottom and sides of the culvert of opposite sign to those produced by the load upon the top of the culvert, and therefore tend to reduce the effect of the top load upon the culvert. Such pressures always exist to some extent, but are not accurately known. It is usual to assume that unit horizontal pressure, when considered, is about one-third the unit vertical pressure. The moments caused by the side pressure will always be much less than those due to the vertical loads and not sufficient to overcome those moments.

If the side pressures be supposed to exist when the vertical loads are not on the culvert, as may be the case with moving loads, the sides will be subject to positive moments and need reinforcing at the inner surfaces.

The existence of side pressures tends to increase the negative moments at the corners, and a box culvert can act as a whole only when the corners are reinforced sufficiently to carry these moments without cracking at the corners.

In case the fill upon the culvert is not sufficient to distribute the load over the whole top of the culvert, the moment will be increased. For a concentrated load at the middle of span, the moments will be about double those for the same total load distributed over the span. In highway culverts which are covered only with the thickness of the road, surface, the distribution of the load may be considered as in Art. 44. In such culverts, the live load should be increased 25 per cent to allow for impact.

When, as is sometimes the case, the corners of the culvert are not reinforced for negative moment, the top becomes a simple beam, resting upon the sides but not rigidly attached to them, and the sides carry only the horizontal earth pressure as simple beams. Such

construction is shown in Fig. 121, which represents a standard section for a highway culvert designed to carry a 20-ton auto truck. The section in Fig. 120 is designed for the same loading.

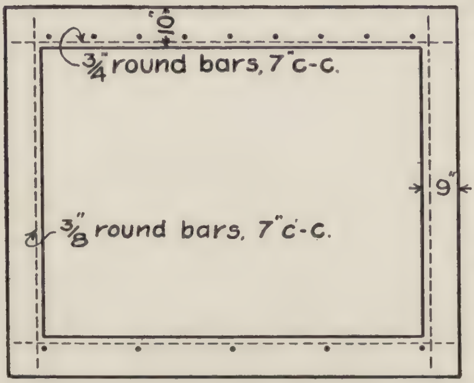


FIG. 121.—Section for Highway Culvert.

208. Arch Culverts.—For locations where sufficient headroom is available, arch culverts are often preferable to those with flat top. Very pleasing and artistic effects may frequently be obtained by careful design of arches for such use. Under fills of considerable height, arch culverts will commonly be more economical to construct than slab top culverts. Fig. 122 shows a section for a standard highway culvert for use under automobile traffic.

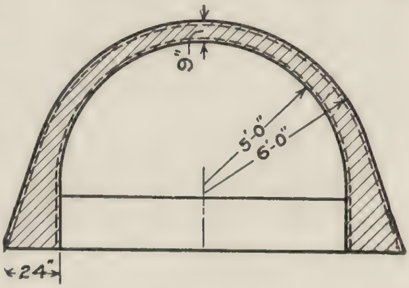


FIG. 122.—Arch Culvert.

The analysis of stresses in arch culverts may be made in the same manner as is given for arch bridges in Chapter X. The horizontal earth pressures on the sides of the arch are usually taken as one-third of the vertical pressures at the same point. These pressures are of greater relative importance than in

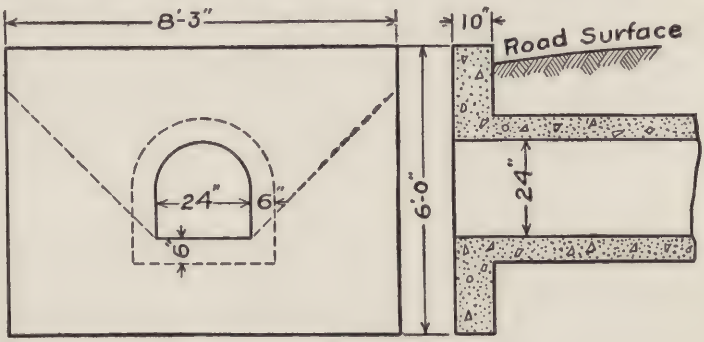


FIG. 123.—Concrete Barrel Culvert.

bridges of longer span. For short spans, plain concrete is commonly employed, while for spans greater than about 8 feet, reinforcement is usually introduced for greater security, although not necessary to carry moments.

ART. 54. CONDUITS

209. Types of Conduits.—Conduits for carrying water may be designed either for gravity flow or for internal pressure. Brick masonry was formerly largely used in the construction of gravity conduits, particularly for larger sewers, but is now being replaced for the most part by the use of concrete. For conduits to carry water under pressure, reinforced concrete or steel pipe is usually employed.

A conduit consists essentially of two parts, the invert, which forms the channel for the water, and the top, usually arched, which covers the channel and carries the weight of earth or other loads which may come upon it. The shape of the invert depends upon the requirements of the service. In sewers, special forms of invert are frequently needed to prevent deposits at times of minimum flow. The designs of sections for various uses may be found in works upon water supply, irrigation, and sewerage.

Sewage may sometimes cause disintegration of concrete, and the inverts of conduits intended to carry sewage are therefore commonly lined with vitrified brick—a method particularly desirable where the sewage is stale or impregnated with chemicals from manufacturing plants. In conduits carrying water for irrigation, injury to concrete may result from alkalis in the soil unless special precautions are taken.

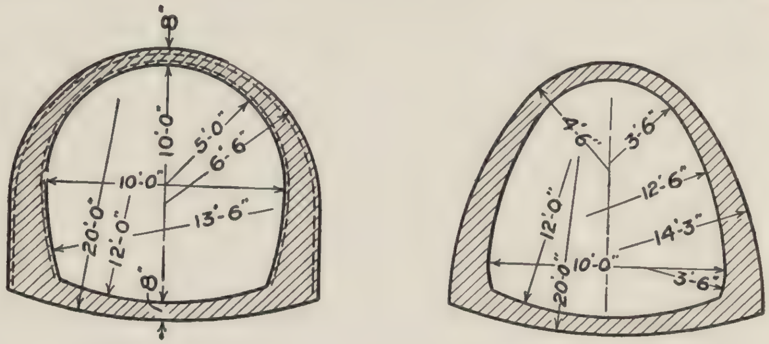
The inverts of carefully constructed concrete conduits usually resist the abrasion of flowing water fully as well as those with brick or stone lining—such resistance depending upon the alignment of the conduit and the amount of sediment carried by the water. With clear water and an undisturbed flow, very high velocities may produce no appreciable damage, while the impact caused by changes in the direction of flow cause rapid wear, particularly when sand and gravel are carried by the stream.

No conduit is absolutely water-tight, and careful attention should always be given to reducing leakages to a minimum. Usually the most serious leakage occurs at joints where one section joins another, although there will generally be some porous spots through which small quantities of water may pass. The leakage may commonly be reduced to very small proportions by careful design and construc-

tion, reinforcing so as to prevent cracks and using dense and uniform mixtures of concrete. This subject is discussed in Art. 23.

Conduits of small size are sometimes made rectangular in section and designed in the same manner as rectangular culverts. Larger conduits are usually of curved form with arched tops.

210. Design of Gravity Conduits.—After determining the size and general shape of conduit required for a given service, the design depends upon the character of the soil upon which it is to be placed and the external loads that it must carry. When the invert rests upon a firm foundation, capable of supporting the structure without sensible yielding, the invert may be considered as fixed in position and the arch may be designed with ends fixed upon the sides of the invert. The design of such arches may be made by the ordinary method used



Horseshoe section. Semi-elliptical section.

Fig. 124.—Typical Sewer Sections.

for arch bridges or culverts. Actual loads, in so far as they can be determined, should be used in such designs. Where the loads are light, such conduits may often be built of plain concrete; usually, however, it is preferable to reinforce arches of more than 4 or 5 feet span. Fig. 124 shows typical forms of standard sewer conduits.

The horizontal earth pressure to which the side of a conduit may be exposed cannot be accurately determined. It is customary to use Rankine's minimum value,

$$\text{unit horizontal pressure} = w\,\frac{1-\sin\phi}{1+\sin\phi},$$

in which w is the unit vertical pressure and ϕ is the angle of friction of the earth. Taking $\phi = 30°$ for ordinary earth, this makes the unit horizontal pressure at any point equal to one-third of the unit vertical

pressure at the same point. In some instances it may be necessary to consider the possible effect of variations in horizontal pressures.

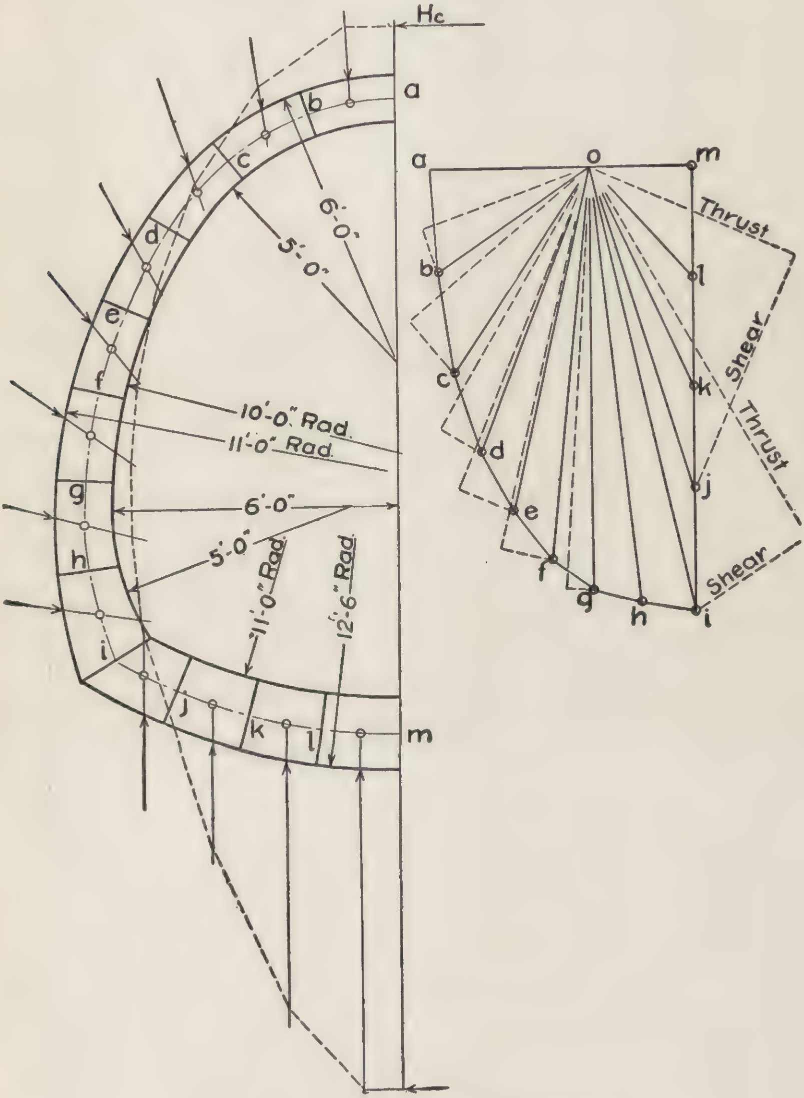


Fig. 125.—Design for Gravity Conduits.

As the tendency of such a structure under vertical loading is to deflect outward upon the sides, it is reasonable to assume that at least this minimum horizontal pressure may always be depended upon, or a

TABLE LXV.—LOADS UPON CONDUIT RING ONE FOOT LONG

Division.	Vertical Loads.				Horizontal Loads.			Sum of Loads.	
	Earth, lb./ft.²	Horizontal Area.	Weight of Arch.	Total on Division.	Earth, lb./ft.²	Vertical Area.	Total on Division.	Vertical.	Horizontal.
		Feet.	Pounds.	Pounds.		Feet.	Pounds.	Pounds.	Pounds.
a–b	2020	2.00	288	4328	673	4.00	269	4,300	00
b–c	2085	1.85	288	4145	695	0.90	625	4,328	269
c–d	2205	1.35	288	3265	735	1.50	1102	8,473	894
d–e	2340	0.95	291	2514	780	1.75	1365	11,738	1,996
e–f	2550	0.70	296	2081	850	1.85	1573	14,252	3,361
f–g	2735	0.35	317	1274	912	1.95	1778	16,333	4,934
g–h	2935	0.08	346	581	978	2.05	2005	17,607	6,712
h–i	3150	0.00	432	432	1050	2.10	2205	18,188	8,717
i–j	−2698	1.90		−5126				18,620	10,922
j–k	−2698	1.60		−4322				13,494	10,922
k–l	−2698	1.65		−4452				9,172	10,922
l–m	−2698	1.75		−4720				4,720	10,922

greater passive pressure if needed. In case of soft, wet earth, the horizontal pressure will be much greater, reaching a maximum when it is practically fluid and exerts normal pressure at all points.

Conduits to be supported upon compressible soil are often designed to act as a whole, assuming that all parts of the structure, including the invert, are equally subject to distortion under the loads. Figure 125 represents a half-section of a conduit of this character. If we assume the middle of the invert, *m*, to be fixed in position, the moments and thrusts in a slice of the conduit 1 foot thick may be found in the manner used for the elastic arch in Chapter X. The axis of the conduit ring is divided into lengths as shown. The lengths of the divisions, coordinates of the centers of divisions with reference to the crown, and thicknesses of concrete at centers of division, are given in Table LXVI.

The loads given (Table LXV), are those due to the pressure of 20 feet of earth above the crown of the arch. The weight of the earth is taken at 100 pounds per cubic foot, and the intensity of the horizontal earth pressure at one-third that of the vertical pressure at the same point. In computing the loads, the unit pressures at the middle of the extrados of the division are considered as acting upon areas equal to the horizontal and vertical projections of the extrados of the divisions. The upward pressures upon the base are considered as acting vertically and uniformly distributed horizontally. The computations of loads and their moments about the centers of division are shown in Table LXV.

The moment and thrust at the crown section, *a*, may be obtained by the use of the formulas of Section 203. As the loading is symmetrical about the crown, m_L and m_R are equal, $V_c=0$, and Formulas (33) and (35) of Section 203 become

$$H_c=\frac{\Sigma my\frac{s}{I}\Sigma\frac{s}{I}-\Sigma m\frac{s}{I}\Sigma y\frac{s}{I}}{\Sigma y^2\frac{s}{I}\Sigma\frac{s}{I}-\left(\Sigma y\frac{s}{I}\right)^2},$$

and

$$M_c=\frac{\Sigma m\frac{s}{I}-H_c\Sigma y\frac{s}{I}}{\Sigma\frac{s}{I}}.$$

Table LXVI gives the computation of the terms needed in these formulas. As the sections are rectangular, no reinforcement being considered in the computations, the value s/t^3 may be used in the formulas in place of s/I.

TABLE LXVI.—COMPUTATIONS OF TERMS USED IN FORMULAS

Division.	x	y	t	s	$\frac{s}{t^3}$	$y\frac{s}{t^3}$	$y^2\frac{s}{t^3}$	m	$m\frac{s}{t^3}$	$my\frac{s}{t^3}$
a–b	0.93	0.08	1.00	1.92	1.92	0.15	0.0	00	00	00
b–c	2.72	0.68	1.00	1.92	1.92	1.20	0.8	7,908	15,183	10,324
c–d	4.22	1.90	1.00	1.92	1.92	3.65	6.9	21,708	41,679	79,190
d–e	5.28	3.43	1.01	1.92	1.86	6.38	21.9	37,204	69,199	237,353
e–f	6.10	5.13	1.03	1.92	1.76	9.03	46.3	53,604	94,343	483,980
f–g	6.55	6.92	1.10	1.92	1.44	9.96	68.9	69,786	100,492	695,405
g–h	6.68	8.82	1.20	1.92	1.11	9.79	86.3	84,827	94,158	830,474
h–i	6.40	10.67	1.48	1.92	0.59	6.30	67.2	95,862	56,558	603,794
i–j	5.42	12.00	1.50	1.64	0.49	5.88	70.6	92,140	45,149	541,788
j–k	3.92	12.04	1.50	1.64	0.49	6.19	78.2	78,889	38,656	488,611
k–l	2.40	13.07	1.50	1.64	0.49	6.40	83.6	70,081	34,340	448,824
l–m	0.83	13.25	1.50	1.64	0.49	6.50	86.3	64,855	31,779	421,697
					14.48	71.43	617.0		621,536	4,841,440

Substituting in the formulas, we have

$$H_c=\frac{4841440\times 14.48-621536\times 71.43}{617\times 14.48-(71.43)^2}=6710 \text{ pounds.}$$

$$M_c=\frac{621536-6710\times 71.43}{14.48}=9830 \text{ ft.-lb.}$$

$$e=\frac{M_c}{H_c}=\frac{9830}{6710}=1.46 \text{ feet.}$$

The load diagram is now drawn as shown, and the equilibrium polygon (or line of resistance) constructed, beginning with H_c at a distance, $e=1.46$ feet, above the middle of the crown section.

The thrusts acting upon the ends of divisions as found from the load diagram may be resolved into normal thrusts and shears as shown by the broken lines. These are tabulated in Table LXIII. The moments at the centers of sections at the end of divisions may be obtained by multiplying the normal thrust upon the section by the distance from the center of section to the point at which the equilibrium polygon cuts the section, or they may be computed by Formula 10 of Section 163, which becomes for symmetrical loading

$$M=M_c+H_c y-m$$

Table LXVII, gives thrusts and moments with the resulting stresses at the extrados and intrados of the sections. These results show that there are tensions at the intrados of the crown section and in the invert, and at the extrados of sections f, g, and h which must be cared for by reinforcement. This reinforcement should be sufficient to carry the tensions in the section without materially changing the position of its neutral axis or the compression upon the concrete. To do this, the stress in the steel should be limited to about fifteen times that shown for the rectangular section, or about 6000 lb./in.^{2} at sections a and g and 9000 lb./in.^{2} at m. Computing the total tension in these sections, we find that an area of about 2 in.^{2} of steel per foot of length is required at a and g and about 4 in.^{2} at m. One-inch square bars spaced 6 inches apart near the intrados at sections a and b, then crossing to the extrados at e and extending along the extrados to section i, with $1\frac{3}{8}$-inch square bars spaced 6 inches apart near the intrados of the invert would answer the requirement.

The maximum shear occurs at section j, the unit shear being about 50 lb./in.^{2}, which is not excessive.

TABLE LXVII.—COMPUTATION OF STRESSES

Section.	e	Normal Thrust.	Bending Moment.	f_c at Extrados.	f_c at Intrados.	Total Tension.	Shear.	Unit Shear.
				Lb./in.²	Lb./in.²	Lb.	Lb.	Lb./in.²
a	+1.46	6,710	+9,830	+457	−363	11,573	00	
b	+1.05	7,500	+7,875	+380	−276		1,970	
c	+0.35	9,910	+3,368	+195	− 85		3,000	**21**
d	−0.20	12,540	−2,508	− 17	+191		2,110	
e	−0.50	14,280	−7,140	−189	+383		2,670	
f	−0.75	16,220	−12,165	−344	+556	9,906	2,440	
g	−0.85	17,450	−14,831	−358	+568	11,460	1,450	
h	−0.90	18,120	−16,308	−303	+497		1,310	
i	−0.75	18,500	−13,875	−152	+312		4,550	
j	+0.60	9,200	+ 5,520	+113	− 47		10,690	**49**
k	+3.25	6,480	+21,090	+420	−360		7,690	**36**
l	+6.20	4,820	+29,844	+575	−531		4,120	
m	+8.05	4,212	+33,906	+648	−608	32,332	00	

It seems probable that this analysis represents the conditions giving the maximum stresses possible in the structure. For a depth as great as 20 feet, the full pressure of the earth would probably not be borne by the structure. For greater depths, these pressures need not be increased, unless the earth is unstable.

The deflection of the conduit under the loads is outward upon the sides, and, if the earth is well packed around the sides of the conduit the earth will resist that deflection and the horizontal earth pressures will probably be greater than those used in the analysis. This will diminish the bending moments at all points and reduce the need for reinforcement.

Longitudinal reinforcement is needed to prevent cracking of the concrete. Usually $\frac{1}{2}$-inch bars, 12 inches apart, are sufficient for this purpose. Where the support of the soil under the conduit may not be uniform, it is desirable to guard against longitudinal deflection by the use of heavier reinforcement near the bases of the side walls.

211. Pressure Conduits.—Conduits to carry water under pressure are usually made of circular or oval form. The stresses caused by internal pressure are all tensile and should be taken wholly by the steel reinforcement.

Let P = the internal pressure per square inch;
D = the internal diameter of the conduit in inches;
f_s = the stress in the steel;
A_s = the area of steel per inch of length.

Then we have, $A_s = PD/2f_s$. Low values of f_s are desirable in order to minimize the possibility of cracks in the concrete. Satisfactory results have been obtained in a number of instances with stresses from 10,000 to 15,000 pounds per square inch. The likelihood of cracks will be reduced by using reinforcement giving mechanical bond, such as expanded metal, diagonal mesh or deformed bar, rather close spaced.

The thickness of concrete, except for small conduits under light pressure, should be at least 6 inches. When the pressure is considerable, it may be possible to reduce the possible leakage by use of a greater thickness with double lines of reinforcement and low tension in the steel.

Pressure conduits must be capable, like gravity conduits, of carrying any exterior loads which may come upon them when empty. They may be analyzed in the same manner as pipes or gravity conduits for exterior loadings.

Longitudinal reinforcement is required in conduits to prevent

cracking due to changes in temperature and shrinkage of the concrete. When the conduit is divided into sections by use of expansion joints, light reinforcement may be sufficient between joints, although closer spacing is desirable than is required for longitudinal reinforcement in bridges or culverts. When prevention of leakage is important, and the probable changes in temperature not too great, continuous closely spaced longitudinal reinforcement may give better results than the use of expansion joints.

CHAPTER XII

FOUNDATIONS ON DRY EARTH

ART. 55. FOUNDATION MATERIALS

212. Examination of Soil.—The stability of any structure requires that it be adequately supported by the ground upon which it rests, hence the nature of the soil upon which the structure is to be placed is the first subject for consideration in designing a foundation, and the local conditions under the surface of the ground must be determined. Numerous instances might be cited of the failure of structures due to lack of adequate investigation of soil conditions, and every effort should be made to obtain an accurate knowledge of the underlying strata.

For shallow foundations, *open excavations* may be made to a depth somewhat greater than that of the substructure, which will give the advantage of permitting the examination of the soil through and into which the substructure must be built and observing its condition. When the excavation is in wet material, pumping may be required to keep down the water and perhaps sheeting to prevent the sides caving in—an expensive procedure if excavating is carried to considerable depth.

Soundings are sometimes made with a rod, or small pipe about an inch in diameter, which is driven into the ground with a maul. When the material near the surface is soft, the depth to rock or other hard material may usually be determined in this manner if it is not more than 20 or 30 feet. Soundings serve to indicate whether resistance increases or decreases, and the depth at which hard material stops further progress. A number of soundings are usually necessary. A sunken log or boulder may stop the rod, and mistakes in interpreting the results of such soundings are easily made.

Borings with earth augers may be easily made for small depths with good results. Ordinary wood augers about 2 inches in diameter have also been used for this purpose, borings 100 feet deep having been made in this manner, though for ordinary work to more moderate depths, the use of earth augers of larger diameters give better deter-

minations. An auger 6 inches in diameter may readily be driven to a depth of 25 or 30 feet by two men with levers. It is held in vertical position by pipes or rods in sections, which may be coupled together as the hole becomes deeper, and is turned by hand with handles at the top 2 to 4 feet long, which are adjustable in position on the rods. The auger is screwed into the soil sufficiently to fill it with earth, and is then brought to the surface and the material examined, giving a good determination of the character of the soil at any depth, but not showing its degree of compactness. Augers for this purpose are sometimes fitted with a cylinder above the cutters to retain the sample of material through which the auger has passed. When the hole passes through material which will not retain its shape, a casing somewhat larger than the auger is driven, through which the boring may be done. When the depth to which the boring must extend is considerable, a block and fall, supported by a tripod, may be used to draw the auger from the hole.

Wash borings may be rapidly driven through soft soil or clay by sinking a casing, with a small pipe or hollow rod inside which carries a jet of water at its lower end. The jet cuts the soil at the bottom and brings up the excavated material through the annular space between the jet pipe and casing. It is usual also to have the bottom of the inside pipe fitted with a bit or chisel, which may aid in cutting into hard material. Both jet pipe and casing are rotated as they descend. When hard material is met, it may be necessary to cut it with the bit by churning the inside pipe. The bottom of the casing is also sometimes flared slightly and fitted with teeth for cutting.

When the depth is not great and only a small amount of work is to be done, ordinary water pipe about 2 inches in diameter is sunk as a casing, a smaller pipe $\frac{3}{4}$-inch in diameter being used inside. Hand appliances may be used in handling these pipes, a tripod with block and fall, levers for turning the pipes, and a hand pump for applying pressure to the jet. On more important work hollow rods for holding the jet and bits, special casings, and pipes with flush joints are necessary. These may be controlled by hand, or machine outfits similar to those used in drilling wells may be employed.

Examination of the materials brought up by the water shows the nature of the underlying strata. It does not, however, reveal the moisture or compactness of the material. It may therefore be desirable to obtain cores of the materials as they occur at certain points in the test holes, which may be done by substituting a cylinder for the jet and bit upon the end of the rod and pressing or screwing the cylinder into the soil at the bottom of the hole until it is filled with a

sample of the material, which is then drawn to the surface and examined. This may sometimes prevent mistakes in judging of subsurface conditions where the wet method of excavation is employed.

Since it is not always possible to distinguish between a boulder and ledge rock, wash borings, to furnish even approximately reliable information, must be taken very frequently over the surface under investigation. If an exceptionally high elevation of rock is found, the immediate vicinity must be explored by a number of borings, to determine whether the general rock surface holds up to this level. If not, it is probable that a boulder has been encountered.

As a general rule, the results of wash borings alone are not considered an indication of the true conditions, owing to the liability of false interpretations. Due to the comparatively low cost of this method, however, which will average one-quarter to one-third that of core drilling, it may prove a valuable adjunct of the latter by interpolating between the core holes and reducing the number of these otherwise necessary.

Core drills are used in testing rock strata. These consist of hollow circular bits, which are rotated so as to cut an annular channel into the rock, leaving a circular core on the inside of the core barrel to which the bit is attached. This core is removed at intervals for examination, and furnishes definite information concerning the character of the material. The core barrel is attached to hollow rods through which water may be supplied to cool the bit.

Several types of bits are used for this purpose; in some the cutting edge is formed of black diamonds or bort; in others, chilled shot are used under a hollow soft steel bit; or steel bits with teeth may be employed. When diamond drills are used, the cores are commonly from 1 to 2 inches in diameter; the other types are usually somewhat larger, varying from 2 to 4 inches in diameter.

Chopping bits are often used in connection with core drills, cores being taken at intervals and the intermediate cutting being done by the chopping drills. In any such work, complete drilling machines are necessary and they should be operated by men experienced in the work.

213. Bearing Capacity of Soils.—Definite values of bearing capacity for various soils cannot be stated with accuracy, because of the variations in character and condition of the same kind of soils and the consequent difficulty in classifying them. The ability of the soil to sustain loads depends not only upon its character, but also upon the amount of water it contains and the degree to which it is confined in position. The location and drainage of the foundation

as well as the character of the soil must therefore be considered in determining its bearing capacity.

Solid rock makes the best and most substantial foundation, and usually is capable of carrying any load that the masonry may bring upon it. The loose and decayed portions of the rock upon its surface need to be cut away, and the surface should be trimmed so that there will be no tendency for the structure to slip upon it.

Clay soils vary widely in character. They may be found in any condition from soft, wet clay, which will squeeze out laterally under light pressure, to hard, indurated clays capable of bearing heavy foundations without yielding. The supporting power is mainly dependent upon the amount of moisture contained in the clay. The tendency of clay to retain water which it may absorb and to soften as the amount of water increases is its most important property. Clays differ considerably in the readiness with which they absorb water. Compact, hard clays may by proper drainage usually be kept dry and capable of bearing heavy loads, frequently 8 to 10 tons per square foot, while wet clay may not safely carry more than 1 ton per square foot.

TABLE LXVIII.—SAFE BEARING CAPACITIES OF SOILS

Material.	Safe Bearing Capacity, Tons per Square Foot.
Rock, limestone or sandstone	15 to 30
Rock, soft or shale	5 to 10
Clay, dry and hard, thick beds	4 to 8
Clay, moderately dry	2 to 5
Clay, soft	1 to 2
Gravel and sand, well cemented	7 to 10
Gravel, coarse	5 to 8
Sand, dry and well cemented	3 to 6
Alluvial and soft soils	0.5 to 1

Sand or *gravel and sand* makes a good foundation when confined laterally so that there is no danger of it being washed out, compact gravel and sand being capable of carrying heavy loads without sensible settlement. Water will not soften it, and it is but slightly affected by frost. Loads of 8 or 10 tons per square foot seem to be conservative for such material under favorable conditions. Fine sand when saturated becomes soft and mushy and is easily displaced; it must be confined laterally to form a good foundation. Dry clean sand may carry loads of 2 to 4 tons per square foot, and

TABLE LXIX.—ALLOWED BEARING VALUES ON SOILS AT VARIOUS CITIES, IN TONS PER SQUARE FOOT

Index Number.	Index Number.
1. Quicksand and alluvial soil. 2. Soft clay. 3. Wet clay and soft wet sand. 4. Moderately dry clay, and fine sand, clean and dry. 5. Clay and sand in alternate layers. 6. Firm and dry loam or clay, or hard dry clay or fine sand. 7. Compact coarse sand, stiff gravel or natural earth.	8. Course gravel, stratified stone and clay, or rock inferior to best brick masonry. 9. Gravel and sand well cemented. 10. Good hardpan or hard shale. 11. Good hardpan or hard shale unexposed to air, frost or water. 12. Very hard native bed-rock. 13. Rock under caisson.

City.	Index Number.												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Albany	....	1	....	....	2	3	4						
Atlanta	....	1	....	....	2	2–3	3–4						
Baltimore	....	1	....	....	2	3	4	6	....	....	12–18*	20	24
Bridgeport	....	1	....	....	2	3	4						
Buffalo	....	....	....	....	....	....	3½						
Chicago	....	....	1½	1¾ to 2¼		2¼							
Cincinnati	....	....	....	2	....	4	4.5	....	8				
Cleveland	½	1	....	....	2	4	3	....	8	6	8	10	
Columbus	½	1	....	2	....	4	4	5	8	....	....	15	25
Denver	....	1	....	2	....	3	5						
Detroit	....	....	....	....	2	3	4						
Grand Rapids	....	....	....	3	....	....	4.6						
Jersey City	....	1	....	....	2	3	4						
Kansas City	....	1	....	....	2	3	4						
Los Angeles	....	1†	....	3	....	3	4						
Louisville	....	....	....	2½	....	2½	4						
Lowell	½	1	....	2	....	....	4	....	8				
Memphis	½	1	....	2	....	3	4	....	8				
Milwaukee	½	1	2	....	....	3	4	5	....	6	....	20	
Minneapolis	....	1	....	....	2	3	4						
Newark	....	1	....	....	2	3	4						
New Bedford	....	1	....	....	2	3	4						
New Haven	....	....	....	....	....	....	4						
New Orleans	....	¾	By tests										
New York	....	1	....	....	2	3	4						
Oakland	....	1	....	....	2	3	4	6	....	10	....	20	
Omaha	....	....	....	....	....	4							
Paterson	....	1	....	....	2	3	4						
Philadelphia	....	....	....	....	....	3½	....	....	6				
Providence	½–1	2–3	....	2–4	3–5	....	4–6	....	8–10	....	10–15	25–50	
Richmond	....	1	2	....	....	3	4						
Rochester	....	1	2	....	....	3	6	....	....	10			
San Francisco	....	1	....	....	2	3	4	6	....	10	....	20	
Seattle	....	....	1	2	....	4	....	....	6	8			
Spokane	....	1	....	....	2	3	4	8	....	10	....	20	
St. Louis	....	....	....	....	....	3							
St. Paul	....	1	....	....	2	3	4.6						
Syracuse	....	1	....	....	2	3	4						
Toledo	½	1	....	2	....	4	4						
Washington	....	1	....	....	2	3	4						

* Under caisson.

† Sandy loam.

when cemented with clay and protected from water it may safely carry loads of 4 to 6 tons per square foot.

When the top soil is loam or made land, foundations should go through such materials to natural subsoil beneath.

The thickness of the layer of material in which the foundation is placed and the nature of underlying strata are important factors in determining the supporting power, as well as the character of the foundation material itself. Foundations in hard clay which is soft underneath may sometimes safely carry $1\frac{1}{2}$ or 2 tons per square foot.

The pressures allowed upon foundations by the specifications of

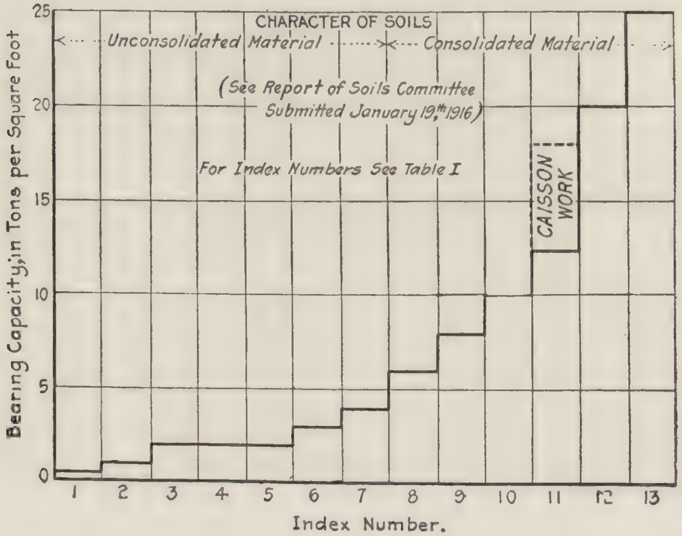


Diagram XVI.—Allowed Bearing Values on Soils of Various Cities. (Proc. Am. Soc. C. E., Aug., 1920.)

various authorities differ quite widely. The values in Table LXVIII represent the range of maximum pressures commonly given.

For the foundations of buildings, local conditions usually lead to a standard practice, and the building codes of the various cities are designed to insure safety under the particular circumstances of each place.

Table LXIX, taken from the 1920 report of the Committee on Bearing Value of Soils of the American Society of Civil Engineers, gives the requirements of various American cities.

The character of the structure to be carried by a foundation may frequently have an influence upon the choice of a limiting value for

the bearing capacity. Where a slight settlement in the foundation may be serious in its effect upon the structure, very conservative pressures should be adopted.

Diagram XVI, taken from the report of the Committee above mentioned, gives allowable values for the bearing capacities of soils with index numbers as listed in Table LXIX (p. 439).

The Committee suggests five types of failure due to soils, namely, compression, flowing, sliding, erosion and chemical changes, and gives the following discussion of each:

(1).—Compression.

"(a) *From Unequal Loading within the Elastic Limit of the Soil.*—Practically all soils possess elastic properties, the amount depending on the character and quantity of the cementing material giving it cohesion and the water content. If unequal loads per unit of area are placed on such soil, unequal settlements will take place. The soil, however, will recover on the removal of the loads unless the latter are great enough to overcome its elastic properties, by breaking down its cohesion. Such action is most apparent at some depth beneath the surface, as displacement is retarded by the restraining action of the surrounding soil. This unequal settlement, though slight, may be sufficient with certain soils and structures to cause trouble.

"As evidence of such action the rebound, or rising, after releasing the jacks, of piles forced into the soil by direct pressure, may be mentioned. The action is also apparent on releasing jacks and wedges when cribwork is used in temporarily supporting heavy loads, as in the underpinning of buildings, etc.

"(b) *From Loss of Cohesion.*—The cementing material is that which sometimes binds a sand or gravel and gives it a certain degree of stability as long as the cementing holds, even though such cementing is not sufficiently firm to permit the material to be classed as sandstone, as it would be if the cementing were more secure. Compression of material of this kind corresponds to the crushing of soft rock. It means the destruction of the structure of the cemented material and the conversion of the mass into a granular material without cementation. This change is normally accomplished by a decrease in volume.

"(c) *From Crushing Edges of the Grains.*—When a granular material is closely packed, weight is transmitted through it on a bearing surface represented by the narrow edges of many grains. When the pressure is increased, some of these edges are stressed beyond the breaking point and break, and this is accompanied by compression of the material. When the latter is composed of hard-grained silicious materials, the amount of compression under pressures commonly used in engineering works is usually so small that it may be neglected. When, however, the granular material consists, in whole or in part, of grains of softer material, compression in this way may be become of practical importance.

"(d) *From Shrinkage of Organic Matter.*—Materials may be considered as containing from 0 to 100 per cent of organic matter, ranging from clean sands deep below the surface, practically free from organic matter, through loams, poor surface soils, and river silts containing from 2 to 10 per cent or more of organic matter, up to rich surface soils and mucks, and, finally, to peat, which

is practically all organic matter. Many of these organic materials are subject to compression under pressure. Such materials are not ordinarily built upon, but they must be included in the classification, for completeness.

"(*e*) *From Loss of Water Content.*—This relates to fine-grained materials, containing so much water in their voids that some of it may be forced out. Under loading such material tends to become compressed, but the rate at which compression takes place is limited by the rate at which the excess quantity of water is able to make its exit, and this depends on the facilities for draining, including the resistance of the material to the passage of water. If the material were clay, so fine in grain as to be perfectly water-tight, the water could never make its exit and the particles would then not be compressible. With a slightly larger grain size the water would be slowly forced out, but it is possible that the rate of exit would be so slow that a gradual settlement might take place, extending even through a long term of years. This condition is found in some hydraulic fills, and it seems to be the most probable explanation of some cases where steady and long-continued slow settlement takes place.

"However, in undisturbed soils that have been long subjected to natural fluctuations in the water level, a stable condition of grain structure may exist, in which, with light loading, no appreciable compression may appear, yet the loss of water content may be considerable. This natural condition has been found in transported soils along the banks and contiguous to some rivers and tidal streams. It is a structural condition, the existence of which should be proven, and is attributable to the manner of deposition of the soil. If subjected to loading without investigation, it might cause trouble.

(2).—FLOWING.

"(*f*) *From Saturation.*—Any granular material temporarily saturated with water may flow. The condition of saturation may be brought about in either of two ways, first by flow of water of sufficient strength through a material to move the grains slightly apart, thus increasing temporarily the volume of the material and making 'quicksand' of it; or second, by the sudden compression or movement of material completely saturated with water.

"(*g*) *From Lack of Cohesion under the Influence of Weight and Pressure.*—This is the most commonly considered case of flowing. It has been more fully treated and is perhaps better understood than any of the others.

"(*h*) *From Exceeding Cohesive Strength.*—This differs from (*g*) in that cohesion plays an important part in limiting the flow. The flow is frequently slow and corresponds to that of an extremely thick or viscous liquid. Such flow ordinarily will not begin until certain limits of stress have been exceeded for that material, but when once begun, it will ordinarily continue under much lower limits of stress than were necessary to start the motion. In this respect the flow of material results from the breaking down of a certain weak structure due to cohesion or cementing, and is to be compared with the movement previously mentioned under (*b*).

(3).—SLIDING.

"(*j*) *From Sliding of Bodies on an Underlying and Usually Inclined Layer of Lubricating Material, Frequently Aided by Water; Particular Case of* (*h*).—This may be illustrated by landslides, such as that of the hillside on which some of the Portland, Ore., reservoirs are located.

"(*k*) *From Sliding of Previously Immersed Material when Water Level is Quickly Lowered; Particular Case of* (*h*).—By floods or a sudden recession of the water level, bodies of saturated materials become unstable and slide when relieved of the balancing water pressure. This has caused trouble in the slopes of dams and natural and artificial waterways.

"(*l*) *From Sliding of a Structure on the Soil.*—When such failures take place and reduction of the frictional resistance between the soil and structure has developed, the latter slides on the soil without any disturbance of the underlying material.

"(*m*) *From Sliding of a Structure Together with the Soil.*—Extensive areas sometimes move, taking with them entire structures without local disturbance due to the structure themselves.

(4).—Erosion.

"(*n*) *From Flowing Water and the Fluctuation of the Water-Table.*—Examples of trouble by reason of flowing water are evidenced at frequent intervals. Notably in the case of water-front and harbor works when tidal water rises behind such structures and causes erosion at stages of low water, and in the case of structures in or adjacent to streams, by erosion around bridge piers, abutments, etc., requiring rip-rap for their protection.

"The flowing of underground water has created sink holes, caverns, fissures, and caused trouble by undiscovered conditions of the soil.

"Failures of dams and levees have been caused by the percolating of water, and the consequent erosive action, by transporting the fine material of the foundation beneath the structure through the body of the dam or levee or overtopping the crest.

"(*o*) *From the Wind.*—The action of the wind causing erosion of the soil adjacent to structures is evident at many points where such structures are founded on the fine sand along lake or seacoast or desert land.

"(*p*) *From Weathering and Frost.*—Erosive action by frost is also apparent in northern latitudes when structures are not founded entirely below the frost range. This causes heaving of the material adjacent to the exposed face, and, on thawing, softening of the material beneath the structure, allowing slight settlement because of the reduced bearing value of the soil.

"When the exposed surface of the soil is inclined, the softening action of the sun and the freezing of the water content causes a tendency for the material to pass on down the slope. This action is sometimes apparent in railroad cuts where overhead bridge structures are built to carry highways over the railroads. Unless the foundations for such structures are carried to considerable depths, it is not uncommon after a few years to find that the movement of the soil from frost and sun combined has exposed a considerable portion of the masonry which was originally beneath the ground surface.

(5).—Chemical Changes.

"(*q*) While chemical changes rarely appear to be the direct cause of soil troubles they are included for the sake of completeness. All movements growing out of changes in the chemical composition of the material, particularly oxidation and hydration of deep-lying materials when exposed to the action of air and water during the progress of the work, may cause trouble.

"It is not to be supposed that any strict classification is possible. On the

other hand, movement of soils will frequently comprise conditions that would be classified under several of the headings, and there will be gradations between the different classes so that no strict lines can be drawn. Nevertheless, a classification of the way in which soils move under stress along some such general lines will tend to crystallize views as to the way in which various deformations take place and as to the means for guarding against them. It is especially to be hoped that discussion will aid in arranging for adequate tests of materials to determine the probability of their movement under the various kinds of stresses that may be brought to act on them, and so connect actual experience with these methods of getting at compressibility, cohesion, and other qualities of materials."

The depth of the base of the footing below the surface of the ground is important in many instances, the weight of the earth upon the sides being relied upon to confine the material and prevent it squeezing from under the footing and lifting the surrounding area. For compressible materials which will pack under the loads upon the footing, such side pressures may be of little consequence, but with materials which give way by being shoved to one side it is essential to have a sufficient weight of earth upon the sides to prevent this motion. According to Rankine's theory of pressures, the horizontal pressure produced by a given unit vertical pressure, p, upon the base will be $p\cdot\frac{1-\sin\phi}{1+\sin\phi}$, in which ϕ is the angle of internal friction for the material. This will cause a lifting pressure at the sides equal to $p\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$ which must not be greater than wx, when w is the unit weight of earth and x the depth below the surface The minimum depth required for stability is

$$x=\frac{p}{w}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2.$$

214. Tests for Bearing Capacity.—Direct tests of the capacity of the soil to support the loads coming upon a foundation are frequently desirable; they should be supplemental to the examination of the site and cannot replace such examination. They are intended to give a more accurate idea of the actual bearing capacity than can be derived from observation of the material upon which the foundation is to be placed and its underlying strata, and should therefore be made in the excavation at the level upon which it is proposed to place the base of the foundation.

The methods of making these tests vary considerably. Sometimes a small area is loaded and observations made of the settlement under varying loads, from which the probable safe bearing capacity

may be deduced. In other instances, a load of about twice that proposed for the foundation is placed upon a small area and settlement for different periods of time observed, with a view to judging the safety of the proposed loading. Usually a platform is employed to carry the load. The platform is customarily supported on a pier of about 1 square foot area, or sometimes upon four legs at its corners. The soil to be tested

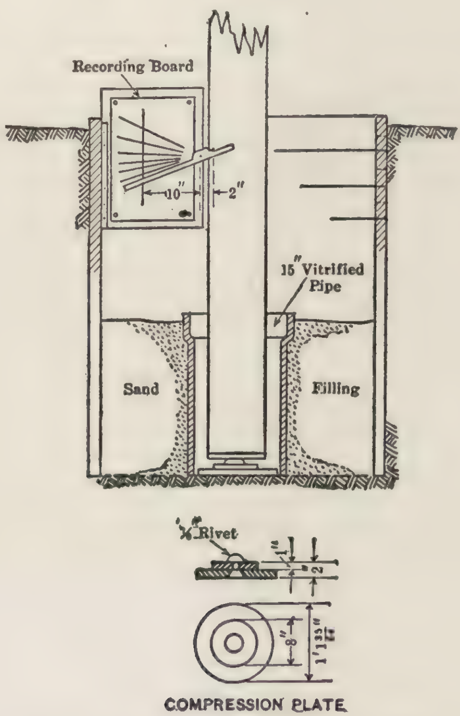


Fig. 126.—Load-testing Apparatus for Soils.
(Proc. Am. Soc., C. E., Aug., 1920. Part of Plate XI.)

is leveled to receive the piers and provision made for observing the settlement of the base of the pier under the loads. The platform must be so arranged as to bring uniform pressure upon the area under test.

The Committee of the American Society of Civil Engineers on the Bearing Value of Soils suggests the use of a standard form of apparatus for making these tests. This apparatus is illustrated in the proceedings of the Society for August, 1920. It brings the

pressure upon the soil through a circular steel compression plate, 1 foot, $1\frac{35}{64}$ inches in diameter. The load is brought upon the center of this plate through a 10 inch × 10 inch-wooden post with a steel footplate, as shown in Fig. 126.

"For measuring the amount of compression of the soil, the Committee suggests a mechanical device, very simple in character, which is preferable to the use of a wire and scale. The use of wire and scale requires that it be constantly watched, and that visual readings of the scale be taken at frequent intervals. The high resistance of most soils makes the reading of the smaller penetrations at different pressures rather uncertain, and a better curve of compression could be obtained by the method suggested. It consists simply of a board, on which is tacked a sheet of paper, with a straight-edge of thin material fastened at a certain fixed distance from a nail on the compression post, against the upper edge of the straight-edge. By drawing a vertical line on the paper, at a definite multiple of the distance between the hinge and nail in the post, a multiplied diagram of the compression for different loads is obtained, reading from a zero line drawn on the paper across the straight-edge before starting the test. At various pressures and compressions, similar lines can be drawn and the time and load noted on the paper, and the actual compression reduced by the ratio of the two distances. By this means a more accurate compression diagram can be made.

"Attention is called to the fact that this machine has practically only one moving part, *viz.*, the vertical post causing the compression of the soil. So long as this post maintains a purely vertical position, it is uninfluenced by any distortion of any other part of the machine. From the behavior of timber and soil under such conditions, the Committee is inclined to believe there will be no difficulty in maintaining the post in a vertical position. It should be borne in mind that this is intended to be a practical field testing machine and no great degree of accuracy should be expected. However, such a machine will take compression tests of soil with a considerable degree of accuracy."

The time element is frequently a matter of importance, settlement in some soils occurring gradually during a period of twenty-four or forty-eight hours, until a stable position is reached. Some soils are elastic under working loads, and the settlement diminishes as the load is decreased after a test.

The resistance offered by the soil to pressure upon a small area is not necessarily the same as that which may exist over a large area, and the results of such tests must be used very conservatively in the design of foundations. These results, however, when combined with careful observations of the character of the materials underlying the foundation, give a basis upon which to form a judgement of safe bearing capacity.

In designing the foundations for the proposed new Union Station in Chicago, a unique method of testing the bearing power of the soil was adopted. It is described by the Chief Engineer, J. D'Esposito, in "Engineering and Contracting" for March 26, 1924.

The piers tested consisted of two concrete cylinders, especially

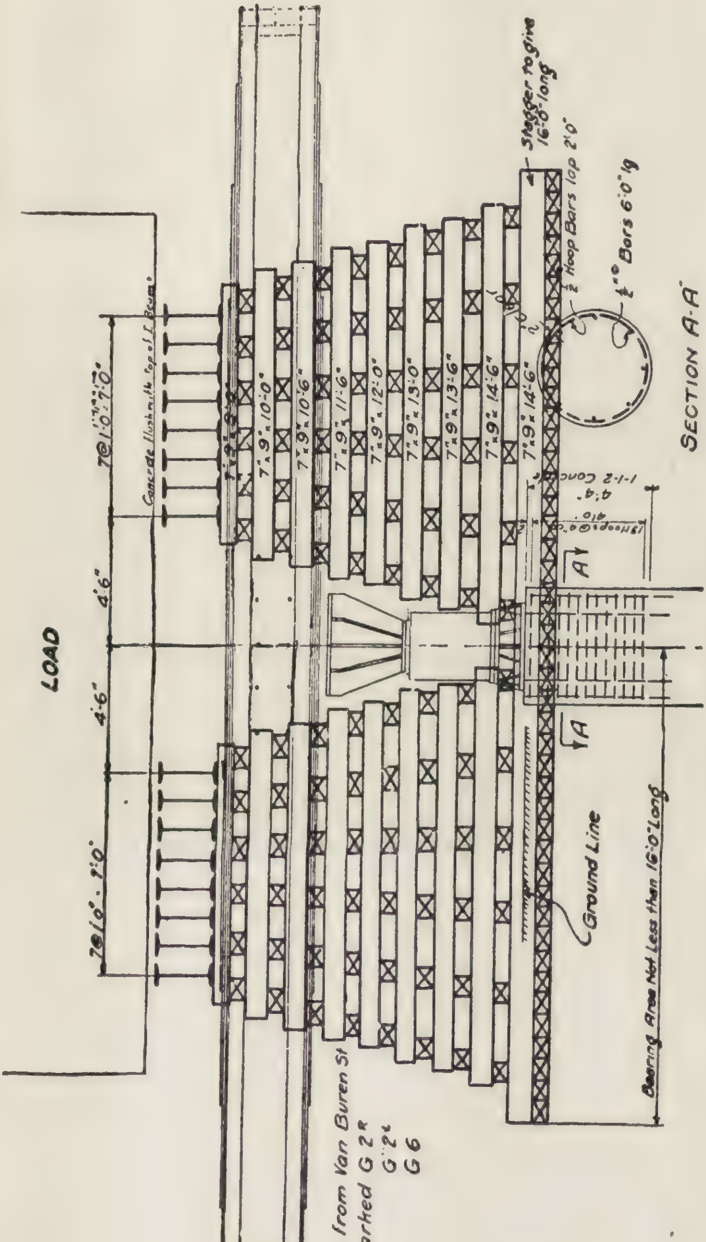


Fig. 127.—Elevation Showing Arrangement of Loads, Grillage and Hydraulic Jacks With Respect to Caisson to Be Tested. (Engineering and Contracting (Buildings), March 26, 1924, page 672.)

built for the test, but by the same labor and methods and with the same materials as were being used in placing the actual foundations. Special caissons were constructed for the purpose in order to secure a site adjacent to a railway track, as the loads to be handled were considerable. These caissons were sunk to hardpan, which is found in this locality at a depth of about 65 feet below the city datum. One cylinder was built with a belled base, expanded to an area of about 56.7 square feet, the other was a straight cylinder, with a base of about 14.5 square feet.

The loading consisted principally of about 1200 tons of steel rails, and the method of applying this load was the novel feature. To balance it on the cylinder pier would have been awkward and uncertain. It was therefore supported upon timber cribwork built on the ground adjacent to the pier, and used as a bearing for an hydraulic jack resting on the cylinder. Pressure up to the weight of the loading could be transmitted to the pier by means of the jack, all without disturbing the position of or varying the amount of the load, and without lifting it off its supports. (See Fig. 127.)

After testing the two piers for bearing pressure, a determination was made of the supporting power of the straight cylinder by surface friction alone. A shaft was sunk at a distance of 40 feet from the pier and a tunnel was driven to its base. From this tunnel all the material under the pier was removed, leaving it supported by the friction on its sides. A settlement of less than $\frac{1}{4}$ inch was observed at a surface friction of 600 pounds per square foot, and $\frac{3}{4}$ inch at 700 pounds per square foot. By this time, however, motion had started, and the value of the friction diminished, so that a settlement of $5\frac{1}{2}$ inches was finally reached with a surface friction of about 535 pounds per square foot.

Assuming an equal value for surface friction in both cylinders, it was found that the settlement of the belled base pier for the same unit soil pressure was greater than that of the straight pier. A load of 13 tons per square foot on the soil produced a settlement of $\frac{7}{8}$ inch in the belled cylinder and $\frac{5}{16}$ inch in the straight pier. Similar settlements for a unit loading of 8 tons were $\frac{5}{16}$ inch and $\frac{1}{8}$ inch respectively. This appears to indicate that the surface friction, assumed equal for both piers, was actually considerably less for the belled cylinder.

The importance of a careful investigation of the site of a proposed foundation cannot be overemphasized. Records are numerous of failures where subsurface conditions had not been properly explored on account of the cost or difficulty of such work, or where designs have been based upon inadequate investigations.

It has been said that 10 per cent of the estimated cost of a projected structure may well be spent in investigating the foundation. While this may be excessive, it is nevertheless true that a failure is always expensive. A definite knowledge of the character of a foundation will secure lower bids, eliminate costly alterations in plans made necessary by the disclosure of unfavorable conditions as the work progresses, and will save the cost of possible damage to the work, due to failure of the foundation. Thoroughness in the preliminary exploration is well worth its cost.

ART. 56. SPREAD FOUNDATIONS

215. Distribution of Loads.—When bedrock is at considerable depth, it frequently becomes necessary to spread foundations over large areas near the surface of the ground by the use of footings at the bases of columns or walls. The method to be employed in such work depends upon the area of soil required to support the loads and the extent of the footings necessary beyond the bases of the walls or columns. When the extensions are small, masonry footings may often be employed to advantage, and this is the most common type of foundations for light buildings upon firm soil. When footings must extend to greater distances beyond the bases of the walls or piers, grillage or reinforced concrete footings occupy less space and are more economical.

In foundations of this type some settlement is usually to be expected, and the object should be to make this settlement as small and as uniform as possible. Spread foundations on compressible soil are expected to settle, and in placing them allowance is made for this settlement. In Chicago, it has been common to allow from about 4 to 7 inches for settlement and some buildings have settled even more than this without serious injury to the structure. The loads to be carried by the different parts of the foundation should be ascertained and the footings so proportioned as to bring uniform pressure upon the soil under the foundation. Inequalities in the settlement of the foundations of buildings are apt to crack the walls, injuring the appearance when not sufficient to impair the stability of the structure. To produce uniform pressure it is necessary that the center of pressure of the load pass through the center of area of the base of the foundation.

In determining the loads which may come upon the footings in the foundation of a building, the dead loads and live loads are separately computed. The entire dead load is always upon the foundation,

while the live load may vary, and only such portion as may reasonably be assumed usually to exist should be used in estimating the load distribution upon the footings, which will depend upon the character of the building. In hotels, office buildings, etc., while the floors of each portion should be designed to carry the maximum live load which could come upon them, only a small percentage of the total of this live load can reach the footings at once, and it is common to neglect it altogether. In churches, theatres, etc., the maximum floor loads are more apt to occur, and a larger percentage should be used in designing the foundations. The building codes of the various cities commonly prescribe the loads to be used in designing foundations for buildings.

Engineers and building codes differ widely concerning the proportion of live load to be used in designing footings. The lowest section of columns should be designed for the maximum live load that may reasonably be expected to occur, and the footings should be capable of carrying this load without exceeding a safe pressure upon the soil. But these maximum loads should not be used in proportioning the relative areas of the various footings. When full live load is used in designing footings, as such load would not usually exist, the interior columns would ordinarily be designed for a larger proportion of live load and may settle less than exterior walls and cause a hump in the building. Where no live load is used in proportioning the footings, the opposite effect may be produced when the building is subjected to live load and a sump caused in the building.

Some leading engineers have adopted the policy of first determining the footing for that column which carries the largest percentage of live load by dividing the sum of the dead load and full live load by the maximum safe value of pressure upon the soil. The dead load plus one-fourth or perhaps one-third of the live load is then computed for this column and divided by the area already obtained. This reduced pressure per square foot is then used in designing the other footings taking the loads as equal to the dead loads plus one-third or one-fourth of the live load as may be selected.

When the exterior walls of a building carry much of its weight, the center of pressure should be somewhat inside the center of the footing, thus avoiding any tendency to tip outward and crack the walls of the structure; a tendency to tip inward will be resisted by the interior walls and floors. The rigid connection of a lightly loaded interior wall with a heavily loaded exterior one often causes an eccentricity of loading in the foundation which produces serious cracks. When a

series of openings one above the other through the wall of a building cause the loads to be brought to the foundation through piers between the openings, the footings should be disconnected and properly centered for each pier, unless the foundation has sufficient stiffness in itself to distribute the loads over its whole base. The walls of many buildings are cracked over the openings by the use of continuous foundations in such cases.

216. Masonry Footings.—For light loads, footings of brick or stone masonry or of concrete are commonly employed. Where suitable stone is available, stone masonry is often the most economical but concrete is now usually preferred. Brick footings are less desirable on account of the likelihood of the deterioration of the bricks when used under ground.

In placing *stone footings*, the stones must be carefully bedded so as to bear evenly upon the foundation soil. The projection of the footing, when of considerable extent, is stepped off as shown in Fig. 128. The width of a step should not ordinarily be greater than two-thirds of the height of the course, and a stone should not project more than one-third of its length beyond the course above. Footing stones under walls carrying heavy loads should be large and roughly squared, and should be set in a thick bed of mortar to give even bearing upon the soil beneath.

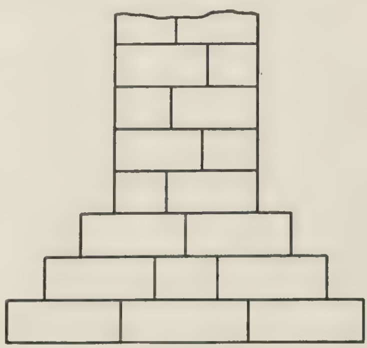


Fig. 128.—Wall Footing.

Plain concrete footings are usually stepped off in the same manner. As the concrete footing is a monolithic structure and capable of carrying small tensile stresses, the projecting step may be considered as a cantilever carrying the upward thrust of the soil upon its lower surface.

Let t = the thickness of the footing at any point;
o = the projection of the footing beyond the point where the thickness is t;
p = the pressure in pounds per square foot on the bottom of the footing;
f = unit stress upon the concrete due to bending.

Then the allowable projection for any given thickness is

$$o = t\sqrt{48f/p}.$$

Thus, if we assume the safe tension on the concrete to be 60 lb./in.^{2}, and the pressure upon the foundation soil as 2 tons per square foot, $o = .85t$, or the projection should not be greater than .85 of its thickness.

The projections for cut stone is which each stone is the full height of the course may be estimated by the above formula, provided the stones may be considered as firmly held in place under the wall. When placed upon compressible soil, however, the pressure will not be uniformly distributed over the base of the stone, and there is likelihood of tipping the block if the projection is too great.

Under brick walls, a bed of concrete is usually employed at the base and the brickwork stepped off on top of this to give the required extensions. The offsets in such work should not be more than three-quarters of their heights, which may be composed of two courses of brick.

217. Grillage Foundations.—When a foundation must be spread over an area which is large compared to that of the column or wall resting upon it, a masonry footing becomes uneconomical and a footing possessing greater transverse strength and requiring less height becomes desirable. For such foundations, grillages of timber or steel or reinforced concrete slabs are commonly employed.

Steel I-*beam grillages* are now very frequently used under heavy buildings. The constructon of foundations of this type was begun in Chicago about 1880. In founding heavy buildings upon the clay subsoil, it was necessary to spread the footings over considerable areas, and room was not available for masonry footings, as the subsoil was soft at greater depths. A footing consisting of several layers of old steel rails encased in concrete was devised and used for some time. This was soon replaced by I-beams of sufficient depth to carry the loads in a single layer, thus saving space and giving better economy in the use of the metal.

A grillage footing as applied to the foundation of a single column is shown in Fig. 129. Such foundations rest upon a bed of concrete and are enclosed by a filling and surfacing of concrete for the protection of the steel. Under heavy loads, the bed of concrete is usually about 12 inches thick and the protective coating from 3 to 6 inches thick. The beams should be held by spaces at least 3 inches apart in the clear in order to permit filling the spaces with concrete. Under a continuous wall, a block of plain concrete is usually employed instead of the upper series of I-beams.

In designing a grillage footing, the loads to be carried and the areas of the walls or piers are known and the grillage must be so

placed as to bring the center of its area in the line of action of the resultant load. The total load may be considered as distributed uniformly over the base, giving uniform upward pressure upon the beams, while the downward thrust of a pier is taken as uniformly distributed over its section. Usually a grillage is centered under each column or wall, proportioned to the load to be carried, but two or more loads may be carried by a single grillage when it seems desirable.

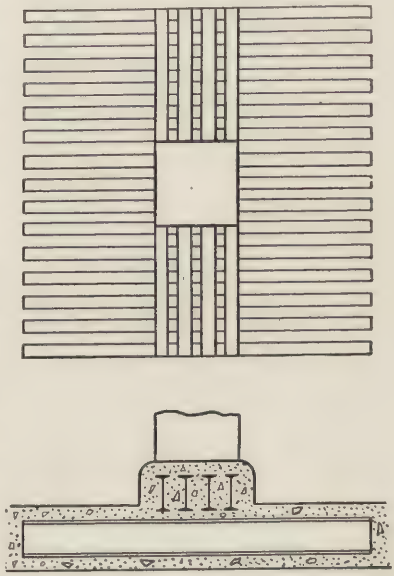


FIG. 129.—Steel Beam Grillage.

Figure 130 shows a footing supporting two piers each 2.5 feet square, one carrying a load of 300,000 pounds and the other 400,000 pounds, spaced 10.5 feet between centers. The soil pressure is limited to 4000 lb./ft.^{2} and an area of 175 ft.^{2} is required. If this area be assumed as 17.5 feet by 10 feet as the center of gravity of the loads is 4.5 feet from the center of the pier carrying the larger load, the piers will

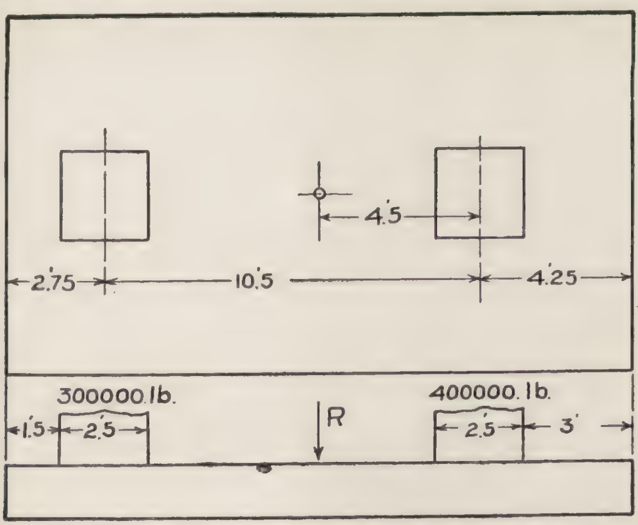


FIG. 130.—Double Column Footing.

occupy the positions shown, when the pressure is uniform upon the soil.

The upper tier of beams under the heavier load carries 400,000 pounds distributed over 2.5 feet at the middle acting downward on its upper surface, and the same load distributed uniformly over the length of 10 feet, acting upward on its lower surface. The maximum moment will be at the mid-section and will be

$$M=200000\times 5/2-200000\times\ 625=375000 \text{ lb.-ft.}$$

If the allowable unit stress in the steel is 16,000 lb./in.^{2},

Section modulus, $S=375000\times 12/16000=281$ in.^{3}, and we might use

2—24-in. 80-lb. I-beams, $S=173.9$ each, flange 7.0 in. wide
2—20-in. 80-lb. I-beams, $S=146.6$ each, flange 7.0 in. wide
3—18-in. 60-lb. I-beams, $S=\ 93.5$ each, flange 6.1 in. wide

The 20-inch beams require less concrete than the 24-inch, and less steel than the 18-inch and may be used, although the spacing is rather wide. The flanges are spaced 10 inches apart and 3 inches inside the block of concrete.

Under the load of 300,000 pounds, S should be 210 in.^{3}, and two 20-inch 65-pound I-beams may be used.

The lower tier of beams carries two loads of 400,000 and 300,000 pounds respectively, acting downward upon its upper surface, each distributed over 2.5 feet as shown, and a load of 4000 pounds per square foot uniformly distributed over its lower surface. There are sections of maximum moment under each load and at some point between them. These sections are where the shear passes through zero. Let y=distance from end of beam to section. Under the heavier load, the shear is

$$4000\times 10y-\frac{400000}{2.5}(y-3)=0, \text{ and } y=4.$$

Then

$$M=4000\times 10\times\frac{4^2}{2}-\frac{400000}{2.5}\times\frac{(4-3)^2}{2}=240000 \text{ lb.-ft.}$$

For the mid-section, $4000\times 10y-400000=0$, and $y=10$.

Then

$$M=4000\times 10\times 10^2/2-400000(10-4.25)=-300000 \text{ lb.-ft.}$$

The greatest moment is 300000 lb.-ft. or 3600000 lb.-in., and the required S is $3600000/16000=225$ in.^{3}

Eleven 9-in. 25-lb. I-beams, $S = 20.4$ each, flange 4.45 inches wide, clearance 7 inches, may be used. Three or four additional beams may be introduced if thought desirable to reduce the clearance. If this is

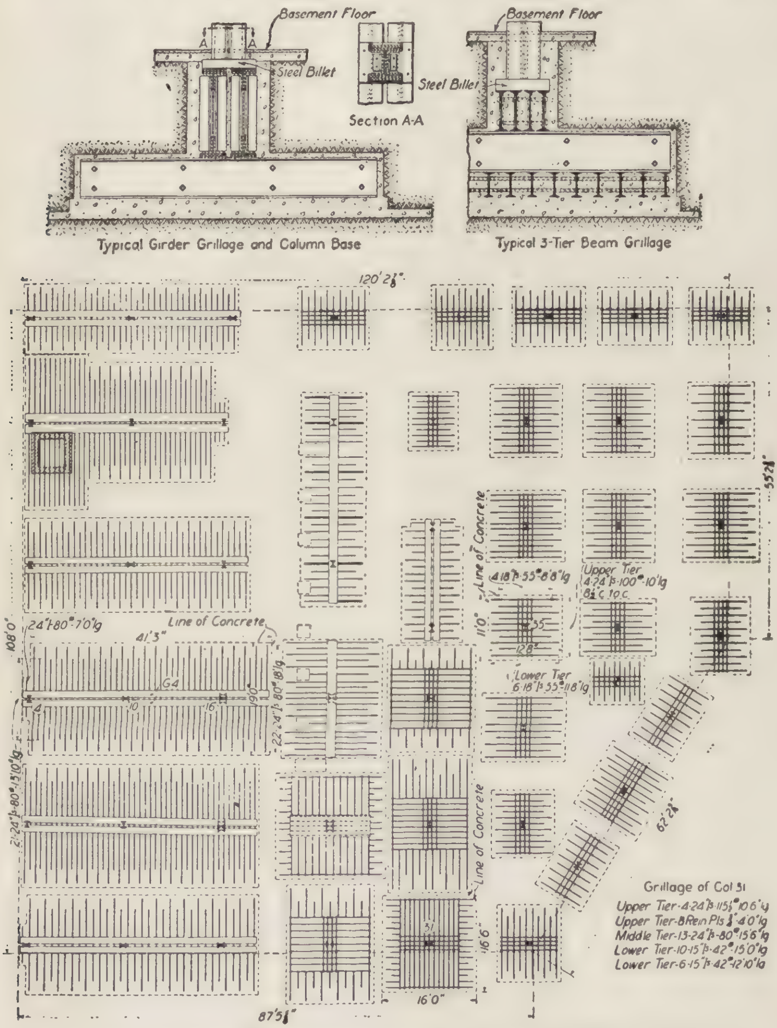


FIG. 131.—Grillage Foundation of the L. C. Smith Building, Seattle. (C. C. Williams, "Masonry Structures and Foundations.")

not done, light transverse reinforcement might be placed in the concrete covering the beams.

The moments might be somewhat decreased and the positive and negative moments made more nearly equal by making the foundation

narrower upon the end carrying the smaller load and widening the other end. The same steel area would then be needed at both ends and the spaces between the beams would widen from one end to the other.

Figure 131 shows the grillage foundation of the L. C. Smith Building[1] at Seattle. "The foundations rest on soil composed of sand gravel, and clay in strata of varying thickness. Borings indicate that this alternate stratification extends to an indefinite distance below the street. The foundations have been proportioned for a load of 5500 pounds per square foot, dead load only being considered. On the north side of the building, the columns are carried on pairs of cantilever girders extending over three column points. Underneath the girders is a single tier of 24- or 20-inch I-beams. This construction was necessary to provide for the required foundation without extending over the lot line and still keep the center of gravity of the foundations concentric with the center of gravity of load. Underneath the bottom of the grillage beams there is a concrete bed 12 inches thick. The grillage beams and girders are filled between and encased in solid concrete.

"The cantilever girders from about 35 to 41 feet long are all pairs of single-web plate girders 54 inches deep, with 6×6 to 8×8-inch flange angles and 14 or 18-inch flange cover plates."

Timber grillages may be employed where the footing is so located as to be continually wet. They are also commonly used for temporary footings which are to be removed in a comparatively short time. These foundations are usually constructed by placing a layer of 2-inch planks on the bed to be occupied by the footing and across these one or more series of timbers in the same manner that the I-beams are used in the steel grillages. The timbers must be capable of carrying the bending moments due to transmitting the loads from the walls or piers to the soil upon which the footing rests. On top of the grillage a floor, usually of 3-inch plank, is placed to carry the base of the masonry. All timber in such foundations must be kept below low water and the spaces between the timbers should be filled with sand or broken stone.

218. Reinforced Concrete Footings.—Reinforced concrete slabs are ordinarily used as footings for the distribution of loads in spread foundations. When used under walls, these consist of a cantilever projecting on each side of the wall; the determination of thickness and amount of reinforcement is made as for a simple cantilever. When used under columns or piers, the load may transmitted to the slab

[1] Engineering Record, July 6, 1912.

through beams, or flat slabs with two-way or four-way reinforcement may be employed.

When beams are used, the moments may be computed by the methods used for I-beam grillages and reinforced concrete beams and slabs with one-way reinforcement designed to resist these moments in the usual manner. If the construction is monolithic, the maximum stresses occur in the sections where the slabs join the beams and in the beams where they join the base of the pier. The stresses in such foundations may be accurately computed in so far as the loads are known, and they are not subject to the assumptions required in the flat-slab computations. Usually these footings are cheaper in cost of materials than flat-slab footings, but require more form work in construction.

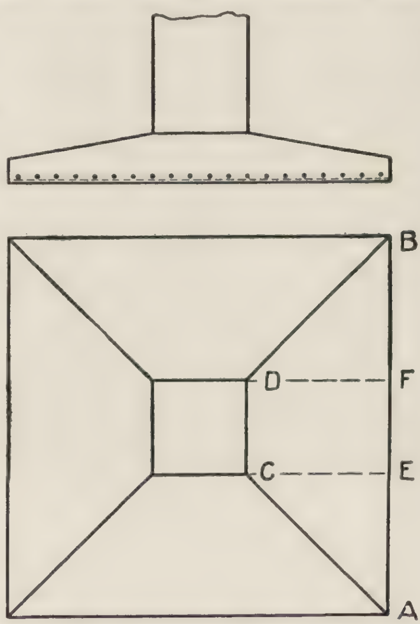


Fig. 132.—Two-way System Flat Slab Footing.

In a flat-slab footing with two-way reinforcement, the maximum moment in the slab occurs in the sections through the face of the pier. In the footing shown in Fig. 132, it is assumed that the section through each face carries the moment between that face and the side of the footing. Thus, the moment of the upward pressures on the area *ABCD* is supposed to be borne by the section *C–D*. These moments are not uniformly distributed over the section, but must be greater in the portion between *C* and *D* than in its ends. From experiments made at the University of Illinois, Professor Talbot [2] concludes that "For footings having projections of ordinary dimensions, the critical section for the bending moment for one direction (which in two-way reinforced concrete footings is to be resisted by one set of bars) may be taken to be at a vertical section passing through the face of the pier. In calculating this moment, all the upward load on the rectangle lying between a face of the pier and the edge of the footing is considered to act at a center of pressure located at a point halfway out

[2] Bulletin No. 67, Engineering Experiment Station, Univ. of Ill.

from the pier, and half of the upward load on the two-corner squares is considered to act at a center of pressure located at a point six-tenths of the width of the projection from the given section.

"With two-way reinforcement evenly spaced over the footing, it seems that the tensile stress is approximately the same in bars lying within a space somewhat greater than the width of the pier and that there is also considerable stress in the bars which lie near the edges of the footing. For intermediate bars, stresses intermediate in amount will be developed. For footings having two-way reinforcement spaced uniformly over the footing, the method proposed, for determining the maximum tensile stress in the reinforcing bars, is to use in the calculation of resisting moment at a section at the face of the pier the area of all the bars which lie within a width of footing equal to the width of pier plus twice the thickness of footing, plus half the remaining distance on each side to the edge of the footing. This method gives results in keeping with the results of tests. When the spacing through the middle of the width of the footing is closer, or even when the bars are concentrated in the middle portion, the same method may be applied without serious error. Enough reinforcement should be placed in the outer portion to prevent the concentration of tension cracks in the concrete and to provide for other distribution stresses.

"The method for calculating maximum bond stress in column footings having two-way reinforcement evenly spaced, or spaced as noted in the preceding paragraph, is to use the ordinary bond-stress formula, and to consider the circumference of all the bars which were used in the calculation of tensile stress, and to take for the external shear that amount of upward pressure or load which was used in the calculation of the bending moment at the given section."

Example.—A column 2 feet square is to carry a load of 300,000 pounds on soil that may safely carry 3000 pounds per square foot. It is required to design a square footing with two-way reinforcement, using concrete of 2000 pounds compressive strength and unit stress of 16,000 lb./in.^{2} upon the steel.

The required area of footing is $300000/3000 = 100$ square feet. A base 10 feet square will be used.

The thickness of footing required for shear at base of column is

$$t = \frac{300000 - 4 \times 3000}{4 \times 24 \times 120} = 25 \text{ in.}$$

Using Talbot's rule, the moment of the load upon *DCEF* (Fig.

132) is $2 \times 4 \times 3000 \times 2 \times 12 = 576000$ in.-lb.; that of the loads DFB and ACE is $4 \times 4 \times 3000 \times 2.4 \times 12 = 1382400$ in.-lb.

Total, $M = 576000 + 1382400 = 1958400$ in.-lb.

The effective width of section is $2 + (2.1 \times 2) + 1.9 = 8.1$ feet.

The depth required for moment is (Formula (9) Chapter VI)

$$d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{1958400}{108 \times 97}} = 14 \text{ in.}$$

If we use the depth of 25 inches,

$$A_s = \frac{M}{f_s jd} = \frac{1958400}{16000 \times .875 \times 25} = 5.6 \text{ in.}^2$$

Nineteen $\frac{5}{8}$-inch bars in the width of 8 feet gives an area of 5.8 in.^{2} and a spacing of about 5 inches. Four additional bars or 23 in all should be used in the full width of 10 feet.

The maximum shear is equal to the load upon the area $ABDC$,

$$\frac{300000 - 4 \times 3000}{4} = 72000 \text{ lb.}$$

and the bond stress is

$$u = \frac{V}{\Sigma o jd} = \frac{72000}{19 \times 1.96 \times .875 \times 25} = 88 \text{ lb./in.}^2$$

This is rather high for plain bars, but deformed bars may be used, and the ends hooked.

According to Talbot's rules, the shear for diagonal tension may be computed on a section distant the depth of footing from the base of the pier, which will give a shear

$$V = [(10)^2 - (2 + 2 \times 2.1)^2]3000 = 184680 \text{ lb.},$$

and a unit shear

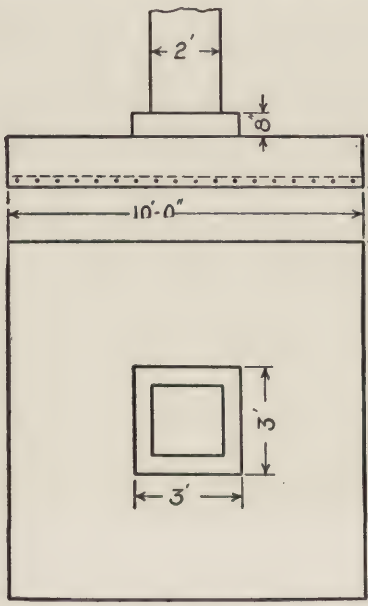
$$v = 184680/[4(24 + 2 \times 25) \times .875 \times 25] = 29 \text{ lb./in.}^2$$

and no diagonal tension reinforcement is necessary.

The volume of concrete in the above footing may be decreased by widening the base of the pier or placing a block of concrete under it as shown in Fig. 133. If a step 6 inches wide be used, making the block 3 feet square, the depth of footing required is found to be 16

inches. Reinforcement for diagonal tension would be required for this depth, but by increasing it to 17 inches the shear may be so reduced as make this unnecessary. This change would decrease the volume of concrete required by about 30 per cent and increase the weight of steel by 20 to 25 per cent.

Four-way Reinforcement.—When a four-way reinforcement is used, each set of bars is supposed to carry an equal share of the moment. As the lengths of the diagonal bars are not the same as those parallel to the sides of the footing, this supposition is only approxi-



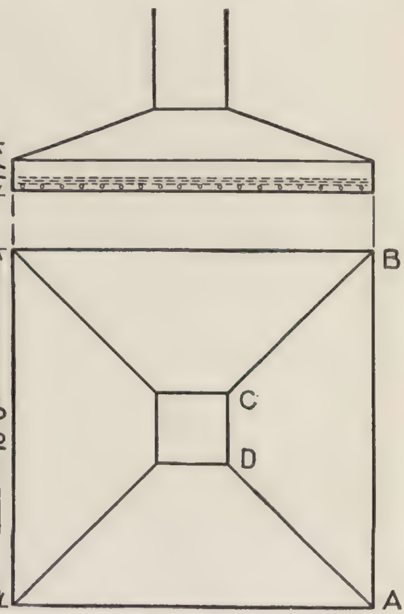


FIG. 133.—Footing with Stepped Base. FIG. 134.—Four-way Reinforcement.

mately correct, but in the absence of more definite information concerning the distribution of stress it may be used in design.

If a four-way reinforcement be used in the example already given, as shown in Fig. 134, the depth required for shear at the base of the pier will be as before, 25 inches. The moment of the upward thrust upon the area $ABCD$ about the section CD is, as before, 1,958,400 in.-lb. If the width of section be supposed to carry all of the compression due to this moment, the depth of section required will be

$$d=\sqrt{\frac{M}{Rb}}=\sqrt{\frac{1958400}{108\times 24}}=28 \text{ in.}$$

The depth to the steel will be made 28 inches at the base of the pier and slope to 6 inches at the edges of the slab, thus giving greater depth than necessary at all intermediate points.

$$A_s = \frac{M}{f_s jd} = \frac{1958400}{16000 \times .875 \times 28} = 5.0 \text{ in.}^2$$

Sixteen $\frac{9}{16}$-inch square bars may be used. The maximum bond stress will be

$$u = \frac{V}{\Sigma ojd} = \frac{72000}{16 \times 2.5 \times .875 \times 28} = 74 \text{ lb./in.}^2$$

Eight bars will be placed parallel to the edge of the footing and eight on the diagonal in each direction; they may cover a greater width than the base of the pier, and will be spaced 5 inches apart, thus making each band 3 feet wide and covering the whole area in a satisfactory manner.

The method for diagonal tension in a slab of this form has not been satisfactorily worked out. If we apply the method proposed by Professor Talbot for slabs with flat-top surface, we have, on a section distant 28 inches from the base of the pier,

$$V = [(10)^2 - (6.67)^2]3000 = 166600 \text{ lb.},$$

and

$$v = \frac{166600}{4 \times 80 \times .875 \times 15.2} = 39 \text{ lb./in.}^2;$$

and no diagonal tension reinforcement is necessary. As the section to resist diagonal tension is increased by the slope of the top surface of the beam, it seems reasonable to employ the method in this instance.

Various modifications of these forms of footing are often employed, depending upon the same principles in design, but varied to suit special needs or to secure greater economy in the use of materials. Ribs may sometimes be used to advantage in distributing the loads upon the slab.

In designing footings for the New York County Court House, at Worth and Center Streets, New York City, a somewhat unique method of placing spread foundations was devised by Messrs. Moran, Maurice and Proctor, Consulting Engineers, consisting of concrete ring girders. The foundation was to be placed on a bed of sand and gravel at a depth of 33 feet below curb grade, the sand and gravel extending to rock at a depth of from 150 to more than 210 feet. The building was to be hexagonal in plan with the principal interior walls nearly concentric circles. To avoid cracking these walls and

arches, it was necessary to secure uniform support under the columns arranged in circles or hexagons. Each ring of columns was therefore supported by a single girder forming a complete circle and made of sufficient strength to carry the loads of any two columns to adjacent parts of the ring if all the support were withdrawn beneath them. The width of the footing under the girders was varied so as to give a uniform load of two tons per square foot on the soil.[3]

219. Combined and Monolithic Footings.—The design of a steel grillage footing to support two columns in the case of a wall column and a heavier interior column has been illustrated in Section 213, and similar combined footings of reinforced concrete are used.

Cantilever Foundations.—When it is necessary to carry the side walls or wall columns of buildings upon footings which cannot project beyond the face of the wall on the outside, cantilever footings are often employed, wherein the wall columns rest upon one arm of a cantilever beam, the other arm of which carries an interior column, the cantilever being so proportioned as to center the total load upon to footing which supports it. Footings of this type are often necessary when the loads upon the wall columns are greater than those upon the interior column, so that the ordinary combined footing is not applicable. The designs for such footings vary widely according to the requirement of the particular foundation and the ingenuity of the designer.

The same results may sometimes be obtained by using a trapezoidal footing, with its center of gravity coinciding with the center of gravity of the column loads. In this case the two columns would be carried on the ends of a uniformly supported beam.

It may frequently be convenient to carry three or more columns upon a single footing. In such a design, when the columns are so placed as to make it possible, two columns are carried upon one beam with the third column on a parallel beam, each beam being arranged to transmit uniform loading to the footing beneath.

When a footing carries more than two loads which are located nearly in a straight line so that they may rest upon a single beam, it is evident that the method of distributing the load by placing the center of footing at the center of gravity of the loads is not exact, on account of the continuity of the beam. Strict accuracy would require that the center of soil resistance be determined by considering the loads as reactions of a uniformly loaded continuous beam. The error, however, would usually be unimportant as compared with the possible variation of the loads themselves from their estimated values.

[3] Engineering News, March 24, 1921.

Monolithic Foundations.—Foundations are sometimes constructed so as to be practically monolithic over the whole area covered by a structure. These usually consist of a slab covering the area of the foundation and carrying a series of beams, or of inverted arches,

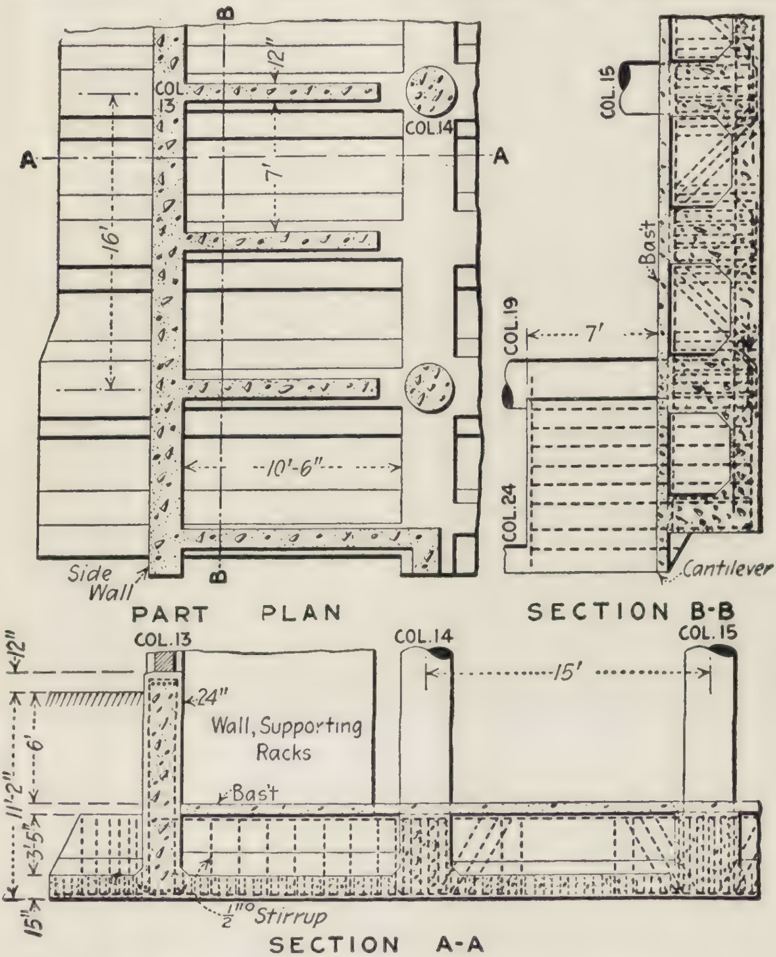


FIG. 135.—Monolithic Foundation under the Felt & Tarrant Building. (Jacoby and Davis, "Foundations of Bridges and Buildings.")

through which the loads brought down by the walls or columns are distributed to the slab. The slab is in effect a floor in inverted position subjected to the upward pressure of the soil and may be designed in the same manner as a floor slab. The beams when used may be considered as T-beams, the slab acting as flanges. Foundations of this type are suitable for buildings upon compressible or unstable

soils, where the loading is distributed fairly uniformly over the area covered by the building.

Figure 135 shows the monolithic foundation of the Felt & Tarrant Building[4] at Chicago. This foundation consists of a series of reinforced concrete beams supported on a continuous slab of con-

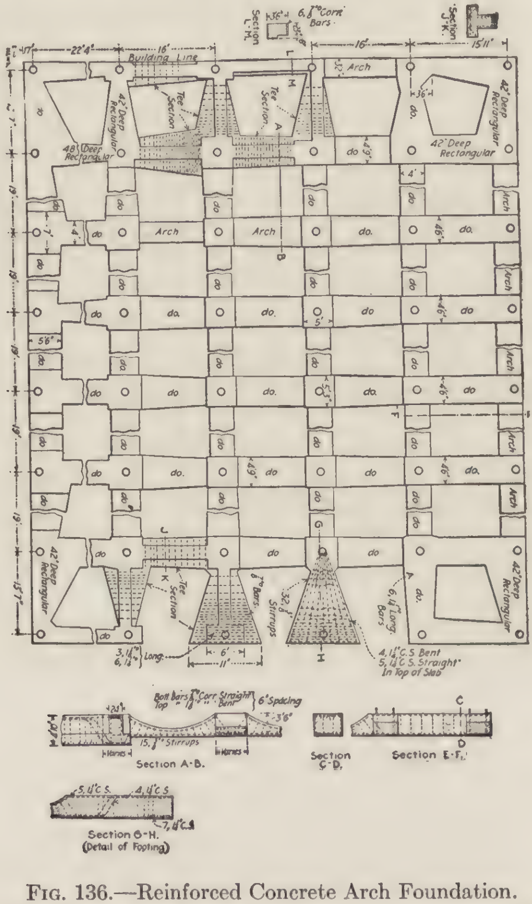


FIG. 136.—Reinforced Concrete Arch Foundation.
(Courtesy of Jacoby and Davis, Foundations of Bridges and Buildings.)

crete. The slab and beams extend beyond the side walls of the building, the extension being greater on the side of heavier loading, to equalize the pressure upon the soil.

In the foundations of a warehouse at 418-426 West 25th Street,[5] it was necessary to use a very shallow foundation and the inverted

[4] Engineering News-Record, Nov. 1, 1917.
[5] Engineering News, December 28, 1911.

reinforced concrete arch foundation shown in Fig. 136 was devised to meet the conditions. The arches were made continuous, practically monolithic, and all of the same thickness, the width being varied for a proper distribution of load to the soil. The total foundation load was 16,450 tons, and the ground area 11,100 square feet. A loading of 3 tons per square foot was taken to be safe and the foundation designed on the basis of this loading.

" The foundation consists of a series of inverted concrete arches running in both directions between columns, as shown in Fig. 136. The arches are 12 inches deep at the crown and 42 inches under the cast-iron column bases, they vary from 4 to 5 feet in width. The reinforcement comprises $\frac{7}{8}$-inch straight round corrugated bars, spaced 6 inches on centers in the extrados, and $1\frac{1}{8}$-inch bent bars, spaced 6 inches on centers, in the intrados.

" The extrados of the arch was designed flat to give even bearing on the soil, and the bars in the bottom (straight) are two spans long and are lapped at alternate columns, so that half the bars are continuous under each column. The splices of the others are long enough to develop the full tension in the bars. It was thought that by following this method, more monolithic foundations would be secured and that the foundation would get the benefit of whatever cantilever action there might be.

"All of the end spans are made of either rectangular or T-shaped concrete beams, in order to provide for the thrust from the adjoining arch, which is taken up by the friction between the soil and the foundation, developed by the loads from the columns and also taken up by the thrust against the outside bank."

ART 57. PILE FOUNDATIONS.

220. Classification of Piles.—A pile is a stick of timber or other material driven longitudinally into the soil for the purpose of increasing its power to sustain loads, or to resist lateral pressures. Piles are usually of timber, concrete or metal, and are further classified according to the methods used in placing them or the uses for which they are intended.

Piles are among the oldest forms of foundation work known. Furthermore, methods of using and placing them have changed very little since ancient times. The use of reinforced concrete and of the steam hammer constitute about the only real improvements. Where permanently covered by water, timber piles are preserved indefinitely. In rebuilding the Campanile of St. Mark's, Venice,

piles that had been in place for 1002 years were found in good condition and allowed to remain in the new foundation.

Bearing piles are those which carry the weight of a structure resting upon them. They may act in either of two ways: (1) When they are driven through a bed of soft material to a firm stratum below, the pile acting as a column and receiving little or no support from the material through which it is driven. (2) When the piles reach no firm material, but are sustained by the frictional resistance of the material through which they are driven and the compacting of the soil about their upper ends.

Batter piles are driven at an inclination to resist lateral forces, and are commonly used where cross bracing of the vertical piles may not give sufficient lateral stiffness to the structure.

Sheet piling consists of piles driven in close contact for the purpose of forming a tight wall to resist the pressure of water or soft material, being commonly used in cofferdams to prevent leakage into excavations.

Guide piles are frequently used to assist in holding a caisson in position during sinking, or to hold in position the horizontal timbers against which sheet-piling is to be sunk.

Screw piles consist of cast-iron or steel pipes with a broad screw upon the bottom for the purpose of giving large bearing area. They are driven by screwing them into the soil, and have sometimes been satisfactorily used in sand or gravelly soils.

Disk piles are pipes with horizontal circular plates stiffened by radial ribs, fastened to their bottom ends, as shown in Fig. 137. They are sunk by the use of a jet of water, which washes the soil from beneath the disk, allowing it to be forced down. The diameters of disks range from about 2 to 4 feet and of pipes from about 6 to 12 inches.

Screw and disk piles are now considered obsolete, except as the latter sometimes find a use in lighthouse foundations, where their resistance to an upward pressure is of value. Both types, however, are difficult to get down, although the water jet will aid materially, and they are not suited to localities where scour may occur, as they cannot be braced below water. In future work, concrete piles will probably take their place.

Sand piles are sometimes used for the purpose of increasing the bearing capacity of the soil by compacting it laterally for short depths below the surface. They are placed by driving short wooden piles, then withdrawing them and filling the holes with sand, which should be damp when placed and should be tamped so as to compact it in the

hole. Sand for this purpose possesses the advantage of transmitting a certain amount of pressure laterally, and adjusts itself to any slight settlement that may take place.

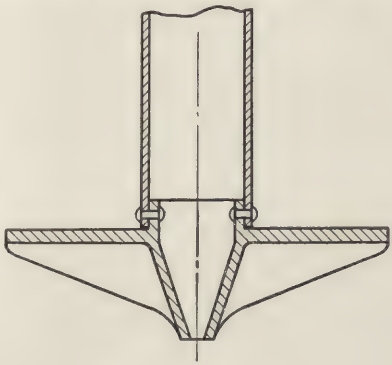


Fig. 137.—Disk Pile.

221. Pile-Drivers.—An ordinary pile-driving machine consists essentially of the leads, the sheaves and the hoisting drums, with their appurtenances. The leads are two parallel uprights between which the pile is held in position while being driven, and which form a guide for the hammer. At the upper end of the leads are the sheaves, over which the lines pass which are used to handle the pile in placing it in position and to raise and lower the hammer in driving. The leads are supported in position by a triangular framework braced with backstays. The platform or

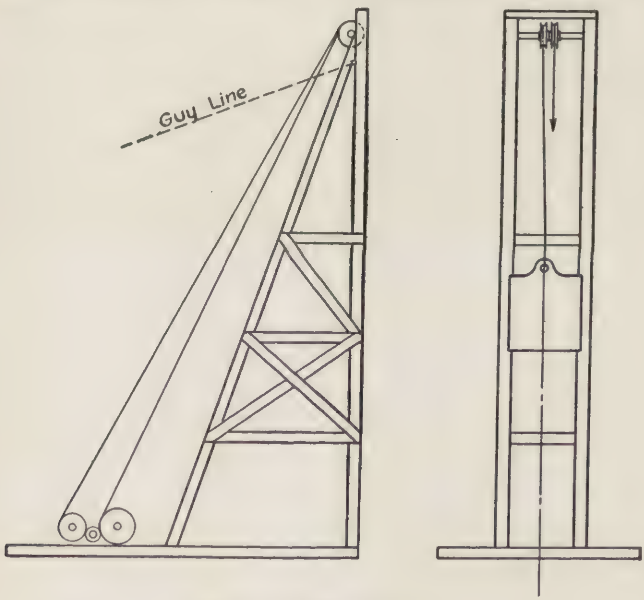


Fig. 138.—Pile-driver.

deck to which the framework is attached also carries a hoisting engine with friction drums for handling the pile and hammer lines. The general arrangement is shown in Fig. 138. The details of arrange-

ment and method of mounting vary widely according to the service for which the machine is intended.

Pile-drivers may be so mounted as to move forward, backward, and to the side by the use of rollers, or made to turn in any direction by mounting upon a turntable. For river work, they are ususally rigidly connected to the deck of a barge which is moved to place the driver in position.

For railway work, drivers are commonly mounted upon cars, and many of them are very carefully designed to render efficient service under varying conditions. The cars are made self-propelling to make the machine independent of locomotive service, and leads which can be quickly raised and lowered are employed. The drivers are mounted upon turntables which permit driving upon either side, and the leads are arranged so that they may be turned to an inclined position for the purpose of driving batter piles. The stability of the machines when driving at the greatest reach from the cars is important and must be carefully considered in design. Combination machines, in which service as pile drivers is added to that as derricks or as excavators are also frequently employed.

A drop-hammer, as used in driving piles, usually consists of a solid casting, which is raised by means of a rope and allowed to drop upon the head of the pile. The hammer slides in guides upon the leads and should be so shaped as to give it a low center of gravity and a sufficient length to cause it to slide in the guides without rocking; it may be given a free fall by the use of nippers which engage a pin upon the hammer and are automatically disengaged at a certain height upon the leads. The more common method, however, is to raise and drop the hammer by the use of a hoisting drum with a friction clutch, the rope being permanently attached to the hammer—a more rapid system, which permits the operator to regulate readily the height of fall. The weight of hammer employed in ordinary work varies from about 2000 to 3500 pounds. For light work in small operations, light hammers may be used, while for heavy service and unusual conditions, heavier ones may be necessary. A heavy hammer with low fall is more effective in driving than a light hammer with high fall, as it may be operated more rapidly and causes less vibration in the machine.

The over driving of wooden piles with a heavy hammer may frequently injure the piles and cause them to become practically worthless for carrying loads. A hammer of 3000 to 3500 pounds' weight should not have a fall of more than 10 or 12 feet, and the driving should stop before the pile refuses to move an inch in the last three

or four blows. Numerous instances have been observed of piles splintered at the bottom, or broken at mid-length where driving had been kept up to an unreasonable extent with the idea that the bearing capacity would thereby be increased.

It has been stated on good authority that more piles have been made unsafe by overdriving than by insufficient driving. Experienced operators claim that overdriving is evidenced by a bouncing of the hammer as it strikes the pile. In a test to determine the safe limits of driving, a 3000-pound hammer with a drop up to 25 feet was used. All piles were afterwards pulled and inspected. Practically every one driven with falls of over 10 feet was damaged.

A steam pile-hammer is one which is raised and dropped by a steam piston working in a cylinder attached to a frame which rests upon the head of the pile. The frame slides in the guides upon the leads, and the striking part or hammer is guided by the frame. In some of the steam hammers the pistons are attached to the striking weight; in others, the cylinders are the moving parts.

Steam pile-hammers are of two types—single acting, in which the weight is raised by the steam pressure and allowed to drop by gravity; double-acting, in which the steam pressure is used to accelerate the downward motion of the hammer and increase the force of the blow. Single-acting hammers are made heavier and of longer stroke than double-acting ones for the same service, and are slower in action. For heavy service, single-acting hammers usually have strokes of 36 to 42 inches and strike 50 to 70 blows per minute, while the double-acting kind have strokes from 12 to 24 inches and strike 120 to 200 blows per minute. Lighter machines may work much faster.

The blows of the steam hammer are so rapidly given that the motion of the pile is practically continuous and under many conditions the effectiveness of the driving is thereby greatly increased. There are few data giving definite information concerning the relative costs of driving by drop-hammer or steam-hammer, but the steam-hammer has seemed to be gradually replacing the drop-hammer in important operations. It has as advantages that of causing less damage to the head of the pile; the driving may be accomplished at a more rapid rate, and more piles may usually be driven in the same time; the wear and tear upon the machine is much less than in the use of the drop-hammer, although the first cost of the steam-hammer is considerably greater.

Water-jet pile-drivers are fitted with appliances for discharging a jet of water at the foot of the pile. The water comes up around the pile, bringing with it much of the material cut from beneath the pile

and lessening the friction resisting its descent. The water-jet equipment is usually a straight piece of pipe, which may be held alongside the pile, with a nozzle at its lower end, the upper end being connected by a flexible hose to a pump which supplies water under pressure. The driver is equipped with leads and hammer, the latter being used to assist in sinking the pile by light blows and to settle it firmly into place after the jet is stopped.

The water jet is especially applicable to driving piles into sand, which usually offers considerable resistance to driving by the hammer alone. It may be used in any material which will be washed up by the jet and puddled about the pile, and frequently effects large savings in costs of driving. The pressure and volume of water required depend upon the kind of material to be penetrated. The pressure must be sufficient to cut the material and the volume enough to bring it up alongside the pile. Pressures of 75 to 150 lb./in.^{2} and volumes from about 50 to 200 gallons per minute are common.

" In driving in sand [6] the jet should be hung on a rope passing over a pulley in the driver so that it may be kept moving up and down with its point near the point of the pile. If this is not done the pipe is likely to 'freeze' fast and cannot be moved. After the pile reaches a depth of 10 or 12 feet the water will sometimes fail to come up around it, breaking out on the surface at a considerable distance, perhaps around a pile driven previously. When this occurs it indicates that the jet has not been kept moving sufficiently, or an auxiliary jet may be needed discharging at some intermediate depth. In any case the jet should be withdrawn at once and immediately put down again, thus usually reestablishing the flow of water along the pile. When piles are sunk 20 feet or more into sand it is advisable to have two jets. One is to be kept moving with its nozzle slightly ahead of the pile, while the other is slowly raised and lowered between the foot of the pile and the surface to maintain the flow along the pile."

In driving pre-molded concrete piles, it has sometimes been found advantageous to churn the pile. A wire bridle is attached near the top of the pile by which to lift it and it is churned up and down with a stroke of from 18 to 30 inches. The hammer is usually allowed to rest upon the top of the pile during the operation and the jet is kept in action. After sinking the pile by this method as far as possible, the hammer is used to settle it in place. In some instances, a hole has been sunk by the water jet, or two or three holes close together to the full depth to which it is desired to sink

[6] Jacoby and Davis, Foundations of Bridges and Buildings, p. 45.

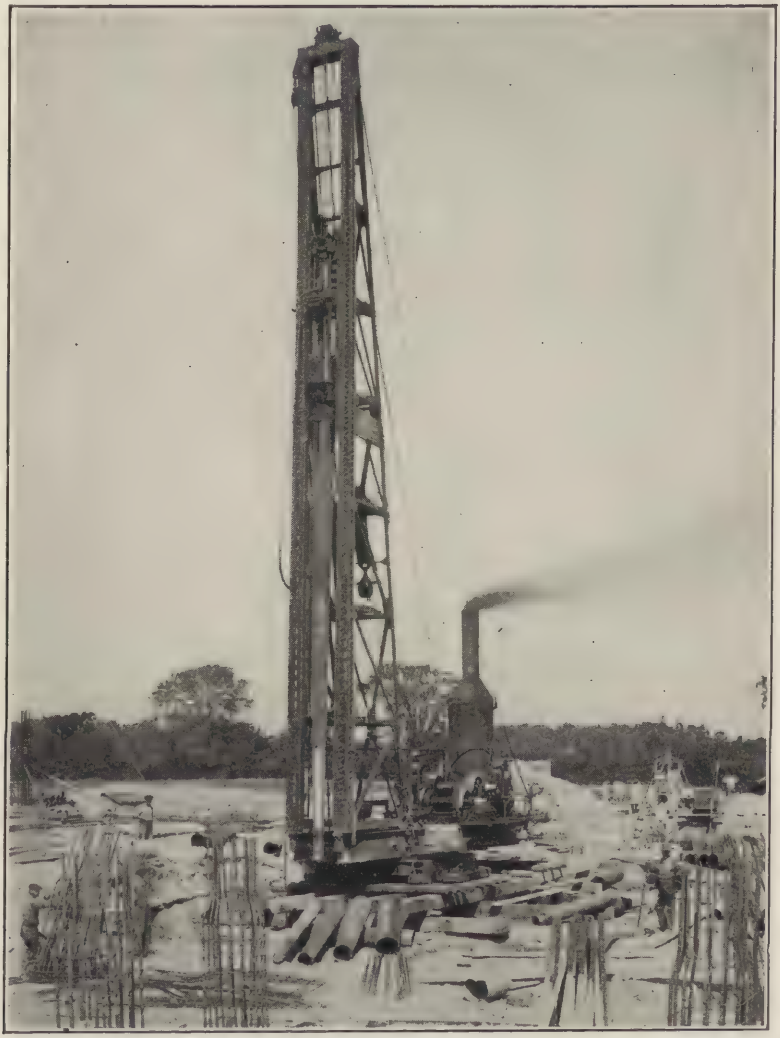


FIG. 139.—An Up-to-date Pile-driver.
(Courtesy of the Raymond Concrete Pile Company.)

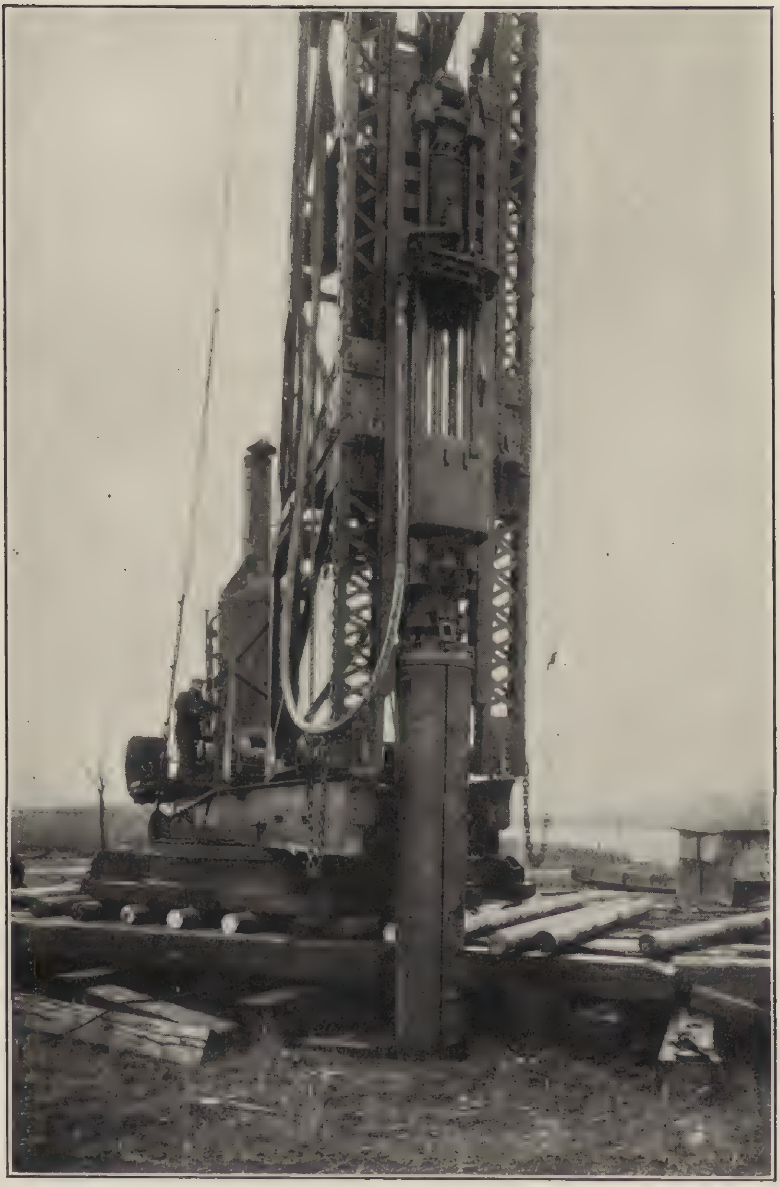


FIG. 140.—Steam Pile-driver—Detail of Hammer.
(Courtesy of the Raymond Concrete Pile Company.)

the pile, and then the pile is driven by the use of a hammer, or by churning. This has sometimes been found to save time in driving.

It was formerly believed that the jet should be attached to the side of the pile as it sank, and the earlier concrete piles were molded with a pipe in the center for a water jet. It has been found, however, that when the jet is attached alongside the pile, the latter will drive off line, towards the jet. Present practice favors a jet operated free of the pile, and sometimes in deep driving, two or even three jets may be used with good effect.

The depths to which piles are sometimes jetted just because this may be easily done is out of all reason. It should be made certain that the pile is below the limits of scour, and that it either reaches a stratum firm enough to bear the load or that the penetration is sufficient for the surface friction to support the load. A greater depth is unnecessary, as even with very easy driving the material will settle firmly around the pile after the jet is stopped. In some cases it has afterwards been found impossible to move the pile with the hammer.

In gravel or porous soil the jet is sometimes ineffective, as the water filters away, taking the finer particles with it. The coarser material that is left makes very hard driving. This condition may ordinarily be remedied by increasing both the volume and the pressure of the water.

In city foundation work the water jet must be used sparingly and with great caution, as the escaping water may undermine other structures and cause their settlement.

The water jet greatly reduces the cost of driving in sand or hard soil. However, it adds considerably to the cost of operating the plant. It is economical, therefore, only where the number of piles to be driven is so large that the resulting saving will exceed the increased cost of operation.

Figure 139 shows an up-to-date pile-driver, while Fig. 137 shows a detail view of the hammer of a steam pile-driver.

222. Timber Piles.—A timber pile is usually the lower portion of the trunk of a tree, from which the branches and bark have been removed. It is nearly circular in section and tapers from butt to tip. Many kinds of timber are employed for the purpose. The conifers—yellow pine, Douglas fir, spruce, and cedar—are commonly obtainable in straight pieces of considerable length. White and post-oak piles are not so straight, but are tough and hard and are suitable

when requirements are severe. Cedar is valuable on account of its durability. For ordinary work in foundations, piles are usually required to be not less than 6 inches in diameter at the tip, and commonly vary from 10 to 18 inches at the butt.

The specifications of the American Railway Engineering Association name the following requirements for timber piles:

Railroad Heart Grade

1. This grade includes white, burr, and post oak; longleaf pine, Douglas fir, tamarack, Eastern white and red cedar, chestnut, Western cedar, redwood and cypress.

2. Piles shall be cut from sound trees; shall be close-grained and solid, free from defects, such as injurious ring shakes, large and unsound or loose knots, decay or other defects, which may materially impair their strength or durability. In Eastern red or white cedar a small amount of heart rot at the butt, which does not materially injure the strength of the pile, will be allowed.

3. Piles must be butt cut above the ground swell and have a uniform taper from butt to tip. Short bends will not be allowed. A line drawn from the center of the butt to the center of the tip shall lie within the body of the pile.

4. Unless otherwise allowed, piles must be cut when sap is down. Piles must be peeled soon after cutting. All knots shall be trimmed close to the body of the pile.

5. The minimum diameter at the tips of round piles shall be 9 inches for lengths not exceeding 30 feet; 8 inches over 30 feet but not exceeding 50 feet and 7 inches for lengths over 50 feet. The minimum diameter at one-quarter of the length from the butt shall be 12 inches and the maximum diameter at the butt 20 inches.

6. The minimum width of any side of the tip of a square pile shall be 9 inches for lengths not exceeding 30 feet; 8 inches for lengths over 30 but not exceeding 50 feet, and 7 inches for lengths over 50 feet. The minimum width of any side at one-quarter of the length from the butt shall be 12 inches.

7. Square piles shall show at least 80 per cent heart on each side at any cross-section of the stick, and all round piles shall show at least 10½ inches diameter of heart at the butt.

Referring to Specification 4, above, a test has shown that in the case of piles cut in December, January, February and March, the respective strengths were as 100, 88, 80 and 62 per cent. As to the peeling, bark is sometimes left in place as a protection against marine borers.

Railroad Falsework Grade

8. This grade includes red and all other oaks not included in Railroad heart grade, sycamore, sweet, black and tupelo gum, maple, elm, hickory, Norway pine, or any sound timber that will stand driving.

9. The requirements for size of tip and butt, taper and lateral curvature are the same as for Railroad heart grade.

10. Unless otherwise specified piles need not be peeled.

11. No limits are specified as to the diameter or proportion of heart.

12. Piles which meet the requirements of Railroad heart grade except the proportion of heart specified will be classed as Railroad Falsework grade.

Piles are driven with the tips down, although in some instances it is desirable to drive the butts down. In certain soils, as quicksand, the upward pressure on the sides of the piles may force the pile upward after being driven with the tip down. Where piles are being driven through soft material to a hard substratum, it may be desirable to drive them with the butts down in order to obtain larger bearing surface at the base.

It has been noted that piles already driven have a tendency to rise when other driving occurs in the immediate vicinity. This is an indication that the soil is thoroughly compacted by the placing of the piles and rises instead of compressing further. The high piles, if previously cut off to grade, may be settled by a few extra taps of the hammer, or if this causes others to rise, they may be recut.

The butt of the pile is cut off accurately at a right angle to its length in order that the blow of the hammer may be uniformly distributed over the section. When the hammer strikes directly upon the head of the pile, it is common to use a hammer with a slightly concave upper surface. This tends to keep the pile centered in the leads, and minimizes the brooming effect of the blow. Heavy blows upon the head of a pile have a tendency to splinter and broom it, and a portion of the energy of the blow is used up in injury to the pile. When the brooming effect has become considerable, the efficiency of the driving is greatly decreased, and a large portion of the work is wasted. It has frequently been observed that when the broomed head of a pile has been cut off, an increase in the penetration under each blow is obtained, the penetration being in some cases more than doubled.

This was shown in the case of a pile that had required an average of 468 blows per foot to drive it from a penetration of 12 feet to 22 feet. The head of the pile was then cut off to remove the brooming, and the next few feet were driven at an average of only 244 blows per foot.

Sometimes, on account of damage, for inspection, or in removing temporary staging or falsework, it is necessary to pull piles. This

is done by tackle on the pile-driver, by means of a loop of chain or wire cable around the pile. The suction and frictional resistance has been known to equal 1800 pounds per square foot, or three times the value of the friction in supporting a load. It may occasionally be necessary to hit with the hammer a pile that is to be pulled, in order to loosen it from the earth that has settled around it.

Pile rings are frequently placed upon the heads of piles to reduce the brooming effects. They are made of wrought iron from 2 to 4 inches wide and $\frac{1}{2}$ to 1 inch thick; the pile is chamfered off so that the ring may be started on and be driven into place by the hammer. The rings are used repeatedly and serve for a larger number of piles.

Pile caps consisting of cast-iron blocks with tapered recesses above and below are used for the same purpose. The head of the pile is fitted into the lower recess and a hardwood block into the upper one. The block is reinforced by a ring at the top and receives the blow of the hammer. The cap fits into the guides of the leads, and holds the head of the pile in place. After the pile is in place, the cap is drawn from its head by being attached to the hammer. Some steam hammers are provided with anvils, which rest upon the head of the pile and receive the blow of the hammer.

In driving piles through hard material, it is often desirable to point the lower end, by cutting the end of the pile in the form of a pyramid, a blunt end 3 or 4 inches square being left at the bottom. A thinner point is apt to be too easily injured.

Shoes, either of cast-iron or formed of strap iron reinforcement for the pile are frequently used with success on piles driven into clay and compact gravel, but in hardpan and cemented gravel a shoe may cause splitting and brooming of the base. In such cases a cast-iron shoe, shaped like an inverted pyramid, with a deep socket to receive the foot of the pile, has given good service. The sharp corners are very effective in cutting hard soil.

When piles are needed of greater length than those available, it becomes necessary to splice two piles together, which is accomplished by the use of fish-plates. The ends of the two piles are cut square and butted together, the sides are trimmed flat for a considerable distance on each side of the splice and long wooden fish-plates are spiked to the sides, four or six fish-plates being commonly used. Another very effective splice is formed of a length of pipe, into which both upper and lower pile are inserted, after being trimmed to fit. This prevents splitting at the splice.

After bearing piles are driven for supporting the foundation of a structure, they must be cut off at the proper elevation to receive the footings. When a timber grillage is to be placed on top of the piles, it is essential that they be cut accurately to the proper level, and that they be low enough to place all timber work below the permanent water level. When concrete footings are to be used, the concrete should enclose the tops of the piles to a depth of about a foot, but it is not necessary that they all be cut off at precisely the same level. For cutting piles under water, a circular saw mounted upon a vertical shaft, which may be raised and lowered from the leads of the pile-driver, is frequently employed.

Where piles are to be driven into deep water and a considerable length afterward cut off, a large saving may be made by driving with a *follower*, which is a length of hardwood that is placed on top of the pile to be driven, and receives the blows of the hammer. When the pile has been driven the proper distance under water, the follower is removed and used on another pile.

Recently, however, steam hammers have been equipped for submarine work, and by means of extension leads on the pile-driver, the hammer is made to follow the pile down to the desired elevation. On the Burnside Bridge over the Willamette River, Portland, Ore.[7], the water is 80 feet deep at the site of the bridge. Piles 110 to 120 feet long would have been required to secure the desired penetration if driven and cut off in the usual manner. Followers were not considered practicable on account of the hard driving. Telescoping leads 70 feet long were rigged, and by means of a submarine hammer piles 40 to 50 feet long were driven until their tops were 10 feet above bottom. About 800 piles were driven at an average rate of 8 piles per $7\frac{1}{2}$-hour shift.

When timber piles are to be used in sea water, it is necessary to use some preservative process to guard against the action of the borers, (the *teredo navalis* and the *limnoria terebrans*) which destroy the timber in a short time if unprotected. In warm climates, like the Gulf of Mexico, the timber may be honeycombed in a few months.

The usual method of preservation is to creosote the timber. This is done by placing the timber in a cylinder where it is subjected to a steam pressure until the wood is thoroughly sterilized and the sap liquefied. The steam pressure is then released and the sap is draw

[7] Engineering News-Record, Jan. 8, 1925.

out with a vacuum pump, the temperature being maintained to cause the sap to vaporize and pass off through the pump. When all the moisture has been exhausted, the cylinder is filled with the creosote oil and pressure is applied and maintained until the specified quantity of oil has been forced into the timber. To secure good results it is nec-

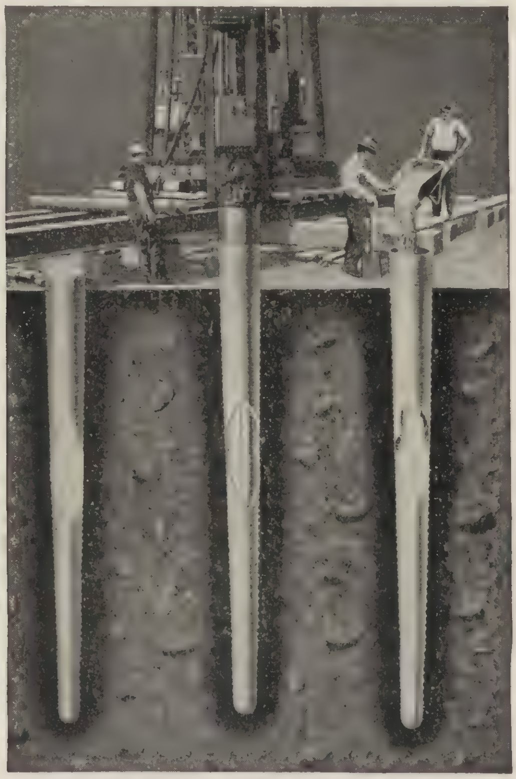


FIG. 141.—Raymond Concrete Pile—Standard System.
(Courtesy of the Raymond Concrete Company.)

essary that the timber be of open grain in order to take the preservative sufficiently. Hard, close-grained timber takes the treatment with difficulty.

223. Concrete Piles.—Timber piles in structures intended to be permanent must be cut off below the water line, while concrete may be used without reference to moisture conditions. In many instances, therefore, the use of concrete piles is more satisfactory and

economical than that of wood, sometimes effecting large savings in excavation. They may be made in any size considered desirable and are not subject to the limitations of wooden piles in this respect.

Concrete piles may be either molded in place or molded before placing and then driven like wooden piles. Those molded in place

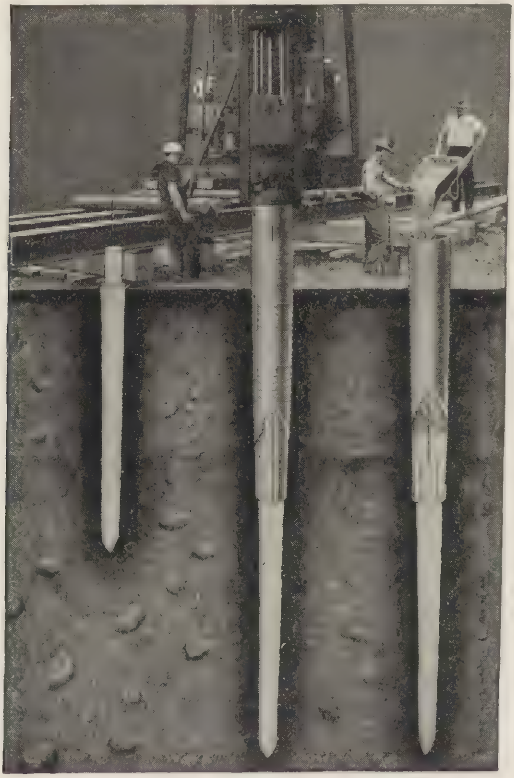


Fig. 142.—Raymond Composite Pile—Wood and Concrete.

(Courtesy of the Raymond Concrete Pile Company.)

are generally not reinforced, while those to be driven after molding must be reinforced so as to resist the stresses brought upon them in handling and driving. The methods employed for molding piles in place are patented, and a number of forms of pre-molded piles are also patented.

The Raymond pile is made by driving into the ground a thin shell of sheet steel with a collapsible core which holds the shell to its form

while driving. When the shell has been driven to the required penetration, the core is withdrawn and the shell filled with concrete. It is made tapering, usually 18 to 20 inches in diameter at the head and 6 to 8 inches at the foot, with a closed boot of heavier steel. They are made in sections for convenience in shipping.

The taper adopted for these piles gives high bearing capacity under ordinary conditions of use. The interior of the form may be inspected before placing the concrete. Difficulty is sometimes met in the collapsing of the thin shell when heavy hydrostatic pressure comes upon it, a fault sometimes corrected by driving a second shell inside the first one.

Fig. 141 shows the Raymond concrete pile, and Fig. 142 shows the Raymond composite pile of wood and concrete.

The Simplex pile is formed by driving into the ground a heavy

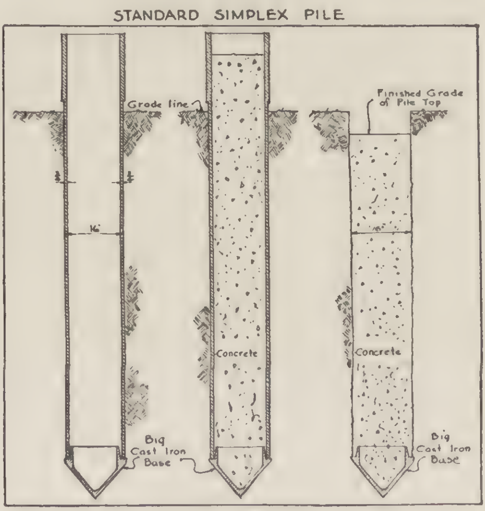


FIG. 143.—Standard Simplex Pile.

(Courtesy of the Simplex Concrete Pile Association.)

steel pipe with the bottom closed by a special jaw. The pipe is driven to the depth required, and is then withdrawn as the hole is filled with concrete. The jaw opens as the pipe is raised, permitting the concrete to pass through, and the concrete is rammed into place so as to fill completely the hole below the end of the pipe, and press the concrete against the earth at the sides of the hole. Sometimes a

cast-iron shoe is used at the bottom of the pipe and is left in the hole when the pipe is withdrawn.

In driving through soft material which will not retain its form after the pipe is withdrawn, it is sometimes necessary to place a form of thin sheet metal inside the pipe and fill it with concrete before withdrawing the pipe. The soft soil then fills around this form and does not mix with or replace the concrete. Figure 143 shows the standard Simplex pile.

The MacArthur Concrete Pile is made in several types. Figure 144 shows the *Pedestal* type which is used in those situations " where poor soils overlie fair strata, which fair strata if punched through by taper or straight piles would find no resistance except at excessive depths and cost." The "pedestal feature shortens the pile length and

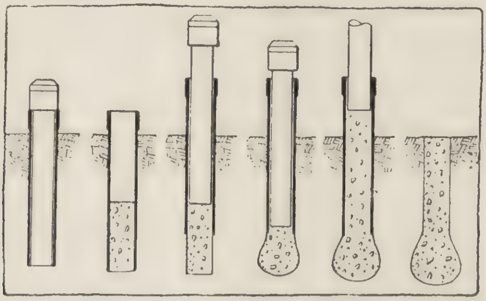


Fig. 144.—Pedestal Type, MacArthur Concrete Pile.

adds to the carrying capacity." These piles are " compressed in place and cannot be disturbed by any underground condition or driving adjacent piles." Figure 145 shows the *Composite* type which is used where piles run over 40 feet. " The concrete section extends below permanent water level, with wood section from there to penetration." " Positive assurance of perfect alignment between wood and concrete" is claimed. " The concrete joint and shaft are compressed in place."

Care is necessary, when using piles molded in place, that injury to the pile may not result from disturbance of the soil around the pile by driving other piles during the period of hardening—a danger which varies with the character of the soil. No pile should be driven near enough to be felt in the earth surrounding a green pile for a week after it is placed, unless the driving can be done before the initial set of the concrete takes place. The extra expense of early strength cement may be warranted in some cases.

All types of cast-in-place piles require very careful attention to secure good results. Any driving in the vicinity of freshly poured piles will greatly impair their final strength.

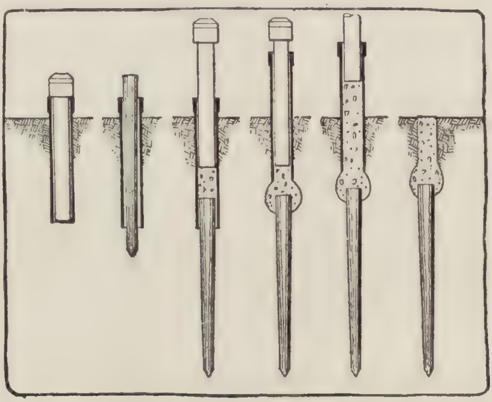


Fig. 145.—Composite Type, MacArthur Concrete Pile.
(Courtesy of The MacArthur Concrete Pile & Foundation Co., Inc.)

The Gow System of Caisson piles and the Gow Undercut Bell Method Caisson Piles are clearly shown in Fig. 146 and Fig. 147 respectively, while another recent type of Caisson pile is shown in Fig. 148.

Pre-molded piles are reinforced like columns with lateral reinforcement of wire hoops, spiral wrappings, or wire mesh, combined with longitudinal steel bars, the cross-section most commonly employed being octagonal or square with chamfered corners. The diameters in general use are from 12 to 20 inches for lengths of 20 to 50 feet, although larger and longer piles are sometimes employed and they are either of uniform section or given a slight taper, according to the service for which they are intended. When to be supported by friction upon their sides, tapering may be of value in increasing bearing power, but at somewhat increased cost of construction.

Pre-molded piles are usually cast with a point at the foot. Frequently a cast-iron shoe is used to facilitate driving and prevent damage to the point of the pile in passing through stiff material. It has been found in a number of instances that the rate of driving has been has been materially increased by pointing the foot of the pile to a diameter of 4 or 5 inches. In some instances the foot of the pile is increased in size to give a larger bearing area, when reliance is

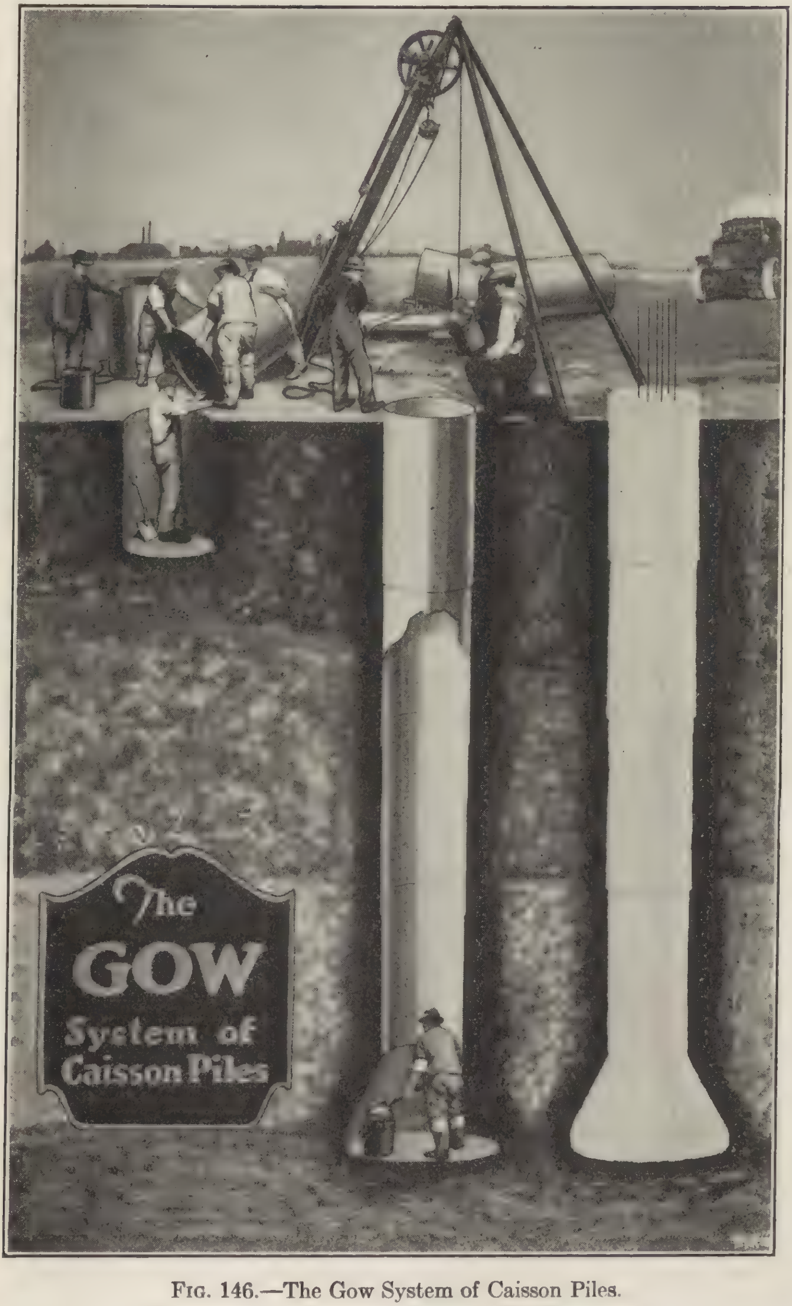


FIG. 146.—The Gow System of Caisson Piles.
(Couresy of the Raymond Concrete Pile Company.)

placed on the resistance of the lower stratum for support. In this case the point is usually made short.

Piles are molded in either horizontal or vertical position. The molding is easier to handle and readily subject to inspection when in horizontal position. When molded in vertical position, the surface of concrete as deposited is normal to the length of pile, but special care

FIG. 147.—The Gow Undercut Bell Method Caisson Piles.

(Courtesy of the Raymond Concrete Pile Company.)

is necessary in placing the concrete to eliminate voids. The reinforcement is connected up and handled as a unit in placing in the forms, to assure its proper position in the pile. During the early period of hardening, special attention should be given to keeping the concrete moist, and it is customary to allow it to harden about thirty days before it is driven, though in some instances the hardening has been hastened by subjecting the piles to a steam bath.

The steel reinforcement in a pre-molded pile must be sufficient

to carry the stresses which occur during handling and driving as well as those caused by the loads which come upon it afterward. In raising the pile from a horizontal position or in moving it horizontally,

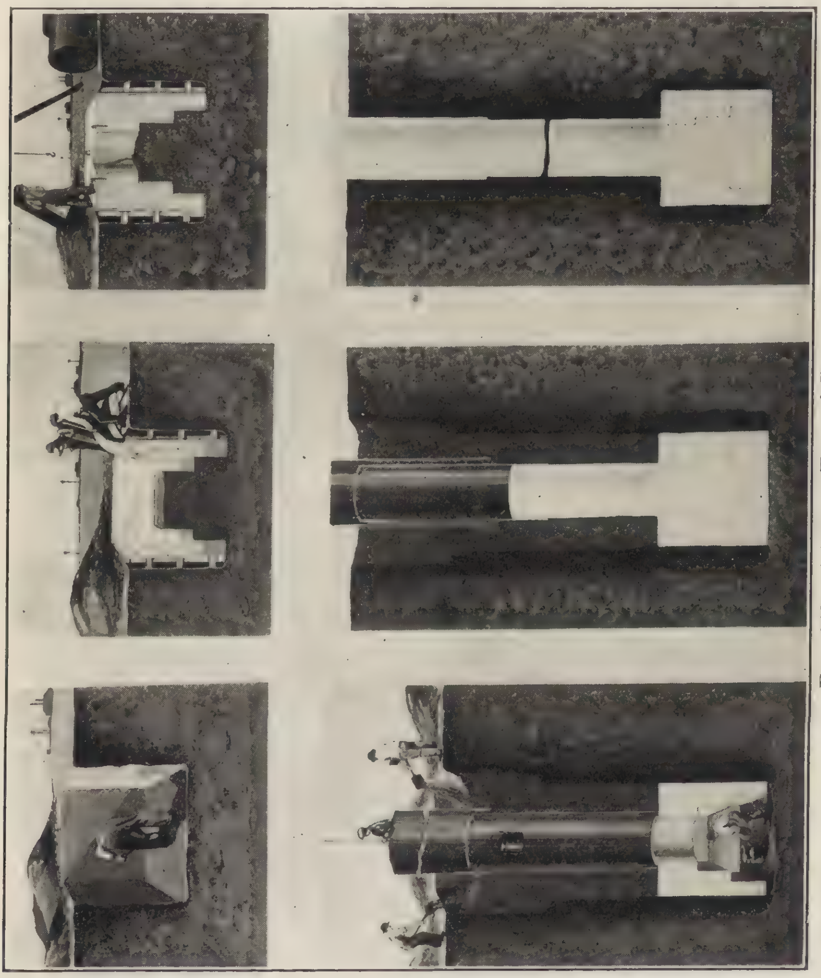


Fig. 148.—A Recent Type of Caisson Pile.
(Courtesy of the Raymond Concrete Pile Company.)

the pile must be capable of carrying its own weight as a beam, supported near the ends or at the middle. Allowance for shocks and impact should be made. After driving, the pile may be in direct compression when supported laterally or it may act as a column when not so supported.

The longitudinal reinforcement is frequently made heavier through the middle third of the pile to provide for the bending moments in handling, and sometimes closely spaced spiral reinforcements are used at the head and foot of the pile to give greater resistance to shocks and diagonal stresses in driving. The reinforcement is usually fabricated as a unit, so that it may be readily and accurately placed in the forms, without danger of displacement while pouring the concrete. 1 to $1\frac{1}{2}$ to 3, or 1 to 2 to 4 concrete is commonly employed, giving a compressive strength of 3000 to 2000 pounds per square inch at the end of twenty-eight days.

On account of the weight of concrete piles, heavy drivers are necessary. Steam hammers are found most effective and occasion less damage to the piles than drop hammers. Heavy drop hammers with low fall give better results than lighter ones with greater fall. Caps of various designs are employed to cushion the blow and prevent shattering the head of the pile. A wooden block receives the blow of the hammer, and layers of old belting, rope ends, or bags of sawdust have sometimes been used upon the head of the pile to prevent the shattering of the concrete. With proper precautions, it has been found practicable to drive pre-molded piles without injury where heavy driving is necessary.

When a jet is to be used in driving, a hole is cast through the center of the pile into which the jet pipe may be inserted—a tapering core, or a collapsible form, being used for this purpose, which is cheaper than casting the jet pipe in the pile. Solid piles are also sometimes driven by use of the outside jet as with wooden piles.

Concrete piles cost more than wooden piles, but usually fewer of them are required and the total cost of a design in which they are used may be less.

224. Bearing Power of Piles.—There are so many variable factors affecting the supporting power of pile foundations that in most instances accurate determinations are not possible. Piles may derive their support either from a hard stratum at the bottom which resists the penetration of the foot of the pile, or from friction of the sides of the pile upon the material through which it is driven. Conditions may also vary widely as to the lateral support afforded the pile between the loaded end and the point of support.

Piles Acting as Columns.—When piles are driven through soft soil, offering slight resistance to lateral motion, and rest upon a hard substratum below, they may be considered as columns. They are fixed in position at the bottom with the top free to move laterally, but held in vertical position by the caps joining them together. Piles

driven in water and not braced depend for lateral stiffness upon being driven into the soil beneath to a sufficient depth to hold them firmly at the bottom. The length of the column in such a pile is to be taken from the cap to a point below the surface of the soil, a distance depending upon the firmness of the soil. In stiff soil a depth of 1 or 2 feet may be sufficient to firmly hold the pile. In less resistant soils, one-third to one-half the total penetration may be required.

When piles project into the air, they are braced laterally, so that no bending can take place and the strength of the pile is that of the compressive strength of the wood, or the resistance to penetration of the soil into which it is driven. The compressive resistance of wooden piles depends upon the kind of wood employed, but is taken at a low value, commonly about 600 lb./in.^{2} When the pile acts as a column, this is reduced to $600(1-L/60d)$, in which L is the length of the column and d is the diameter at its middle point.

Piles Supported by Friction.—Numerous attempts have been made to state in a formula the relation between the penetration of a pile under a hammer blow of given energy and the load the pile may bear without yielding. The effective work done upon the pile by the hammer in striking the blow should equal the work done by the resistances in stopping penetration. There are, however, so many indeterminate losses of energy in the operation of striking the blow that a rational formula is not feasible—there is loss of energy in the friction of the hammer in the guides; some energy is consumed in brooming the head of the pile; the elastic compression of the pile consumes a part of the energy; the effectiveness of the blow is affected by the height of fall and velocity of the hammer. The impossibility of evaluating these and other data affecting the resulting penetration renders any formula obtained by discussion of the theory of the subject rather useless. Mr. Ernest P. Goodrich has made a very elaborate and interesting study [8] of the subject in which is produced a formula of very complicated form. This formula is reduced by evaluating experimentally many of the terms, but the result seems to show that a usable rational formula cannot be produced.

Engineering News Formula.—This formula was suggested in 1888 by Mr. A. M. Wellington, the editor of *Engineering News.* When the drop-hammer is used this formula is $P=2Wh/(s+1)$, in which P is the safe load in pounds, W is the weight of the hammer, h is the height of fall in feet, and s the average penetration under the last blows in

[8] Transactions, Am. Soc. C. E., Vol. XLVIII, p. 180.

inches. When using a steam hammer the formula suggested by Mr. Wellington is $P=2Wh/(s+0.1)$. For the case of a double-acting steam hammer this may be modified to $P=2(W+Ap)h/(s+0.1)$, in which A is the area of the piston and p is the steam pressure.

These formulas are the only ones in common use. They are empirical formulas obtained by studying all available data derived from tests of bearing power. It is assumed that the blows have been struck upon sound wood and commonly it may be necessary to cut off the head of the pile to remove the wood splintered or broomed by previous driving before making the tests. There must be no visible rebound of the hammer in striking the blows, and if such rebound occurs, it indicates that the fall is too great or the hammer too light, and the full effect of the blow is not communicated to the pile. The hammer must always be heavier than the pile, and should be twice as heavy, in order to strike an effective blow. The formulas are supposed to give a factor of safety of about six.

Eytelwein's formula is frequently used for reinforced concrete piles, on account of the greater weight of such piles. This formula takes into account the relative weights of pile and hammer. With a factor of safety of six the formula is

$$\text{Safe load} = \frac{2W_h H}{s(1-W_p/W_h)}$$

in which W_h is the weight of hammer, W_p the weight of pile, H the height of fall and s the penetration. For the steam hammer this would become

$$P=\frac{2W_hH}{s+0.1W_p/W_h}, \quad \text{or} \quad P=\frac{2(W_h+Ap)H}{s+0.1W_p/W_h}.$$

It is desirable that the blows used for measuring penetration be struck with a hammer having free fall, as considerable loss of velocity may result from the resistance of a rope and friction drum. It is also necessary that the penetration under the last few blows be uniform and fairly represent the state of resistance of the pile. The penetration should be not less than one-half inch, as less penetration may indicate injury to the pile rather than resistance to penetration.

For piles sunk by the water jet or those cast in place, these formulas will not apply. A rough estimate of supporting power of such piles may sometimes be made by assuming values for the bearing power of the soil at the bottom of the pile and the friction of the soil on the surface of the pile. This friction is very indefinite

and difficult of determination, varying with the character and condition of the soil. In general, the friction per square foot of surface of the pile may be taken at from 5 to 10 per cent of the bearing value for the same soil.

When piles are driven into soft or plastic materials, the resistance to penetration usually increases with time after the driving ceases. A rest of twenty-four hours may be sufficient to cause the material to settle against the surface of the pile so as to develop a resistance several times that existing when the material was disturbed by the operation of driving. Numerous instances are recorded in which it was found that the penetration under a blow had been decreased by a rest of a few days to from one-third to one-sixth of that at the end of the original driving. In case of driving into material of this kind, it is desirable to examine the effect of rest upon the bearing power and piles upon which tests are to be made should have a period of rest before the final test is made. Piles easily sunk by light blows or even by static pressure frequently carry loads a few days later much greater than those required to sink them. In coarse sand or gravel, the time effect is of less importance, if it exists at all.

Piles are frequently tested by applying static loads until movement occurs. Usually a load is balanced over a single pile, although sometimes a platform resting upon several piles is loaded. The pile is allowed to stand under the load at least twenty-four hours before being examined for settlement. It is desirable that the load be added in increments, each being allowed to stand for twenty-four hours, until a load is obtained which produces settlement.

In any test of bearing power, it is essential that the pile be tested under the same conditions that will afterward apply to the foundation. The determination of the requirements in any particular instance is largely a matter of judgment on the part of the engineer, but such judgment should be exercised with knowledge of all conditions that may be evaluated and in accordance with the principles underlying such work.

The spacing of piles in a foundation is a matter of importance because of its possible bearing upon the supporting power of the individual piles. In general, piles should not be closer than 3 feet center to center, although they are sometimes driven $2\frac{1}{2}$ feet apart. When piles are closely spaced over the area of a foundation, a considerable compression of the soil between them must result. The effect of this disturbance of the soil depends upon its character, but too close driving impairs the bearing capacity of all of the piles, and they cannot be considered as individually carrying loads up to their normal

bearing capacity. It is important that the piles be so spaced that they shall be uniformly loaded. The center of gravity of the loads should coincide with the center of resistance of the piles, thus making any settlement uniform over the area of the foundation.

225. Sheet Piling.—Sheet piles are made to fit closely together and are driven in contact with each other so as to form a wall to prevent the lateral flow of soft materials, and find their greatest use in enclosing areas which are to be excavated, or guarding foundations against undermining by currents of water. They are made of timber, steel, or concrete.

The simplest and most common form of sheet pile consists of a thick plank sharpened (as shown in Fig. 149) to a point at one side

FIG. 149.—Plank Sheet Piling.

FIG. 150.—Wakefield Piles.

so as to cause each pile to drive closely against the one previously driven. When heavy timbers are employed, they are sometimes arranged with tongue and groove, which may be planed into the edges of the planks, or made by nailing strips to the edges. In some instances these are made to dovetail together.

Wakefield sheet piling is formed by bolting and spiking three planks together so as to form a tongue on one edge and a groove on the other, as shown in Fig. 150. The patent upon this pile has expired. They have been quite extensively used in this country and for heavy work are preferred to the other forms of wooden piles. They are made of planks from $1\frac{1}{2}$ to 4 inches in thickness, depending upon the strength needed in the work, and are bolted together by pairs of $\frac{1}{2}$-inch or $\frac{5}{8}$-inch bolts, 6 or 8 feet apart, and spiked between the bolts. The

planks are 12 inches wide and the tongue is made as wide as the thickness of plank, but not less than about $2\frac{1}{2}$ inches for the thin planks. This type is economical, as there is no waste in forming the tongue and groove, and it is easily made of stock sizes of plank. It is also a very flexible style, as it can readily be made into shapes for corners, angles, etc. In driving, the tongue should be kept in advance. Otherwise the open groove might become filled with earth and obstruct the driving of the next pile.

Clear, straight-grained timber (usually pine or fir or sometimes hemlock) should be used for sheet piling which is to stand heavy driving or to be subjected to heavy pressure. Wakefield piling has the advantage that any defects in the timber will not extend through the pile. In driving, the bottom of the pile is beveled toward the one previously driven so as to cause the piles to draw together, the groove in the pile enclosing the tongue of the previous one.

Steel Sheet piling is made in a number of forms either built up from standard rolled sections, or rolled in special sections so that the piles may interlock. A few of these forms are shown in Fig. 151. In form *A*, known as the Jackson pile, two channels bolted together. with pipe separators are used alternately with I-beams. The Fricstadt piling, *B*, consists of alternate channel bars interlocking with channels having Z-bars riveted to them. Form *C* is made up of I-beams held together by a special locking bar. Forms *D* and *E* are special rolled sections, the ends of which are designed to interlock, and may be used in work curved in plan.

For temporary work, where the piling is to be removed, steel sheet piling is largely used and is often more economical than timber piling. The interlocking edges hold the piles together in driving, and give a certain amount of transverse strength to the wall. In hard driving, the steel piling is less injured than timber piling and may be repeatedly used.

Steel sheet piles have a further advantage over timber in that one wall will secure watertightness in a cofferdam where two of timber would be necessary. Also splices are easily made on account of the interlock, by simply breaking joints on adjacent piles.

It will facilitate the removal of steel sheet piling that is to remain in place for some time before pulling, to lubricate the joints with graphite. Where concrete is to be laid in contact with a wall of steel piling, the latter should be protected by tar paper or light matched wood sheeting, to permit easier withdrawal of the piling.

Steel sheet piling has been used to penetrate old cribwork, logs, stone, brick and other filling materials in made ground. For driving

through logs, a chisel is sometimes first used to penetrate the wood, after which the pile is driven.

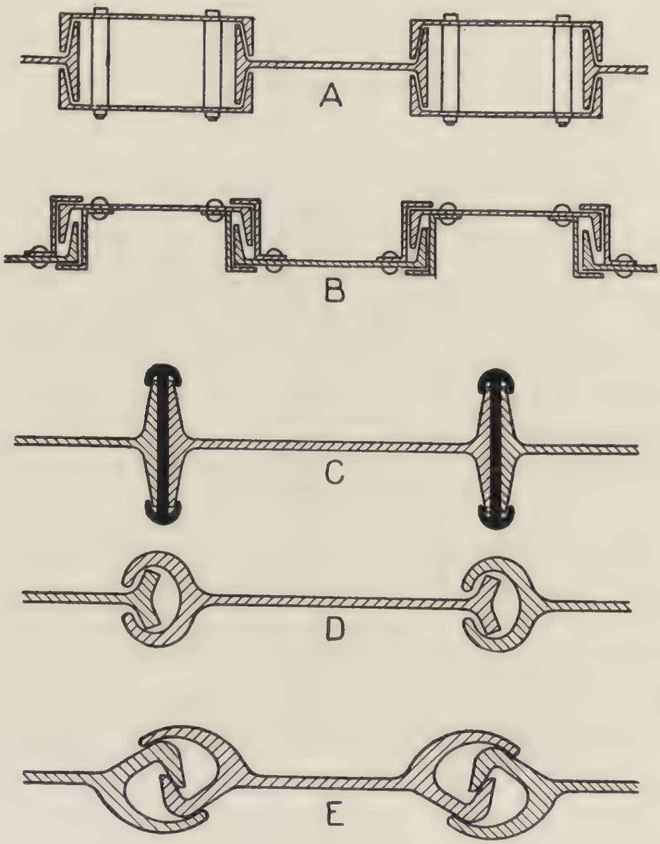


FIG. 151.—Types of Steel Sheet Piling.

Reinforced concrete sheet piles, shaped like wooden piles, either rectangular or with tongue and groove on the edges, are often used on important work where the piling, is to be left permanently in the structure, and are often reinforced with longitudinal bars to resist the stresses occurring in handling and driving. The loads coming upon them after driving are in a transverse direction and the piles should be designed for hydrostatic pressure, being supported laterally by the waling.

Concrete sheet piling is sometimes made interlocking by setting interlocking steel bars in the pile edges, the interlocking parts being

FIG. 152.—Precast Concrete Piles.
(Courtesy of the Raymond Concrete Pile Company.)

then enclosed in concrete after driving. In some instances semicircular grooves are left in the edges of the pile, the circular opening between the piles being filled with concrete after driving.

In driving sheet piling it is necessary to first drive a row of guide piles to which may be attached horizontal timbers, or wales, against which the sheet piling may be driven. The driving of ordinary sheet piles is much lighter work than driving bearing piles, and light steam hammers are used for the purpose. These are frequently operated from a derrick without leads and may be handled with greater rapidity and less injury to the piles than the ordinary heavy driver.

Concrete sheet piles are more durable than timber or steel, and find a considerable use in permanent work on waterfronts, bulkheads, dock and quay walls, etc.

Figure 152 shows a yard devoted to the manufacture of precast concrete piles with gantry crane for handling them. Finished interlocking reinforced concrete piles are shown in the foreground.

CHAPTER XIII

FOUNDATIONS BELOW WATER

ART. 58. COFFERDAMS

226. Types of Cofferdams.—A cofferdam is a structure intended to exclude water and soft materials from an enclosed area, in order to permit the water to be pumped out and the work of placing a foundation to be done in the open air. This method is applicable only to rather shallow foundations, and for depths greater than about 30 feet other methods are more economical. Cofferdams can be used only where the soil at the bottom is fairly impervious, so that an excessive flow of water under the dam does not occur.

The type of structure for this purpose varies with the depth of the foundation and the character of the soil upon which it is to be built. Earth, sheet piling, timber cribs, or combinations of these arranged to meet special conditions, are the materials employed.

Earth cofferdams are banks of earth surrounding the area of the foundation, and are made thick enough to sustain the pressure of the water and to prevent excessive leakage into the inclosed space. The use of plain earth dams for this purpose is limited to shallow water without currents; where danger of washing from a light current exists, a wall of bags filled with clay and gravel or a revetment of such bags upon the exposed face of the embankment may be employed. The top of the dam should be at least 2 feet above the water surface, and the top width not less than 3 feet. A row of sheet piling is sometimes driven and inclosed in an earth dam for the purpose of reducing the size of the embankment needed, or of cutting off a flow of water through the soil under the dam. Earth cofferdams may be used to a depth of 5 or 6 feet where the current is slight.

Sheet-pile cofferdams are constructed either of timber or steel piles in single or double rows, and are supported by guide piles, timber frames, or cribs. Where a double row of sheet piling is used, a filling of earth between the rows is necessary. The use of sheet piling is commonly limited to depths of 25 or 30 feet, although steel sheet piles are sometimes used successfully to depths of 40 or 50 feet.

A crib cofferdam consists of a timber crib built so as to be watertight and is floated into place and sunk around the site of the foundation.

Movable cofferdams which may be removed after using and sunk again have been employed in a number of instances. These may be cribs with watertight compartments, or framework supporting sheet piling.

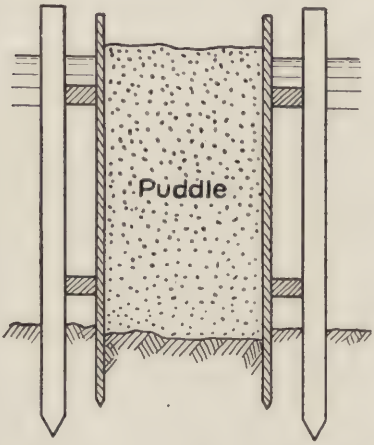


FIG. 153.—Double-wall Cofferdam.

227. Sheet-Pile Cofferdams.—When timber sheet piling is used the most common form of cofferdam consists of two rows of piles with a filling of puddled earth between them—a system of construction shown in Fig. 153. Two rows of guide piles are first driven. Horizontal timbers known as wales are attached to these, and the sheet-piling driven inside against the wales, the tops of the guide piles being tied together to prevent spreading when the puddle is put in. The guide piles should be driven to a firm bearing in order to develop the transverse strength of the pile in resisting the water pressure. Horizontal braces across the area to be drained may sometimes be used to assist the cofferdam against lateral pressure, and when this is not feasible,

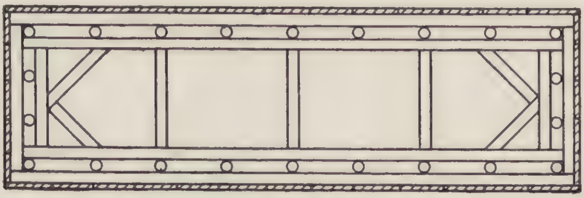


FIG. 154.—Single-wall Cofferdam.

the width of cofferdam must be made sufficient to provide lateral strength..

The sheet piling must be driven into a fairly impervious stratum to prevent leakage under the dam, and pervious material overlying such stratum between the rows should be excavated sufficiently to give the puddle contact with the impervious material below. The puddle needs to be both impervious and stable, and a mixture of gravel

and clay is desirable for the purpose. Clay is impervious but washes easily if the water finds an opening through it, while gravel or coarse sand mixed with the clay tends to prevent such washing. The thickness of puddle required depends upon its quality and upon the pressure to be resisted, a thickness of one-fourth to one-sixth of the depth being usually sufficient. For best results, the puddle should be placed in thin layers and well tamped in damp condition.

A single wall of sheet piling is often used supported by guide piles or by an interior framework—a method which requires less space than the puddle wall type and is preferable where it is important not to restrict the water way. Plan of a cofferdam of this type for use in constructing a bridge pier is shown in Fig. 154. The guide piles are first driven, wales attached, and the sheet piles driven against the outside waling. Braces from wall to wall across the opening are used to assist in resisting the lateral pressure. Such bracing when needed may be placed at lower levels as the water is pumped out and excavation proceeds.

When guide piles cannot be driven to firm bearing, timber frames are sometimes used to act as guides and support the sheet piling against lateral pressure. These frames may be built upon the ground, floated to the site and sunk into position. The sheet piles are then driven around the frame so as to inclose it.

Interlocking steel piling is often employed for single wall work because of its greater strength and tightness. Timber piling for such use should be tongued and grooved. Wakefield piling has most frequently been used. When the depth is considerable, steel sheet piles may be used with much greater facility and usually with more economy than timber piles. The resistance in driving is less and the salvage usually considerably greater. Steel sheet piles may ordinarily be used three or four times when carefully handled, and if the use be not too severe.

Some leakage is always to be expected in cofferdams, and in many instances special precautions are necessary to exclude water. The possibility of meeting difficulty in preventing leakage is the principal objection to this method of construction. Banking clay against the outside of the cofferdam is a common expedient to prevent leakage through or immediately under the dam. When it is feared that channels may open under the piling, gravel may be deposited around the base of the dam to close such incipient openings. Tarpaulins are often employed to cover the outside of the dam, or spread out upon the bottom outside the base of the dam and weighted with gravel. When the bottom is rock, it is sometimes necessary to cover the whole area

inside the cofferdam with a layer of concrete to prevent inflow of water through seams in the rock. While concreting, water from a

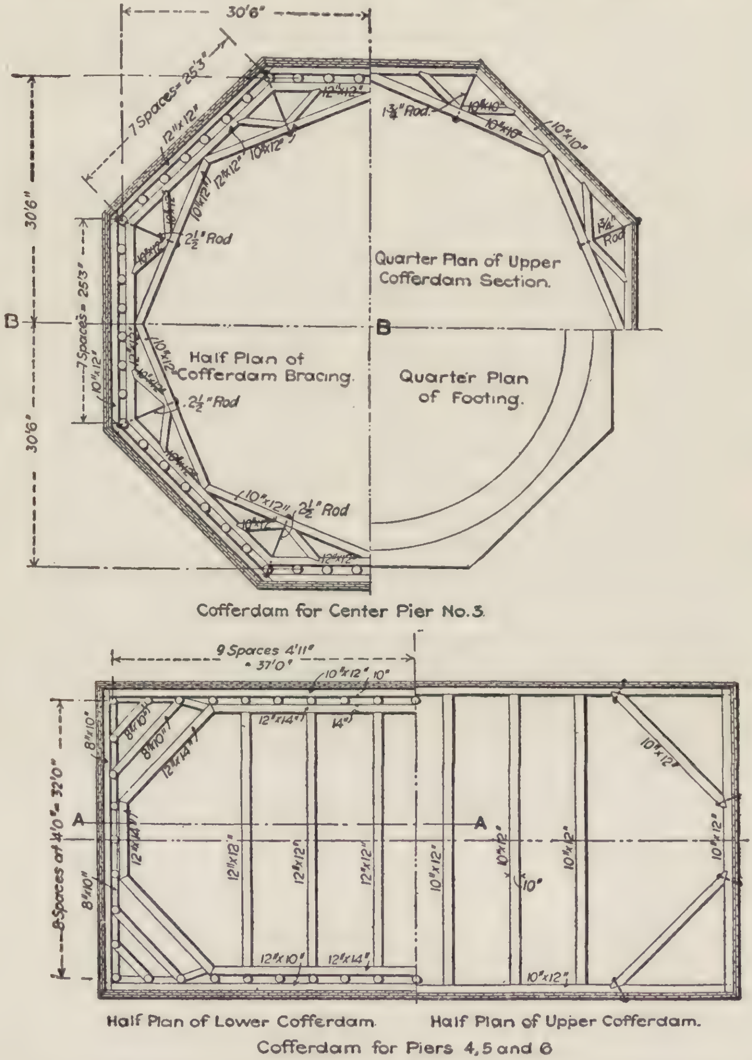


Fig. 155.—Cofferdams with Single Walls of Timber Sheet Piling Supported by Wales and Guide Piles, for Piers of Illinois Central Railroad Bridge over Tennessee River, at Gilbertsville, Ky.

(Jacoby and Davis, "Foundations of Bridges and Buildings.")

leak may be led away through a pipe, which is closed or capped after the concrete has set.

Where a leak through a cofferdam wall is persistent, and cannot

be closed by ordinary methods, the operation known as *stock ramming* may prove successful. A 3- or 4-inch pipe is sunk through the filling or puddle, ending as near as possible to the location of the leak. Then cylinders of clay, about a foot long, and cut to enter the pipe readily, are inserted and rammed home until no more can be forced in. Sometimes the rammer for this purpose is rigged to a pile-driver, and pushed into the pipe by the weight of the hammer.

Even with the best construction, it must be expected that there will be some leakage, which will have to be handled by pumps. It has usually been found cheaper in the long run to do a moderate amount of pumping than to incur the expense of a perfectly water-tight cofferdam.

Concrete in a cofferdam will usually have to be deposited under water. If there is a leak, causing a current, or if the water is stirred up by the pumps, the bottom should be cleaned, ready for the concrete, then the cofferdam should be flooded, and the concrete placed through the still water. Furthermore, a mixture one-third to one-half richer should be used for the concrete, as there will be some washing out of cement, with a resulting loss of strength. A very satisfactory device for placing concrete under water is a metal tube, with a door at its lower end. Without this latter, the tube cannot be entirely filled before the first concrete is dumped, and some washing will occur. To prevent this, the tube may be filled for the first time with concrete in paper bags. After the flow is started, the lower end of the tube should be kept buried in the concrete, and it should be kept full, so that the concreting is a continuous operation.

Figure 155 shows a cofferdam with single walls of Wakefield piling, supported by guide piles and wales, as used for piers of the Illinois Central railroad bridge [1] over the Tennessee River at Gilbertsville, Ky.

" For the river piers the bottom was dredged about 17 feet below low water to a stratum of very hard cemented gravel into which shod piles were driven 16 to 20 feet and cut off 2 feet above the bottom with a submerged 42-inch circular saw. Cofferdam piles were then driven, 10×12-inch waling timbers bolted to them outside, and 10×12-inch triple-lap sheet piles driven against the latter, penetrating the gravel from 4 to 6 feet and forming a cofferdam with the top 15 feet above low water."

The cofferdams were braced with horizontal braces engaging the waling pieces. It was sealed at the bottom with a 3-foot layer of concrete and then pumped out.

In constructing foundations for the Chicago, Milwaukee and.

[1] Engineering Record, May 20, 1905.

St. Paul Railway bridge over the Wisconsin River at Kilbourn, Wisconsin,[2] a single row of Wakefield piling supported by a framework (see Fig. 156) was used. The bottom of the river was rock, covered with several feet of loose shifting sand, containing some loose rock, and the current was swift.

The frame was weighted down with scrap rails and secured in place with wire guys to the banks of the river. After driving all

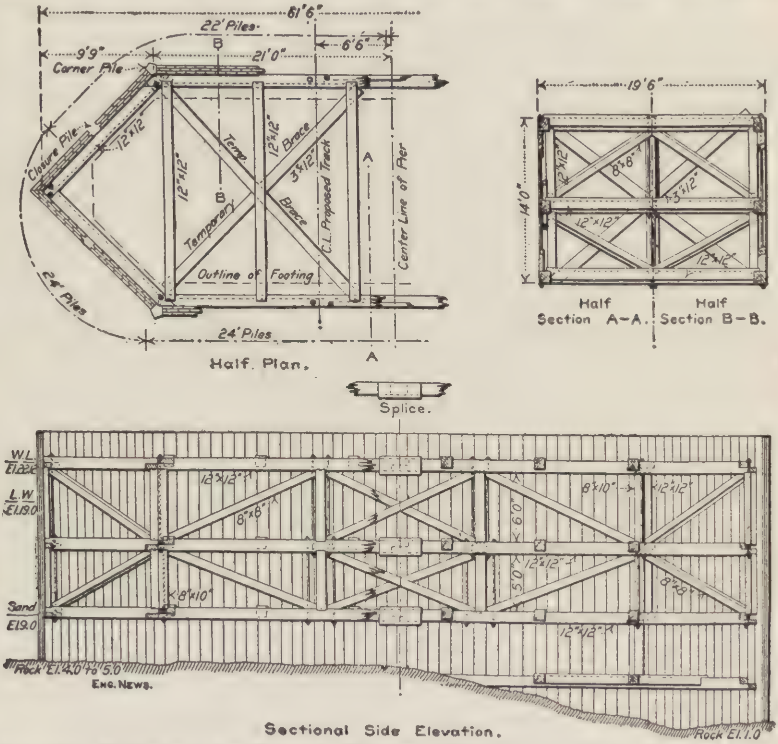


FIG. 156.—Cofferdam for Pier of Chicago, Milwaukee, and St. Paul Railway, Kilbourn, Wis.
(Jacoby and Davis, "Foundations of Bridges and Buildings.")

the sheet piles, they were hammered lightly to broom the bottom and cause then to fit more accurately upon the rock.

The outside of this dam was covered with canvas which lay flat on the bottom for 8 feet out from the dam and extended above water level. The outer edges of the canvas were weighted with sand bags and old rails. Fifty car loads of gravel were deposited on the canvas. The cofferdam was then pumped out, and later required only light pumping to keep clear of water.

The type of cofferdam to be used in any particular instance will

[2] Engineering News, March 30, 1905.

necessarily depend upon the local conditions surrounding the work. The external pressures of water and earth may be roughly estimated provided the character of the materials is known, and the wales and interior framing in a sheet pile cofferdam must be designed to carry these loads, while the sheet piles must act as beams between the wales. It is usual to consider the wales as simple beams between bearing piles or cross-beam supports. The outside earth pressure may be estimated in the same manner as for a retaining wall. In a puddle wall cofferdam, the piles and framing must carry the outward pressure of the puddle.

In streams where a swift current exists, it is desirable to use cutwaters on the upstream side of the cofferdam. This may be a pointed end upon the cofferdam, or a separate cutwater of planks placed above the cofferdam. Sometimes a row of sheet piling pointing upstream above the dam may be necessary to deflect drift or ice. In shallow water, a triangular crib of logs, or other available material may be sunk and filled with stone.

228. Crib Cofferdams.—Timber cribs are often used as cofferdams where the depth is not great. These cribs are commonly built upon the land and floated into place and sunk. Before sinking a crib, it is usually necessary to excavate the site so that the crib may rest evenly upon the bottom of the stream. For soft materials and shallow depths, a spoon dredge may be used, which dips up the material in bags or buckets by the use of a derrick. For stiffer materials a scraper dredge is used in much the same manner.

When the crib is to rest upon a bed of rock, the bottom of the crib may sometimes be shaped to fit closely upon the rock surface.

The crib consists of either a single or double wall of timber built so as to enclose the area to be included in the foundation. A single wall crib is usually made of squared timbers laid on top of each other and drift bolted together, the walls being tied together and braced with framework to hold them rigidly in place. A double wall crib consists of two walls, usually of planks upon frames which are rigidly connected by braces and ties, extending around the area upon which the foundation is to be constructed without materially obstructing this area with braces.

In sinking a crib, it is usually necessary to construct anchorages from which the crib may be held accurately in place by lines while being sunk. These may be piles or small cribs weighted with rock sunk upstream from the foundation area.

After the crib is sunk, it is necessary to find means of making the bottom watertight. Double wall cribs are sometimes filled between the walls with clay puddle to render them more watertight. Sheet

piles are sometimes driven about the base of the crib to cut off water flowing through the soil under it, when the crib does not lie on the rock or other impervious material. Tarpaulins fastened near the bottom are sometimes used to prevent leakage under the crib. A deposit of clay puddle around the base of the crib is usually sufficient to seal the bottom against excessive leakage in ordinary work. Sometimes the bottom is puddled for some distance from the crib when there is considerable leakage through the soil.

In shallow foundations, cribs built of old timber which may be available have sometimes been found economical. The availability of materials must always be considered in deciding upon the methods to be adopted. The ability to devise means for using materials or machinery readily available has often proved of great value in such work. Success depends upon careful study of all conditions surrounding the work and the skillful adaptation of all available means in securing the desired results.

Cribs are sometimes made so that they may be removed and repeatedly used. These have sometimes been used for bridge piers, being made in two parts joining together on the short sides so that they may be taken from around the foundation after it is constructed. Watertight compartments are provided which may be pumped out when it is desired to float the cribs. Sometimes sheet piling is used around these cribs, which may be withdrawn before raising them.

Heavy timber cribs are frequently used to support sheet piling where the foundation is too large for internal bracing, or under conditions where a strong current makes the construction of an ordinary sheet pile cofferdam difficult. These cribs are built of an open cribwork of heavy timbers made wide enough to give the necessary stability. A portion of the crib is floored near the bottom to hold the rock required to sink and hold the crib in place. A single row of sheet piling outside the crib may be used, or when necessary, a double row of piling between two cribs with a filling of puddle between.

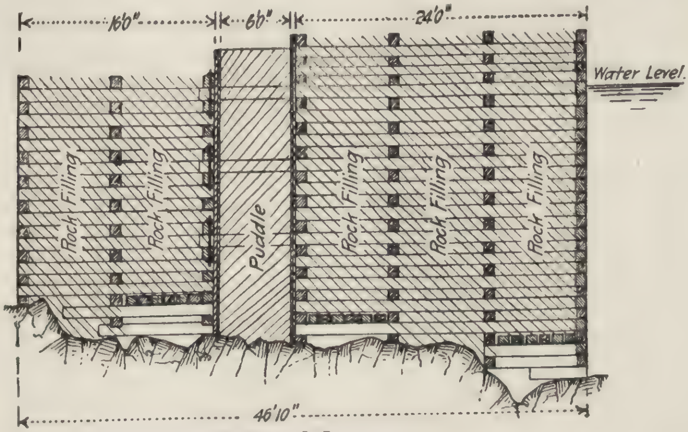
" The cofferdams for the Niagara Power Plant [3] of the Electrical Development Co. of Ontario, furnish an example of exceedingly strong and rigid cofferdams placed under the most trying conditions. In some places the current had a velocity as high as 17 feet per second which made it difficult to study the nature of the bottom and the depth of water previous to placing the cofferdams.

" The widest part of the cofferdam consisted of two lines of parallel, rock-filled timber cribs with a space between, sheet piled

[3] Jacoby and Davis, Foundations for Bridges and Buildings, Second Edition, p. 227.

and filled with puddle as shown in Fig. 157. Both cribs were built of squared timber with the outside wall of the outer crib laid solid. The width of the cribs varied to meet the variation in depth and the bottom of the cribs was made to fit the irregularities of the rock surface. In shallow water the cribs were built in place but elsewhere they were constructed in the river upstream, and by means of cables from the shore they were floated into place and were sunk by filling with rocks the wells which had bottoms. For further details of this interesting cofferdam the reader is referred to Engineering News, vol. 54, page 561, Nov. 30, 1905.

229. Cellular or Pocket Cofferdams.—This is a comparatively recent type. For stability it depends upon its cellular form, which



Section C-D.

Fig. 157.—Typical Section of Crib Cofferdam. Niagara Power Plant, Electrical Development Company of Ontario.
(Courtesy of Jacoby and Davis, Foundations of Bridges and Buildings.)

is possible only in the steel sheet piling, with its interlocking joint. This is capable of resisting the tension due to the bursting pressure of the filling in the pockets. The cofferdam is therefore self-supporting if properly designed, and requires only the sheet piling and the earth fill for its construction

The first large dam of this character was used for the U. S. lock at Black Rock Harbor, Buffalo, N. Y., which was constructed in 1908 and 1909. The dam inclosed an area 200×880 feet, and was built of Lackawanna steel sheet piling, which on test developed a strength in the interlocking joint of 10,000 pounds per linear inch of pile. Two walls were driven, 30 feet apart, and the space between them was divided by cross walls into pockets 30 feet square. A

FIG. 158.—Cellular Cofferdam for Black Rock Harbor.
Interior View Showing Concrete Ship Lock Erected up to Last Section.
(Courtesy of the Bethlehem Steel Company.)

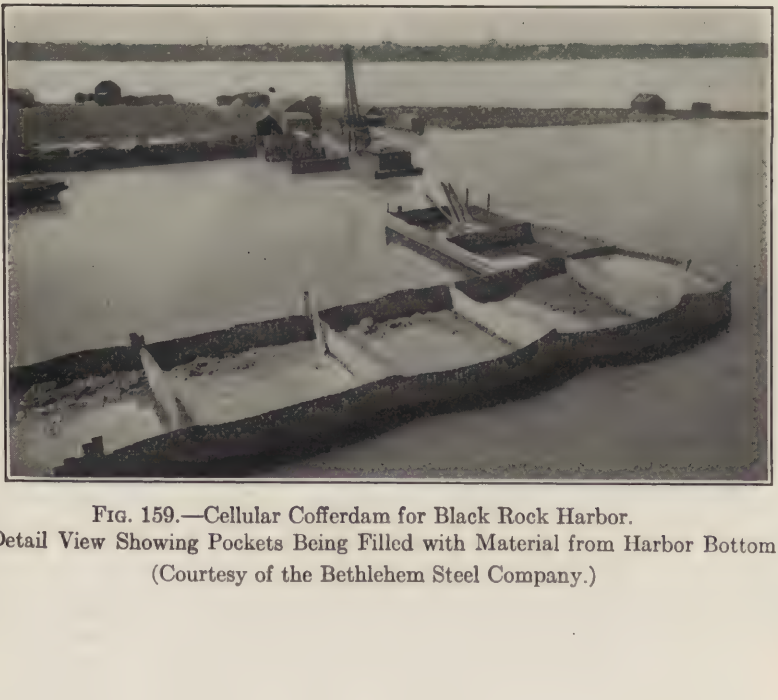


FIG. 159.—Cellular Cofferdam for Black Rock Harbor.
Detail View Showing Pockets Being Filled with Material from Harbor Bottom.
(Courtesy of the Bethlehem Steel Company.)

15-inch, 40-pound channel was bolted to the top of the piling as a waling piece. The head retained was 40 feet of water.

The pockets were filled with clay, and a toe of sand and gravel 25 feet high was left inside. The cofferdam retained the head successfully until the completion of the work, although the piling bulged considerably, both along the waling channel and vertically, between this and the earth surface.

Figure 158 is an interior view showing the concrete ship lock erected up to the last section, and Fig. 159 is a detail view showing the pockets being filled with material from the harbor bottom.

The success of this type of cofferdam at Black Rock led to its adoption in modified form for raising the U. S. S. " Maine " from Havana Harbor. Here the head to be retained consisted of 30 feet of water and 30 feet of very soft mud, making the total unsupported depth of piling about 50 feet. In order to place the joints of the piling entirely in tension, the pockets were made of a full circular form, each complete in itself, and tangent to the adjacent circles along the circumference of a large ellipse, surrounding the wreck. On account of the greater depth, the diameter of the pockets was increased to 50 feet. The space between the large circles was closed by an arc of sheet piling driven on a 9-foot radius, connection with each cylinder being made by a special pile, formed by a half-pile riveted to the side of a full section.

The pockets were filled with material dredged from the harbor, which material proved much softer and more difficult to drain than had been anticipated. Efforts to increase its stability by pumping water from well points driven into the cylinders were not highly successful. As unwatering progressed, the cylinders became badly deformed as a result of the outside pressure. A toe of stone was dumped against the inside wall, and finally bracing was placed between the cylinders and the wreck. Thus supported, the cofferdam stood while the wreck was exposed, examined, and finally caulked for floating.

Figure 160 is a plan view of the cellular cofferdam used for raising the Battleship " Maine," showing the arrangement of the cylinders and connecting arcs. Figure 161 is a detail view of the 50-foot cylinders and the connecting arcs.

Benefiting by the experience in Havana Harbor, the cellular type was further modified and used in the construction of the lock and dam in the Hudson River at Troy, N. Y., which was built by the U. S. Engineer Department-at-Large. The Troy cofferdam was practically a compromise between the one used at Black Rock and that used for raising the Battleship " Maine."

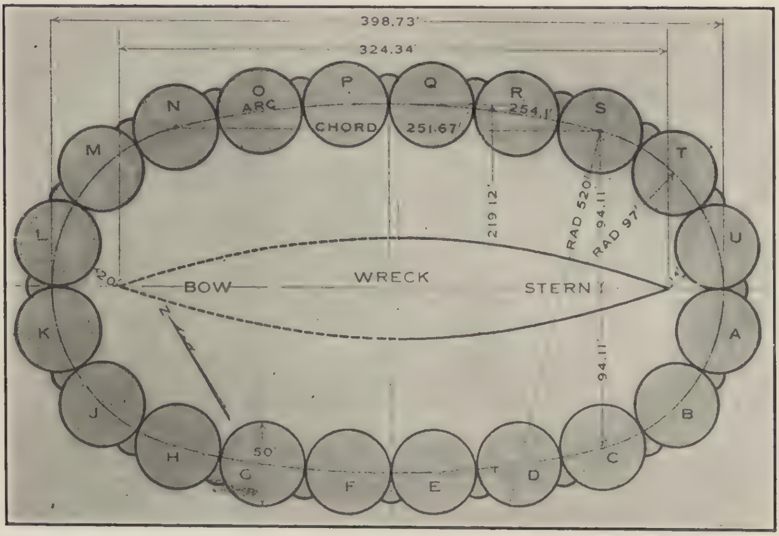


FIG. 160.—Cellular Cofferdam for the Battleship "Maine."
Plan View Showing the Arrangement of Cylinders and Connecting Arcs.
(Courtesy of the Bethlehem Steel Company.)

FIG. 161.—Cellular Cofferdam for the Battleship "Maine."
Detail View Showing 50-Ft. Cylinders and Connecting Arcs.
(Courtesy of the Bethlehem Steel Company.)

FIG. 162.—Cellular Cofferdam Used for Lock and Dam at Troy, N. Y.—East Side, Looking North. (Courtesy of United States Engineer Office, First District, New York City.)

FIG. 163.—Cellular Cofferdam Used for Lock and Dam at Troy, N.Y. Excavation for Upper Cross Walls of Lock—Looking West.
(Courtesy of United States Engineer Office, First District, New York City)

The pocket form with single cross-walls as used at Black Rock was adopted, but the main walls between supports were driven on a parabolic curve, with the bulge outward. This is the curve assumed by a member in tension, loaded uniformly along its horizontal projection. The result was a form toward which both former types had tended under load. The head was not as great as in the other cofferdams, so the test cannot be considered quite so severe, but the final deformation in use was very small. It appears that this form is based on correct principles.

Figures 162 and 163 are photographic views of the cellular cofferdam as used at Troy, N. Y.

The most recent use of the cellular cofferdam is that built by the Frazier-Davis Construction Company for the excavation of the intake and pump pits of the new waterworks for the City of St. Louis on the Missouri River near Hines Station, Missouri. This self-supporting cofferdam is 140 $\times$ 112 feet, and in its construction about 2000 tons of Lackawanna steel sheet piling, $12\frac{3}{4} \times \frac{3}{8}$ inches. Straight-Web section in 56-foot lengths were used. The work was carried out under the supervision of Mr. Edward E. Wall, Water Commissioner, and Mr. C. M. Daily, Designing Engineer.

ART. 59. BOX AND OPEN CAISSONS

230. Box Caissons.—A caisson is a watertight casing within which the work of placing a foundation may be done. The casing forms a shell which contains and usually remains a permanent part of the foundation. Caissons are of three general types: Those closed at bottom, known as *box or erect caissons*; those open at both top and bottom, known as *open caissons*; and those closed at top, called *pneumatic or inverted caissons.*

Box caissons of timber are commonly employed when masonry foundations are to be placed upon piles cut off under water. These caissons are watertight boxes, open at the top, which may be floated into position over the piles upon which they are to rest and then sunk by building the masonry inside them. The floor and lower part of the caisson is usually a permanent part of the foundation, but the sides which extend above the water are intended to act as cofferdams during construction of the masonry and are removed upon completion of the work.

The construction of box caissons varies with the depth of water in which they are to be sunk and the shape and dimensions of the foundations. For light work, timber studding with plank sides and

bottom may be sufficient, while in heavier work, a bottom of two or more thicknesses of 12×12-inch timbers, with sides built up of similar timbers on top of each other, and drift-bolted together, or timber framework with vertical staves may be used. The bottom must be capable of carrying the load of masonry required for sinking and the sides must resist the water-pressure or the outward pressure of material with which it may be filled. The caisson may be built on land, launched and floated to the side of the foundation, or when heavy timbers are to be used for a floor, it may more easily be built afloat.

Timber box caissons are occasionally used as a base for foundations upon fairly firm soil. The excavation must be made to the depth required before the caisson is sunk. Such caissons were used in the foundations of the south pier of the Duluth Ship canal. "They [4] were from 24 to 36 feet wide, 21 feet high, and from 50 to 100 feet long. The floor was 8 inches thick laid close, the channel side had a solid wall of a double thickness of 12×12-inch timbers, while the opposite side was composed of a single thickness of 12×12-inch timbers laid close. Connecting and bracing the two walls were transverse bulkheads of 10×12-inch material spaced 4 feet center to center horizontally.

"The caissons were built in the harbor, towed to the site, and sunk by filling with rock and gravel. After sinking, the caissons were covered with a layer of heavy timbers, in which was built the concrete pier, the top of the caisson being slightly below low water level."

For work of this kind, a timber crib or grillage which is not watertight is sometimes used for the lower part of the foundation, the top of the crib being below low water. A box caisson is then sunk on top of the crib. The floor of the caisson carries the masonry superstructure, and the sides, which are intended only to exclude water during construction, are removed when no longer needed.

In recent practice, reinforced concrete box caissons have come into quite general use. They have the advantage that they may become a permanent part of the foundation above low water, and in sea water they are not subject to the ravages of the teredo. These caissons are usually built upon ways on land and are launched and floated to the point where they are to be used. They are boxes with bottom and sides of reinforced concrete, and sometimes cross-walls or buttresses to assist in carrying the pressures brought by the water upon the sides, and the bending moments caused by lack of uniform support at the bottom. These caissons are sometimes used in placing

[4] Jacoby and Davis, Foundations of Bridges and Buildings, p. 264.

foundations on pile sub-structures. They are especially suitable when a foundation is to be placed on a good gravel or sand base in a moderate depth of water where the current is light.

Box caissons of small size have sometimes been sunk several feet into soft material by the use of water jets under the bottom. A number of pipes are run through the bottom to carry the water, which washes the material from underneath and allows the caisson to sink.

231. Types of Open Caissons.—An open caisson consists of a casing, with one or more openings extending through from top to bottom, intended to be sunk through soft materials which may be displaced by the weight of the caisson or removed by dredging through the openings. The caisson is always an integral part of the foundation. It may be simply a shell to contain concrete upon which the main reliance for strength is placed, or the caisson itself may be designed to bear the loads coming upon the foundation and the filling for the purpose of sinking and anchoring it.

The open-caisson method is extensively used and has been employed in placing foundations when the depth is too great for any of the other methods in common use. The caissons may be made of timber, steel, or concrete, and vary widely in design, according to the size and character of the foundation to be constructed. Three types of open caissons are in use: (1) Single-wall caissons of timber, consisting of an outer watertight wall with the bracing necessary to enable it to hold its form; (2) cylinder caissons, consisting of a single or double cylindrical shell of steel or concrete with a single opening at the center; (3) caissons having several openings or wells, with double walls between and around them. The double walls are joined at bottom into cutting edges, and the spaces between them filled with concrete or other materials to aid in sinking.

Caissons of the first type are used where the depth of sinking is small or the material through which they are to be sunk is soft. They are frequently employed for piers where the foundation is upon rock with little or no soil above it, and a shell is needed within which the concrete body of the pier may be formed. Cylinder caissons are used for foundations of small area which must be sunk to considerable depths through soft materials. The method with several openings is used for larger foundations requiring sinking to considerable depths.

232. Single-wall Timber Caissons.—Single-wall caissons are constructed in the same manner as box caissons, without the bottoms. The walls are commonly built up with 12×12-inch timbers

or with 12-inch plank laid flat. They are sunk upon a bottom of rock or other firm material which has been prepared to receive them. It is then only necessary to provide an outer wall of the form desired for the foundation, with bracing to resist the water pressure when pumped out, and a means of carrying sufficient load to sink it.

When the site is covered with soft material, sinking is accomplished by weighting the top of the caisson with some material which may afterward be removed, by dredging the soil from inside the bottom, and sometimes by using a water-jet to wash the soil from under the walls. After sinking, the bottom is sealed with concrete deposited under water and the caisson pumped out, after which the concrete filling may be placed in the open air. In some instances the filling is all placed through the water without pumping out the caisson, in which case, it would not be necessary that the caisson be watertight, but it must be capable of holding the concrete filling.

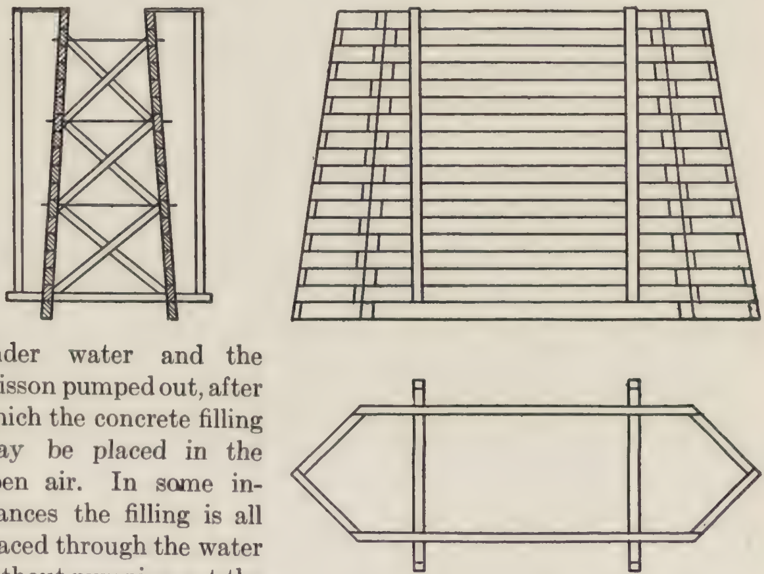


Fig. 164.—Open Caisson.

The permanent portion of a timber caisson usually terminates below low water, the part extending above the water being removed after serving as a cofferdam within which the masonry has been constructed.

In constructing the Columbia River Bridge of the North Coast Railway,[5] open caissons were used to provide forms for the construction of concrete piers upon the hard bottom of the river. The

[5] Engineering News, Oct. 5, 1911, p. 392.

depth of water was about 30 feet and velocity of current seven miles per hour. The construction of the caissons is shown in Fig. 164. The walls consisted of 12×12-inch timbers framed and braced to conform to the shape of the pier. The caisson was sunk by weighting with rails held in racks upon the sides in order to keep the load near the bottom and prevent capsizing. The concrete was deposited through the water to the depth of 32 feet, large buckets with movable bottoms being used for the purpose.

In the construction of a pivot pier on rock foundation, the Engineering Department of Boston used a single-wall circular caisson 60 feet in diameter and 30 feet high as a form for the concrete body of the pier. The caisson was built of about 145 courses of 3×12-inch yellow pine planks, 8 feet long, laid flat and breaking joints. The ends were beveled to make radial joints, and each plank secured to those below it by 1-inch oak tree nails 9 inches long, two at each end of each plank. In addition, the planks were well spiked to the lower courses throughout their lengths with 6-inch spikes. The courses were also secured together by 4×12-inch vertical planks opposite alternate joints.

Before placing the caisson, the site was dredged to rock. "There [6] was no attempt to construct the crib so that on the bottom it should conform to the variations in the rock surface. Instead, the bottom of the crib was made level and it was sunk until it took bearing on only a portion of the lower edge at the highest rock level. Then to provide continuous bearing to all parts of the circumference, and especially to complete the enclosure of the crib and confine the concrete that was afterward deposited within it, wooden boxes of varying sizes, but averaging about 4 feet square and 4 feet deep, were filled with lean concrete, lowered to the bottom and placed by divers under the edge of the crib to form a continuous wall. After the concrete boxes were placed, the excavation outside the crib was back-filled with gravel until the whole crib was surrounded by filling to about 29 feet below low water or some 2 feet above the bottom courses of plank. This back-filling formed an effectual seal to retain the concrete which was deposited in water inside the crib without unwatering the crib."

233. Cylinder Caissons.—The method of sinking wells by using a curbing of brick masonry which sinks as the earth is excavated from the bottom has been in common use for many years. A wooden cutting edge is constructed and the brickwork started on top of this and built up as the sinking progresses. This method has been used for a

[6] Engineering Record, Aug. 2, 1913.

long time in India for bridge foundations and in a number of instances in Europe. Usually work of this character has been of small diameter and sunk to comparatively shallow depths, but in some instances large shafts have been sunk by this method, and depths of over 200 feet have been reached.

Circular caissons of metal and reinforced concrete have come into use more recently and are frequently employed where foundations of small area are feasible, and in a few instances for foundations of larger area where circular piers are to be constructed. They are frequently used for the foundations of highway bridges where considerable depths must be reached, a pair of cylinders braced together being employed for each pier. Circular caissons of small diameter are constructed with single walls and a cutting edge at the bottom, those of larger diameters having double walls with space between the walls for loading with concrete.

Steel walls are commonly used for circular caissons in this country although small sections are frequently of cast-iron pipe resting upon a steel cutting edge. In the foundations of the California City Point Coal Pier, 4-foot cast-iron pipe was used in lengths of 12 feet bolted together.[7] A conical steel section 8 feet in diameter was used at the bottom to give large bearing area, and the concrete filling in the pipe was reinforced with vertical steel.

In constructing foundations for torpedo boat berths at Charleston, S. C., steel cylinders 8 feet in diameter and 42 to 52 feet long were used as cofferdams.[8] The cylinders were sunk through a bed of sand and about 4 feet into a bed of blue clay, which sealed the bottom, the soil inside being then excavated to near the bottom. Some wooden piles 45 feet long were driven inside the cylinder and the bottom section 5 feet deep filled with concrete, inclosing the tops of the piles. A form was then set up inside the cylinder and 4-foot reinforced concrete columns constructed to the top, the forms and cylinder above the bottom section being then removed.

Cylinders 8 feet in diameter were used for the foundations of the bridge over the Atchafalaya at Morgan City, La. (see Baker's "Masonry Construction"). These were sunk to a depth of 120 feet below high water and from 70 to 115 feet below the mud line. Below the river bottom, the cylinders were of cast-iron $1\frac{1}{4}$ inches thick and above of wrought iron $\frac{3}{8}$ inch thick.

A double-wall caisson of steel was used for the pivot pier of the Omaha Bridge and Terminal Company's bridge across the Missouri

[7] Engineering News, June 27, 1908.

[8] Engineering News, Nov. 11, 1915.

River at Council Bluffs, Iowa.[9] The caisson was of steel, 40 feet outside and 20 feet inside diameter and was sunk through sand and clay and coarse sand to the rock 120 feet below low water. The spaces between the walls were filled with concrete to furnish weight for sinking. In sinking, the material was dredged from inside the caisson, and water jets were used upon the outside to reduce the friction. Twenty 3-inch vertical pipes were carried down inside the outer cylinder to the cutting edge to provide for operation of water jets.

Reinforced concrete walls are gradually coming into use for cylinder caissons and seem to offer advantages for the purpose. The weight of concrete is of help in sinking and obviates the necessity of so much temporary loading, which is an item of considerable expense, while the greater durability of the concrete as compared with steel is also of value. Gravel filling may sometimes be employed in a reinforced concrete cylinder, while the steel cylinder should be filled with concrete.

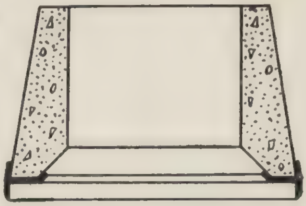


FIG. 165.—Reinforced Concrete Caisson.

Concrete cylinders are cast in place by using adjustable forms for building up the upper end as the cylinder is sunk. In some instances, however, they are cast in sections off the work and placed in position after hardening.

Reinforced concrete cylinder caissons were used in the foundations of the lumber docks at Balboa, Canal Zone.[10] The caissons were made 8 feet in outside and 6 feet in inside diameter and were pre-cast in sections 6 feet long. The bottom sections had conical exterior surfaces, widening to 10 feet in diameter and fitted into a cutting edge made of steel plates as shown in Fig. 165. The sections were reinforced with vertical bars and horizontal rings of steel, and were fastened together by means of six 1-inch anchor bars 12 feet long, which pass through cores molded in the shell. The rods were fastened together by the use of sleeve nuts which were adjusted in niches molded in the shell for the purpose.

The caissons were sunk 60 to 70 feet to rock, by laborers excavating inside of them, the water being kept down by pumping. The cutting edge was embedded about a foot in the rock, and a conical depression was blasted out of the rock in the center to give the concrete filling a strong bond.

Caissons having shells $6\frac{1}{2}$ feet in outside and $4\frac{1}{2}$ feet in inside

[9] Engineering Record, Jan. 24, 1903.

[10] Engineering Record, July 20, 1912.

diameter were used in the foundations for the Penhorn Creek Viaduct of the Erie Railroad.[11] They were reinforced with $\frac{1}{2}$-inch horizontal rings spaced 6 inches apart. The caissons were built in place in sheeted pits, 12 feet square and 15 feet deep, collapsible steel forms 5 feet long being used and 29 feet of caisson built at one operation, which after being allowed to set was sunk and another section added, depths of about 70 feet being reached in this way. The concrete was allowed to harden six days before sinking, which was accomplished by dredging with an orange peel bucket, and sometimes using a water jet. The jets were usually necessary below depths of about 40 feet. Four $1\frac{1}{2}$-inch pipes suspended from the derricks and guided by hand were employed. The jets were used around the upper part of the exterior faces of the caissons to within about 20 feet of the bottom.

Concrete cylinder caissons 6 feet in outside diameter, 8 inches thick and from 33 to 55 feet deep were used in the foundations for the storehouse of the Boston Army Supply Base; 577 of these piers were placed in 110 working days.[12] Pits 12 feet deep and 10 feet 4 inches square were dug and concrete cylinders 22 feet high constructed in the pits. The caissons were sunk below the bottoms of the pits by men digging the earth from inside them and forcing them down by the use of jacks. The forms were removed and excavation begun twenty-four hours after pouring the concrete. When a sufficient depth could not be reached by this method, the concrete cylinder was continued at the bottom in open cut behind poling boards. After the concrete shell reached solid clay, the hole was belled out below the end of the shell to give a larger bearing to the base of the pier and the whole filled with concrete. Ground water was kept down by constant pumping.

234. Dredging through Wells.—When foundations are to be sunk to considerable depths through soft materials, the method of dredging through wells is very commonly employed, wherein caissons of wood, steel, or concrete are built with vertical openings, or wells, extending through them. The body of the caisson surrounding the wells is filled with concrete to provide weight for sinking, and the soil at the bottom is removed by dredging through the wells, or by men in open excavation when the water can be kept down by pumping.

When the foundation is to be sunk through deep water, the caissons may be built on land or on barges and floated to the site. When the site for the foundation is on land or in shallow water, the caisson

[11] Engineering News, Oct. 13, 1910.

[12] Engineering News-Record, Sept. 24, 1918.

may be started in place, in an open cut or inside of cofferdams, and built up as the sinking proceeds. As the position of the caisson cannot be accurately controlled in sinking it is necessary to make the horizontal area covered by it larger than that of the foundation it is to carry. In a large caisson, the descent is guided by the manner of excavation, when resistance is met upon one end which tends to tip the caisson, the excavation is confined to that end until it is righted. If the caisson is narrow and the wells in one line, the control in a transverse direction is often difficult. Obstructions, such as boulders or sunken logs under the cutting edges, offer the most serious obstacles to work of this kind. These are not met at great depths and are commonly removed by divers, or sometimes by the use of a water jet.

This method is usually employed for foundations too deep for use of the pneumatic caisson, and frequently for those of less depth when the depth is too great for cofferdams and the conditions are such as to make the cost of pneumatic caissons unnecessary. In sinking a caisson by this method it is desirable to have rows of dredging wells on each side of the caisson as near the edge as is feasible to assist in guiding the caisson in sinking when inequalities are met in the material through which the caisson must pass.

Timber caissons have been employed more frequently than metal ones in this country.

The first use of deep open caissons in America was in the construction of the foundations of the Poughkeepsie bridge over the Hudson River.[13] The largest of these caissons was 60×100 feet in plan for the bottom 40 feet, narrowing to 40 feet in width at the top. There were 14 wells, each 10×12 feet, separated by one longitudinal and six transverse walls. The cutting edges at the bottom were 12×12-inch white-oak timbers, and the walls were of hemlock, solid and triangular in shape for the lower 20 feet, widening at that height to their full widths. The end walls and longitudinal walls were hollow above this height. The six transverse walls were solid and 2 feet thick for the full height.

The hollow walls were filled with gravel in sinking the crib and the soil was excavated through the wells with a clam-shell bucket. The caisson was 104 feet high and was sunk until the top was 23 feet below low water, the last dredging being done with the top submerged. The wells were then filled with concrete, and a box caisson with bottom of grillage 6 feet thick was sunk on top and the masonry of the pier built up in this as a cofferdam, the sides being removed when the masonry was above water.

[13] Transactions, American Society of Civil Engineers, June, 1888.

Open timber caissons were used in the foundations of the bridge of the Oregon Railway and Navigation Company across the Willamette River at Portland, Oregon.[14] They were 36×72 feet with six well holes, each 9 feet square, the cutting edges being made of steel plates inclosing the bottom timbers, with $6 \times 6 \times \frac{3}{4}$-inch angles at the bottom. The lower 11 feet of the crib was of solid timber, triangular in shape, with vertices at the cutting edges. Above that height, walls 12 inches thick were carried up and the entire spaces around the wells filled with concrete as the caissons were sunk. The caissons rest upon cemented gravel 120 and 130 feet below low water, and when in final position the wells were filled with concrete. The crib proper ends at 20 feet below low water and the upper parts of the caissons were built to be used as cofferdams within which the superstructure of the pier could be constructed.

" In the construction of each of these piers a substantial dock was first constructed in the river, consisting of about 100 piles well driven down, capped, and braced together. Borings were then made around the entire perimeter of the crib at spaces about 8 feet apart, and the elevations of hard material at all points were determined. It was found to be on a considerable slope, showing a difference of elevation of 22 feet for opposite diagonal corners. When these elevations were determined, pipes were successively sunk at numerous points around the perimeter and in the location of the cross-walls, and holes drilled in the hard material to a common level some 2 feet below the lowest elevation of the top of the cemented gravel. As soon as the drilling at each hole had been completed to the proper elevation, a cartridge of black powder and dynamite in a sheet-iron case was lowered to the bottom of the hole and discharged by an electric battery. This process was repeated at such frequent intervals as it was deemed would produce a bottom uniform in character throughout the entire area of the crib. Thus the blasting for leveling the cemented gravel was carried on before an excavation was made through 50 feet of gravel and sand. In the meantime the steel cutting edge had been set up and riveted together on ways in a shipyard convenient, and enough timber put on to float the crib, which in this condition was some 30 feet high. It was then floated into position in the dock already prepared and other piles driven on the open end of the dock, entirely inclosing the crib."

For the Thames River bridge of the N. Y. N. H. and H. Railway at New London, Conn., some of the piers were sunk by the open

[14] Railway Age-Gazette, July 14, 1911.

caisson method.[15] These caissons have cutting edges of steel filled with concrete, both under the outside walls and under the center and cross-walls (see Fig. 166). One of these caissons reached a depth of 142 feet, and the crib extended up to 30 feet below mean low water, where the masonry of the pier was started.

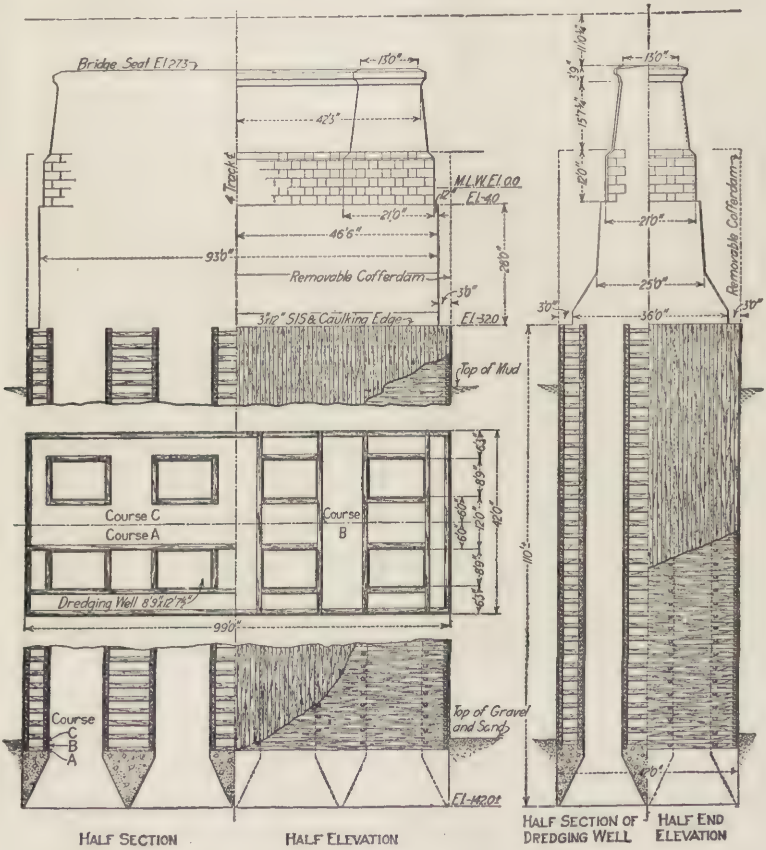


FIG. 166.—Open Caisson for the New London Bridge.
(Jacoby and Davis, "Foundations of Bridges and Buildings.")

In constructing the caisson, the cutting edges were assembled on a false bottom resting on ways on shore, and the timber work carried up about eight courses. The concrete was then placed in the cutting edges and up to a depth of one course on the cribwork. The caisson was then launched and towed to the site. The timber

[15] Engineering Record, December 16, 1916, p. 737.

structure was then built up to keep the top above water when sunk upon the bottom of the stream. The false bottom was removed by filling the dredging wells with water and weighting the bottom with sand through the wells, the bottom being recovered by the use of lines attached to it before sinking.

When the bottom has been removed the caisson floats on account of the air space above the concrete and between the dredging wells and outer walls and is sunk by filling this space with concrete. The crib is built up as the sinking progresses until the level is reached at which the masonry of the pier should start, and above this level a cofferdam is constructed within which the masonry may be built.

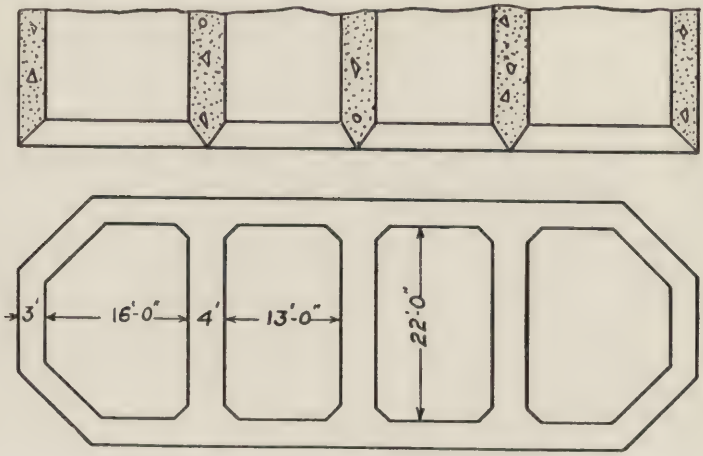


Fig. 167.—Concrete Caisson.

Jet openings are provided on the outside of the cutting edge a few inches above the bottom as well as on the bottom. These are arranged in sections separately connected with pipes so that they may be used to assist in guiding the caisson in sinking.

Concrete open caissons are rapidly coming into use and possess many advantages where they may be built in place and started in the open air. When the wells are filled with concrete, it makes a monolithic structure, with no parts subject to decay or corrosion, and when heavy walls are used, the weight is of advantage in sinking. Reinforcement is usually employed, although in a few instances heavy shells have been sunk without reinforcement. Light reinforcement seems desirable in nearly all cases as having additional security against cracking.

The method of dredging through wells was used in sinking con-

crete caissons for the foundations of the Pittsburgh & Lake Erie Railroad bridge at Beaver, Pa. The caissons were 80 feet long and 28 feet wide with semicircular ends. The shell was 7 feet thick with two cross-walls each 5 feet thick, and was tapered in the lower 9 feet to the cutting edge of steel. Rectangular cofferdams were constructed around the site in about 7 feet of water and pumped out. The caissons were then built inside the cofferdams and sunk through about 38 feet of sand and gravel to the rock. When they had been sunk nearly to the rock by dredging through the open walls, they were transformed into pneumatic caissons and bedded upon the rock by the pneumatic process.

Figure 167 shows a reinforced concrete caisson used in the foundation of a pier of the American River bridge of the Southern Pacific Railroad.[16] It was 76 feet long, 28 feet wide, and 22 feet high, with a shell 3 feet thick and three cross-walls each 4 feet thick, and was built when the stream was dry in a pit dug to the level of ground water, being sunk by dredging through the four wells. When the top of the caisson reached the ground level, a timber cofferdam was constructed on top. The sinking was then continued until the stratum of cobbles and boulders upon which it was to rest was reached. The wells were then filled with concrete and the pier built up in the cofferdam. This caisson was very light in weight and the sinking so slow that it was found more economical to construct the other piers by excavating inside of sheet-iron cofferdams.

In constructing the channel pier of the North Side Point bridge over Allegheny River at Pittsburgh, a concrete caisson $83\frac{1}{2} \times 23$ feet was used, with 4 wells 10×9 feet spaced 19 feet between centers.[17] When the caisson reached a height of 31 feet, with the cutting edge 17 feet below the bed of the river, a transverse crack extending from top of caisson to below the river bed occurred near the mid length, probably due to tension in the top of the caisson caused by unequal dredging. This caisson was unreinforced. It was blasted out and replaced by one reinforced by longitudinal bars.

Iron and steel caissons have been extensively used by English engineers, but in this country the use of metal has usually been restricted to the cylindrical form, as timber has generally been found cheaper and more satisfactory.

The foundations of the Hawkesbury bridge in Australia is a notable example of this method. The caissons were of wrought iron, 48 feet long and 20 feet wide, with semicircular ends. Three

[16] Engineering Record, Aug. 27, 1910.
[17] Engineering News, Oct. 17, 1912.

circular dredging wells were used, 8 feet in diameter and 14 feet between centers. The pockets around and between the wells were filled with concrete to aid in sinking the caisson, and the sides and filling were continually built up as the sinking progressed. The caissons were bedded upon sand, the maximum depth reached being 162 feet below high water, through 108 feet of mud and silt.

The bottoms of the caissons were made flaring for the lower 20 feet, making the bottom 2 feet wider all around, this arrangement being intended to reduce friction on the sides, but it was found to increase seriously the difficulty of guiding the caisson. When the soil is not uniform over the base, there is a tendency to travel toward the firmer material, which was obviated by making the surfaces of the wider bottom sections vertical instead of flaring, with an offset about 20 feet above the cutting edge.

CHAPTER XIV

PNEUMATIC FOUNDATIONS

ART. 60. PNEUMATIC CAISSONS

235. The Compressed-air Method.—A pneumatic caisson as ordinarily employed consists essentially of an air-tight box, or working chamber, open at the bottom, which may be filled with compressed air to keep back the water and permit the excavation of the soil from below the bottom of the caisson by men working in the compressed air. The working chamber is ordinarily at the bottom of a crib, constructed in a manner similar to an open caisson and arranged to be filled with concrete to aid in sinking, Shafts with air locks connect the working chamber with the outside air, and provide means of entering the working chamber and transporting materials to and from it.

The pneumatic caisson has an advantage over the open-well caisson method in the fact that the excavation is made in the air and the conditions at the base are in plain view. When obstructions are met in sinking, such as boulders or logs, they may be removed from under the cutting edge by the workmen in the working chamber without the difficulties met when working through a considerable depth of water. When the caisson is in place, the bed upon which it is proposed to place the foundation is open to inspection and may be properly prepared to receive the concrete.

The concrete is also placed in the air, thus eliminating the danger of injury to the concrete from washing in passing through the water and providing a better and more reliable foundation.

The disadvantage of this process is that the men are obliged to work in compressed air, which involves short hours and many precautions to prevent injury to health. It is also necessary to provide machinery and appliances for maintaining the air pressure, and locks through which both men and materials may pass to the outer air, making work rather slow and expensive.

This method is frequently employed for foundations to depths within which men may safely work in the compressed air, about 110 feet below water surface, and is sometimes combined with the open-

caisson method, being used where obstructions may be met in sinking the open caisson, or for bedding a caisson which has been sunk by open dredging. The caissons are constructed of timber, metal, or concrete. The use of timber caissons has been quite common in the United States, although concrete seems to be coming into more general use, while in Europe, iron and steel are preferred.

The earliest use of the pneumatic principle was in the diving bell, which consisted of a steel cylinder closed at the top and connected with an air pump above the surface by means of a hose. This was followed by the use of pneumatic cylinders, originally known as pneumatic piles. These were cast-iron cylinders which extended above the water and had air locks to permit the passing of men and materials to and from the compressed air chamber at the bottom. From these beginnings, the steel and concrete cylinders, and the larger caissons now in use have gradually developed.

For depths greater than about 110 feet, open caissons must be used on account of the inability of men to work in air under greater pressure. In some instances, where obstructions may be met at less depths, the pneumatic process is used at first and the locks are removed and the sinking continued as open caissons when the depths become too great for the pneumatic process.

The pneumatic system was first applied to large foundations in this country by Col. James B. Eads in the construction of the St. Louis arch bridge over the Mississippi in 1870, where a depth of 109 feet below water surface was reached. This was followed by the Brooklyn bridge foundations, in which the caissons were very large in plan and sunk to a depth of 78 feet. Since that time, pneumatic caissons have been used in a large number of structures, with rapid improvement in the methods of handling the work, and in preventing injurious effects upon men working in the compressed air.

Pneumatic caissons have been quite extensively used for heavy building foundations, particularly in New York City, where it has been found necessary to carry the foundations of high buildings through unstable materials to solid rock, without undermining older buildings on more shallow foundations. In such construction, separate caissons of small area are sunk for individual piers, although frequently two or more piers rest upon a single caisson. The layout of a foundation for a heavy building is a matter requiring very careful study. The first instance in which this method was applied to a building in New York was in the Manhattan Life Insurance Company's building by Kimball and Thompson, architects, in 1893.[1]

[1] Engineering Record, Jan. 20, 1893.

Since that time the use of pneumatic caissons has become quite common.

236. Cylinder Caissons.—Pneumatic cylinder caissons are usually quite similar in construction to open cylinder caissons, with the addition of an air-lock and means of sustaining the air pressure. The air-lock is formed by two horizontal diaphragms about 7 to 10 feet apart and fitted with doors. The facility with which cylinder caissons may be changed from open dredging to pneumatic caissons is frequently an advantage when obstructions are met or quicksand is encountered. When a caisson has been sunk without difficulty by open dredging, the introduction of an air-lock and the use of a compressed air working chamber to aid in properly preparing the bed upon which the caisson is to rest may be of much value in giving stability to the foundation.

Bridge piers are frequently formed of a pair of cylindrical caissons sunk side by side to a firm bearing and extending upward to carry the trusses; the upper part of the pier being stiffened by connecting the two cylinders with braces. Some of the earlier caissons were of cast iron, although the present practice is to use steel or reinforced concrete.

For the foundations of the Merrimac River bridge [2] at Newburyport, Mass., cast-iron cylinders 8 feet in diameter were used. These were $1\frac{1}{2}$ inches thick, and were cast in 8-foot sections having inside flanges, which were planed and bolted together, the joints being made tight with a mixture of red lead and linseed oil.

For handling the pipe, a staging was built by driving eight piles; two at each corner of a 10-foot square, capping them with 12×12 timbers, and bracing them together. The bottom section was without a flange and used as cutting edge. They were sunk by open dredging through the pipe with an orange-peel dredge until the cylinders reached a stratum in which boulders were imbedded in coarse sand and gravel. An air-lock was then placed on top and the water blown out. The bottom was reached by the use of ladders and examination made of the stability of the stratum. It was then decided to use this stratum as a foundation. To solidify the base under the cylinders, the air pressure was allowed to drop a little and a foot or so of water to enter the bottom of the cylinder. Portland cement was then mixed with this water into a thin grout and the air pressure increased, forcing the grout into the gravel below. This seemed to be quite effective in solidifying the base under the caissons.

[2] Engineering Record, Aug. 20, 1904.

In the cylinder piers of the Atchafalaya bridge [3] of the Texas and Pacific Railway, steel pneumatic cylinders were employed.

Each pier consists of a pair of 8-foot steel cylinders filled with concrete and braced together at the top by a stiffened web plate or diaphragm about 20 feet high (as shown in Fig. 168). Each cylinder has a concentric 5-foot inner cylinder with a conical section uniting it to the cutting edge, and closing the lower end of the annular space. There are four stiff webs riveted to both cylinders to connect them together and keep them concentric. The inner cylinders extend above

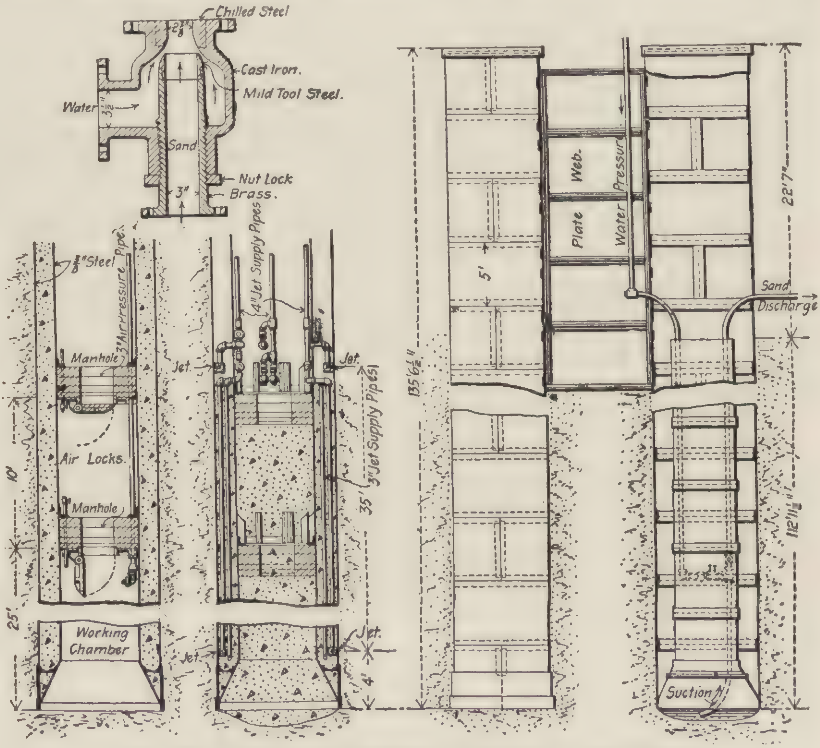


Fig. 168.—Pneumatic Cylinders, Atchafalaya River Bridge.
(Jacoby and Davis, "Foundations of Bridges and Buildings.")

the bottom of the river, but terminate about 22½ feet below the top of the outer cylinder. These have a total length of 135 feet and are made with 5-foot rings erected in 10-foot sections, which were successively riveted on the tops of the cylinders with single lap splice plates.

Four feet above the cutting edge there was made a row of holes through the outer shell around the cylinder, and 35 feet higher was a similar row. All these holes were for hydraulic jets to decrease skin friction and diminish resistance to sinking.

These were arranged in quadrants, each connected to separate pipes, and could be used to assist in guiding the cylinder in sinking

[3] Eng. Record, April 8, 1899, p. 421.

The working chamber was 25 feet high, roofed by a 2-foot oak diaphragm made of four thicknesses of timber with a 2-foot circular hole closed by a heavy cast-iron door. Eight feet above this deck a similar diaphragm formed the roof of the air-lock.

The excavated material was removed by a sand pump through a 4-inch pipe discharging above the bottom of the river.

In making a settle, the material would ordinarily be pumped out of the working chamber to near the cutting edge. Then the pressure gang, consisting of three men, would come outside. The jets would then be operated two minutes and then the pressure was suddenly reduced to two-thirds or one-half of the working pressure and the cylinder would sink. The working chamber would be found partly filled with the material encountered, which would be pumped out and sinking resumed.

The amount of material excavated was probably three or four times greater than the displacement of the cylinder. This was due to drawing material down from the outside.

The piers of the Columbia River bridge at Trail, B. C. (designed by Waddell and Harrington of Kansas City), were [4] composed of cylinders 9 feet in diameter at the bottom and 6 feet at the top. The caisson was of steel plates $\frac{5}{16}$ to $\frac{7}{16}$ inch thick, with an inner shell 3 feet in diameter. A cutting edge and working chamber were formed by splaying the inner shell out to meet the outer one from a point 8 feet above the bottom. Two diaphragm doors were used to form the air-lock, one 13 feet above the cutting edge and the other 16 feet higher. Later, during the sinking, another door 16 feet above the second one was added to form a new air-lock, the lower one being available in case of emergency.

The cylinders were filled with concrete, and were joined above low water by two $\frac{5}{16}$-inch vertical plates 2 feet apart braced to each other and the space between filled with concrete.

In a caisson of this kind it is necessary only to carry the inner shell high enough to provide for the air-locks and to allow sufficient weight of concrete to be placed between the shells to sink the caisson. The outer shell must be kept above the water to serve as a cofferdam and acts as a form to hold the concrete filling for the upper part of the pier.

The plate girder spans of the Bronx Viaduct for the New York Connecting Railway were supported by piers founded upon ninety-one cylindrical concrete caissons varying from 10 to 18 feet in diameter and sunk to an average depth of 55 feet.

[4] Engineering News, December 5, 1912.

They had angle iron cutting edges [5] and vertical and horizontal reinforcement of rods $\frac{3}{4}$ inch in diameter, and were built in steel forms on the surface of the ground. They were sunk by dredging through the center well with a $\frac{1}{2}$-yard orange-peel bucket. They were sunk 8 to 12 feet through fill to blue clay by hand excavation, the ground water below 3 feet from the surface being removed without difficulty by ordinary Cameron pumps. The fill was underlaid with a 25-foot stratum of blue clay, which sealed the caisson and excluded most of the ground water, enabling the work to be carried on by hand if desired; but it was found generally more desirable to excavate there with the orange-peel buckets.

Below the clay the caissons were sunk through 8 to 10 feet of quicksand into

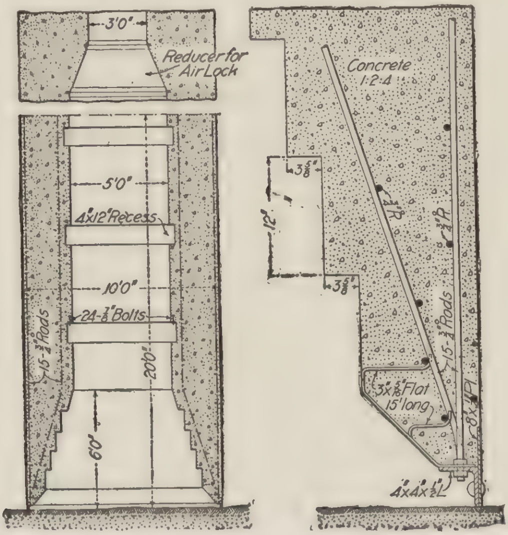


FIG. 169.—Pneumatic Caissons of Reinforced Concrete for Bronx Viaduct of New York Connecting Railway.
(Jacoby and Davis, "Foundations of Bridges and Buildings.")

hardpan or rock stratum. As soon as they entered the quicksand, the shafts were closed with air-locks and the remainder of the sinking was completed under pneumatic pressure.

As the work progressed the height above the cutting edge at which the air-locks for the successive caissons were required could be estimated in advance, and at that level the fifteen vertical reinforcement rods terminated, with two ends engaging the bottom flange of a conical riveted steel reducer 3 feet in diameter at the top to which the Moran air-lock was connected.

Whenever the caissons showed a tendency to get out of plumb they were easily corrected by placing one, two, or three inclined wooden struts against the downhill side of the caisson and securing

[5] Engineering Record, Sept. 20, 1913.

them rigidly to it. As the sinking continued, the lower end of each strut being fixed, the upper end revolved through the arc of a circle, forcing the caisson back into the vertical.

Telescopic Caissons.—The use of pneumatic caissons in a restricted space has been accomplisehd by the use of telescoped caissons. The caisson is made in several sections each inside the one above, and designed to fit tightly against the one above when driven to place. In the Central Union Trust Company's building, 80 Broadway, New York, foundation for a vault was put in without disturbing the foundations of the building by this method under the direction of Mr. T. Kennard Thomson.[6] The several sections, varying from 5′ 4″ to 6′ 6″, were placed inside each other in an open excavation, the depth of the upper section, and an air-lock placed at the top. They were then driven by hydraulic jacks and the material inside excavated as the sinking progressed. Shafts about 4 to 5 feet in diameter were driven 44 feet to a bed of hard pan. This method is particularly applicable to the underpinning of an old wall adjacent to a new foundation.

237. Timber Caissons.—The essential parts of a pneumatic caisson are the *working chamber* and the *air-lock*. To these must be added shafts for connecting the air-lock with the working chamber and the outside air. A crib is commonly used on top of the working chamber to hold the filling required in sinking the caisson, which later becomes an integral part of the foundation. The cofferdam is an extension upward of caisson or crib to hold back the water while the masonry of the pier is being constructed on top of the crib.

The method of construction to be adopted varies with the size of the caisson and the conditions under which it is to be sunk. When a considerable depth is to be reached below the bottom of a stream, the crib is commonly carried up to near the bed of the stream, and above that the pier is built of masonry or concrete inside the cofferdam which is later removed. Sometimes when there is a comparatively shallow depth of water, the pier masonry may be begun directly on top of the working chamber without the use of a crib, and sometimes without a cofferdam; the masonry being carried up as the caisson sinks, and its top always kept above the water surface.

Timber caissons have been extensively used in the United States, although recently there has been a tendency to substitute reinforced concrete on account of the high cost of timber.

[6] Engineering News-Record, Aug. 21, 1919.

Working Chamber.—The working chamber is usually an opening at the bottom of the caisson surrounded by side walls, which rest upon a cutting edge and carry the roof and weight of caisson and crib. The height of the working chamber is commonly from 6 to 8 feet. It must provide room in which men may work in excavating the soil for sinking the caisson. In sinking into stiff materials the excavation will usually be in advance of the cutting edge and this gives head room for working. When soft materials are encountered, in which the cutting edge will advance more rapidly, the excavation will usually be made by pumping without labor of men in digging.

Cutting edges.—Cutting edges may be either sharp or blunt. For light caissons, as in the cylinder caissons discussed in the last article, sharp edges are usually employed. For heavy caissons, however, most engineers seem to prefer blunt cutting edges, on account of the difficulty of getting sufficient strength in a sharp one to stand the stresses induced in sinking, without losing its shape when hard materials are met.

It is desirable that the cutting edge have a form which will permit the caisson to sink readily, but it should not drop suddenly when soft materials are encountered. It must pass into the material sufficiently not to permit air to escape under it; a vertical plate is sometimes used extending 4 to 6 inches below the edge to cut off escape of air.

Many engineers at present favor the blunt cutting edge [7] in preference to the sharp one. T. K. Thomson's experience is, that when the knife edge is needed, *i.e.*, in hard material, it would cost too much to make the cutting edge strong enough, and where the material is soft a knife edge is not needed.

The cutting edge of the caisson used in the Kinzie Street drawbridge, Chicago, was formed with an 8-inch channel iron laid horizontally with flanges turned up as shown in Fig. 170. The same general form was used on the caissons of the Broadway bridge, Portland, Ore. (Fig. 171), the only difference being that in the latter case the cutting edge timber extended out to protect the bottom of the sheathing, while in the former case the channel iron served this purpose. This form of metal cutting edge is the most economical and was designed in 1901 by T. K. Thomson.

The Side Walls may be either vertical walls braced at intervals by struts, or double walls, in which inclined struts with sheathing form a sloping roof (see Fig. 173). In the earlier caissons, inclined walls were usually employed, sometimes formed of solid timbers, drift-bolted together; but more recently vertical walls are often preferred on account of making excavation easier near the sides of

[7] Jacoby and Davis, Foundations for Bridges and Buildings. Second Edition, p. 321.

the chamber and close to the cutting edge. The side walls of the caisson and crib are usually sheathed on the outside with plank running vertically to diminish friction in sinking. Sometimes a double sheathing is used, one thickness being diagonal and the other vertical.

The side walls must be strong enough to carry the weight of the caisson and the masonry of the pier to be built inside it. They may also be subjected to lateral thrust and shocks due to unequal support under the cutting edge or to sudden drops in sinking. The outside of the walls should be vertical to aid in guiding the caisson

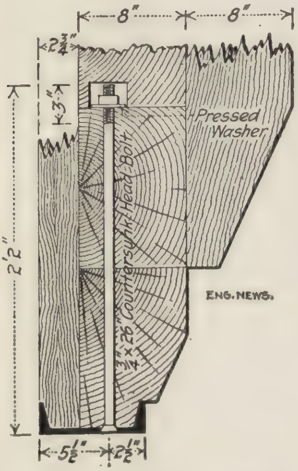


Fig. 170.—Blunt Cutting Edge.

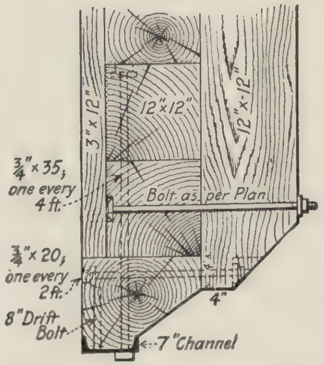


Fig. 171.—Blunt Cutting Edge.

(Jacoby and Davis, "Foundations of Bridges and Buildings.")

in sinking. Attempts to reduce frictional resistance in sinking by flaring the walls at the bottom have not proved successful on account of the difficulty in controlling the alignment of the caisson.

In large caissons, cross-bracing will usually be required to hold the side walls in place and to aid in supporting the roof. This bracing may consist of bulkheads running across the working chamber in either or both directions. These may give support to the roof, but will also obstruct freedom of working in the chamber. In one of the caissons of the new Quebec bridge, the area of 55×180 feet was divided into eighteen compartments, by a longitudinal and eight transverse bulkheads. The longitudinal bulkhead was 24 inches

and the transverse ones 12 inches thick. This caisson was unique in that the weight of the roof and filling above were supported upon these bulkheads instead of on the side walls and cutting edge as is usual. There were 40 sand jacks bearing against the roof, and about double this number of sets of blocking under the bulkheads or

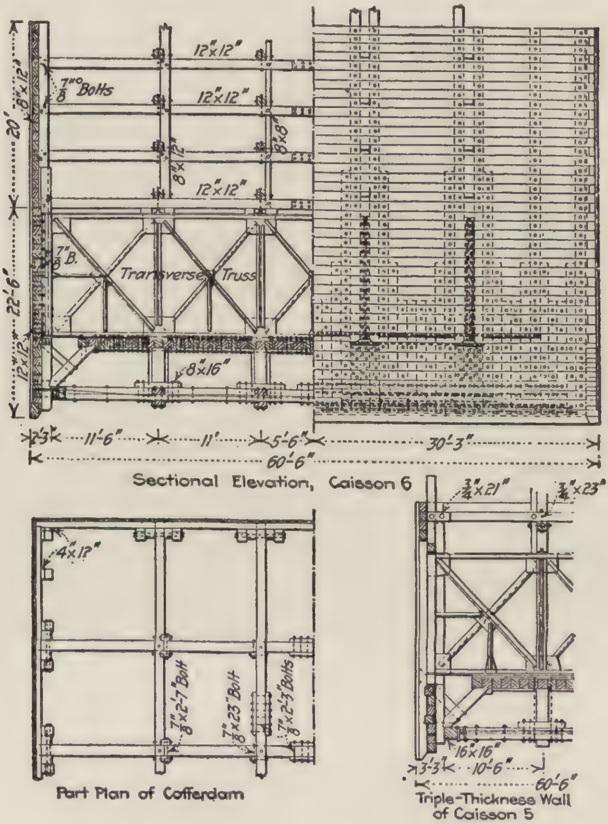


FIG. 172.—Details of the Metropolis Bridge Caisson.

(Eng. News, March 22, 1917, p. 465.)

cross walls. The jacks consisted of 29-inch steel cylinders, with plungers of hard wood. At the lower end of each cylinder were two holes, on opposite sides, with slides to close them. When the caisson was to be lowered, a trench 2 or 3 feet deep was excavated under the cutting edge and filled with clay, to lubricate the caisson and prevent

leakage of air. The blocking under the cross walls was removed by undermining it and was reset at a lower level. The caisson was then lowered by opening the slides in the jacks and washing the sand from under the plungers with a hose. The jacks were then refilled about two-thirds full of sand and the wood plungers inserted and blocked against the roof. The caisson was thus sunk evenly, and in the correct location.

In the Municipal bridge, at St. Louis, the bottoms of the side walls were braced by 12×12-inch timbers about 10 feet apart, both longitudinally and transversely, with adjustable tension rods on each side of each strut; at intersections, they were braced by vertical 12×12-inch timbers and pairs of $\frac{7}{8}$-inch rods extending to the roof of the caisson. A similar method was used in the caissons of the Ohio River bridge, at Metropolis, Ohio (see Fig. 172).

The caissons of the Union Pacific Railway bridge at St. Joseph, Mo., are shown in Fig. 173. The walls of the working chamber are braced at mid-height by one longitudinal and five cross struts; each strut is a 12×14-inch timber, with a $1\frac{1}{2}$-inch rod on each side.

The Roof of the Working Chamber carries the weight of the crib and filling. This roof must be made strong enough to carry these loads and to resist shocks incident to sudden changes in the rate of sinking. The extent to which the weights of cribwork and filling may cause bending stresses in the roof depends upon the arrangement of the cribwork and the type of filling used. In some of the earlier caissons, when masonry for supporting the piers was begun directly upon the roof, it was found necessary to make the roofs very heavy in order to carry these loads safely.

The roof of the caisson under the Brooklyn bridge was made 22 feet thick of solid timber. The caisson was 102×172 feet, and the roof was made of 12×12-inch timbers thoroughly drift-bolted together. In the St. Louis Arch bridge, the east abutment had a caisson with roof about 5 feet thick. Two timber bulkheads were used longitudinally under the roof and two transverse iron girders above to assist in supporting it. This caisson was 82×$72\frac{1}{2}$ feet with the corners cut to give it a hexagonal form.

In later caissons, the tendency has been to use thinner roofs and to make the crib self-supporting by using reinforced concrete on top of the timber roof. In the roof of a caisson used for the Union Pacific Railway bridge [8] at St. Joseph, Mo., this method was adopted (see Fig. 173).

[8] Engineering News-Record, Oct. 25, 1917.

The caissons have the sides of the cribs and the roof of the working chamber built of 12×14-inch timbers thoroughly drift-bolted and caulked. The working chamber has sloping sides, the ends of which are shaped for the attachment of the steel cutting edge. The roof is designed of sufficient strength to take the weight of a reinforced concrete beam built over it. This beam takes the load of the pier during the sinking, until the caisson is landed and sealed.

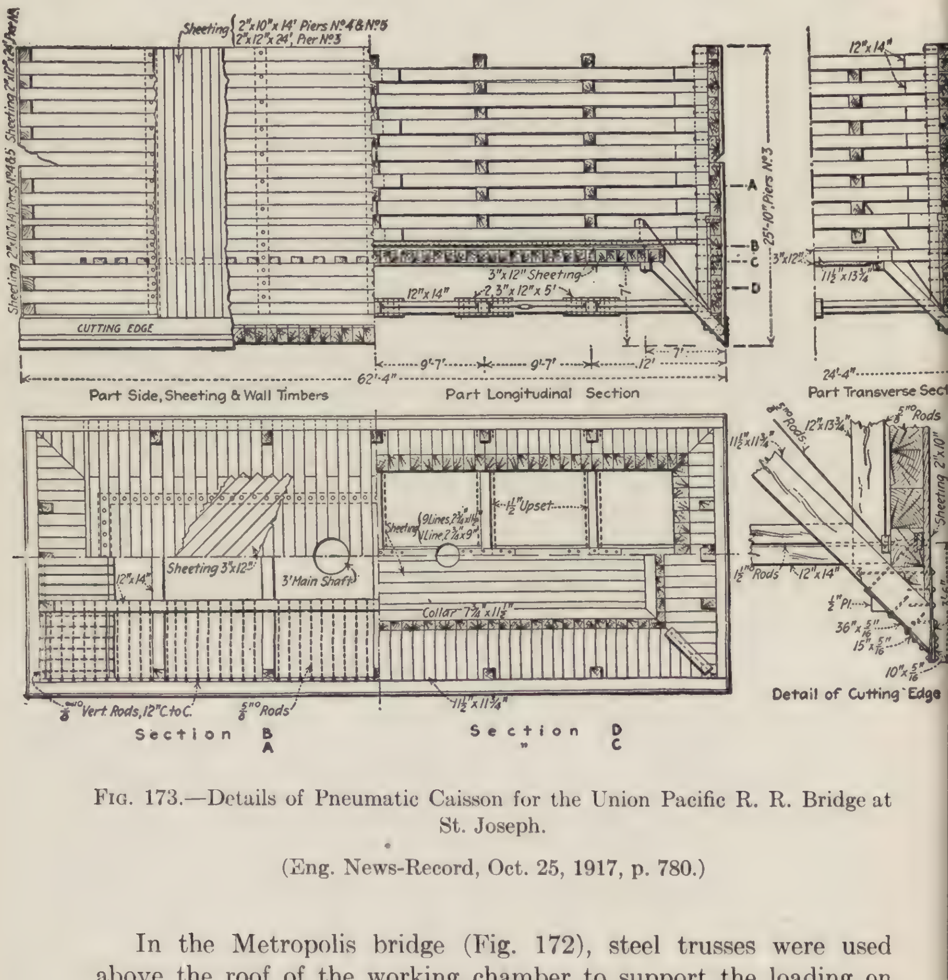


FIG. 173.—Details of Pneumatic Caisson for the Union Pacific R. R. Bridge at St. Joseph.

(Eng. News-Record, Oct. 25, 1917, p. 780.)

In the Metropolis bridge (Fig. 172), steel trusses were used above the roof of the working chamber to support the loading on account of the large size of the caisson. (60′ 6″×110′ 6″).

The roofs of the caissons for the Municipal bridge over the Mississippi at St. Louis consisted of a single layer of 12-inch timbers with sheeting of 3-inch planks, on both upper and lower surfaces, placed diagonally and well caulked. This acted as a form for the

concrete filling, which was reinforced near the lower surface with 1-inch bars, 6 inches apart both longitudinally and transversely.[9]

In small caissons for foundations of buildings, temporary roofs are sometimes employed, which serve as forms for the concrete filling, and are removed before the working chamber is filled with concrete, in order to make the construction monolithic, with no separation between the bottom concrete and the filling.

When stone masonry is used, the weight to be carried by the roof is usually taken as that of a prism of masonry whose sides make an angle of 60° with the horizontal; the arching action of the masonry being supposed to transfer the weight of the masonry above this line to the side walls.

Great care must be used to make the roof and walls of the working chamber air-tight to prevent the escape of the compressed air. This is usually accomplished by using one or two layers of 3-inch sheathing (tongue and groove), and caulking with oakum.

The Crib is a structure built on top of the working chamber for the purpose of carrying the filling required in sinking. It becomes a permanent part of the foundation when filled with concrete, being carried up to a proper level to support the base of the masonry of the pier. In some instances, when the depth is not great below the bottom of the stream, the crib may be omitted and the masonry begun directly on the roof of the working chamber, the weight of the pier masonry being used in sinking the caisson.

The crib as ordinarily constructed consists in continuing the outside walls of the working chamber upward, sometimes modifying the construction as may be desirable to resist the diminishing press sures. Interior cross braces must be provided capable of safely resisting the external water and earth pressures, and of holding the concrete filling when placed.

The walls of the crib are commonly 12×12-inch timbers placed horizontally and drift-bolted together, and bolted to verticals of the same size. The bracing of the crib, when to be used under considerable pressure, is sometimes a grillage of timbers at right angles to each other spaced 8 or 10 feet apart in each layer (see Fig. 173). In other instances, the cross braces are placed several feet apart vertically, with vertical stanchions at the intersections (see Fig. 172). The outside of the walls of the crib is usually sheathed with vertical plank, caulked to make the walls watertight.

For a pier in a stream, the crib usually terminates just below the bottom of the stream and the masonry of the pier begins at that point,

[9] Engineering Record, Oct. 15, 1910.

in order to give a section which will disturb the flow of the stream as little as possible.

The Cofferdam is used above the top of the crib to hold back the water during the construction of the pier masonry. The walls of the cofferdam are usually of the same type as those of the crib and are arranged so that they may be disconnected from the crib and removed after the construction of the pier.

In construction of the foundation for the New York Tower of the Manhattan bridge across East River, a caisson 144 feet long, 78 feet wide and $47\frac{1}{2}$ feet high, including the crib, was used.

Above the caisson there was a cofferdam 50 feet high [10] whose walls were made with solid courses of one thickness of timber spiked together and bolted to vertical timbers. At the joints between the caisson and cofferdam the horizontal courses were not spiked together, but the vertical timbers were continuous across them to form solid splices. Flanges were bolted to the verticals just below the top of the caisson to engage vertical screw-ended rods attached to the top of the cofferdam. After the masonry was constructed, the verticals were sawed through horizontally in the plane of the joint between the caisson and the cofferdam. This left the two attached by means of the vertical rods only, and when these rods were unscrewed from the top, the cofferdam floated up and was removed.

Air-locks are used for the entrance and exit of men, and for passing materials into and from the working chamber. An air-lock commonly consists of an enlargement of the shaft with an airtight door at top and bottom, similar to the arrangements described in Section 236 for pneumatic cylinders. In large caissons where the depth is considerable and an elevator is used for transporting men to the working chamber, an auxiliary chamber is sometimes employed for the air-lock. In this lock, the shaft leading to the outer air is not directly over the lower shaft and the air-lock is alongside the bottom of the one shaft and the top of the other, with doors opening horizontally into each shaft, or if the air-lock be at the top, the upper shaft would not be needed.

The air-lock for materials is placed at the top of the shaft for convenience in handling. The air-lock for men is either at the top or some distance up from the bottom in order to leave the lower end of the shaft open as a refuge for the men in case of an accident, which might trap the men if the lock were too near the working chamber.

When the air-lock is at the top of the shaft, it becomes necessary to remove and replace it in extending the shaft as sinking proceeds, For this purpose a door is usually placed at the bottom of the shaft which may be closed to retain the pressure in the working chamber when the lock is being moved.

[10] Engineering Record, March 12, 1904.

The shafts are usually of steel plate from 3 to 6 feet in diameter. They are made with flanged ends which may be bolted together. Where separate shafts are used for materials, they are usually smaller, about 2 feet in diameter. Sometimes collapsible tubes are used for shafts, which may be removed after the concrete filling has been placed around them.

Design of Caissons.—The dimensions and shape of a caisson must be fixed by the character of the structure it is to carry and the nature of the materials through which it is to be sunk and by which it is to be supported. The height of the caisson and crib will be fixed by studies of the materials underlying the site and selection of the stratum upon which it is to rest.

The horizontal area of the caisson must be sufficient to distribute the loads carried without excessive pressure upon the soil at its base. It must be large enough to carry the base of the pier which is to rest upon it with reasonable allowance for inaccuracy of position in sinking. When it is to be supported by rock or other firm material the caisson must be of sufficient strength to carry the weight of the structure to be supported by it.

The load to be transmitted to the soil at the base of the caisson includes a number of items which are ordinarily somewhat difficult of determination. They include:

Weight of caisson, and concrete filling when used;
Weight of concrete filling in working chamber;
Weight of pier masonry resting on caisson;
Weight of structure resting on pier, including live load;
Weight of water and earth on top of caisson;

From these may be deducted:

An allowance for skin friction in caisson;
For buoyant effect of water in some cases;
For buoyant effect of earth displaced.

The resulting total load on the base of the caisson divided by the area of the caisson gives the soil pressure per square foot, which must be within the safe bearing resistance of the soil.

The allowance for skin friction is difficult to determine unless the results upon similar structures in the same materials are available. Some idea of the probable value of this friction may be obtained from the table given in Section 239. Very conservative values should be used. The measurement of skin friction usually has been derived from the weights required to overcome this friction in sinking caissons.

These are, in general, probably considerable less than the resistance offered after the materials have time to become well settled and compacted about the caisson. This difference has been observed in sinking concrete caissons when one section is sunk and allowed to stand while another is being built and given time to harden. Sometimes it is difficult to start the caisson for the second sinking.

The buoyant effect of water can be exerted only when there is opportunity for the water to get under the caisson, and this effect will vary with the permeability of the soil. For a caisson resting upon rock or impermeable clay, there will be no chance for the water to exert upward pressure on the caisson. In permeable soil, any amount of pressure will be exerted and will vary with the density of the material. The full static head of water will never be exerted upon the whole area of the bottom of the caisson.

The greater resistance of the soil to displacement under load when at a considerable depth below the surface is frequently taken into account by allowing a greater unit pressure than would be allowed for the same material near the surface of the ground. When this is not done, the buoyant effect of the weight of earth displaced may be considered. This would usually be done in the same manner as in computing increased bearing power. (See Section 213.) In some instances it has been the custom to allow an increased bearing value on account of the depth and also to make allowance for the buoyant effect of the displaced earth, although they are actually different statements of the same thing.

In small caissons under buildings, when the caisson rests upon rock, the size required for the caisson may often be fixed by considering it as a column supporting certain loads and transferring them to the rock beneath.

After determining the general dimensions and shape of the caisson, the design of its details and various working parts are controlled by the way in which it is to be handled in placing and sinking.

238. Building and Placing the Caisson.—The method to be followed in constructing and placing a caisson for sinking must depend upon the local conditions under which it is to be used.

Launching from Ways.—For bridge piers in deep water, the most common method is to construct the caisson on ways on shore from which it may be launched. To do this, there must be deep water near the shore into which the caisson may be launched, and the distance that the caisson needs to be transported to the site where it is to be sunk should not be great. When built in this manner the caisson is sometimes given a false bottom to reduce the depth of

water required for floating it. The depth of flotation should be determined in advance and the crib built high enough to be sure of being well above water when the false bottom is removed. When the caisson is in position for sinking, the crib must be built up enough to be out of water when the caisson is on the bottom, then concrete is placed in the crib and the sinking begins.

For the Vancouver bridge over the Columbia River, the caissons [11] were launched from ways on the Vancouver shore after they had been built up to a height of 20 feet. The launching ways had an inclination of about six horizontal to one vertical.

> We tried two methods of launching; one to build the caisson in vertical position using suitable blocking, and another to build it in an inclined position. There was no practical difference between one and the other method, but we finally decided to launch most of them in an inclined position.
>
> The working chamber of the caissons had sides 3 feet thick with 3-inch planking on the inside, the roof being 2 feet 6 inches thick with 3-inch planking on the under side. The entire caisson was planked outside with two layers of 3-inch planking, one diagonal and one vertical. After the caissons were properly caulked and launched, they were towed into place and landed on bottom in the usual manner. In order to reduce the tendency to scour, the cribs above the bed of the river were made with curved ends.

Figure 174 shows a caisson for a pier of the Norfolk Berkley bridge on the ways ready for launching, and Fig. 175 shows the launching of another caisson for a pier of the same bridge.

Building in Place.—With a small depth of water, where no great change in water level is to be expected, a construction platform on piles driven around the site in which the caisson is to be sunk, may be the most convenient method of handling the work. The caisson may then be built on this platform and launched, or it may be constructed in a suspended position between platforms surrounding the site and sunk directly by lowering from the platforms

Sometimes barges are used instead of platforms for this purpose; the caisson being constructed between a pair of barges, which are rigidly connected with each other, floated to the site and sunk. This method could be used where possible changes in the depth of water might make the platform method undesirable. In the construction of the Willamette River bridge of the Northern Pacific Railway caissons 61×21 feet were used. There was no suitable place for launching the caissons from shore and barges were employed for the purpose.[12]

[11] Journal, Association of Engineering Societies, Sept., 1912.

[12] Journal, Association of Engineering Societies, Sept., 1912.

FIG. 174.—Norfolk Berkley Bridge—Caisson Ready for Launching. (Courtesy of The Foundation Company.)

FIG. 175.—Norfolk Berkley Bridge—Launching the Caisson. (Courtesy of The Foundation Company.)

Two barges were used for that purpose, placed far enough apart to allow the caissons to come between them. Suitable bents were built on the barges and the caissons suspended from them (see Fig. 176). After the caissons were built to a height of 20 feet long screws were attached and the caissons lowered into the water. To obviate the unequal motion of the barges, due to the wash from passing boats, two heavy trusses were built tying the barges together, one truss being placed at each end.

Launching from Pontoons or Barges.—Pontoons, or floating docks, are sometimes used for constructing and launching caissons. The caisson is constructed in the pontoon, and then water is let into the pontoon and it is sunk so as to float the caisson, after which the pontoon is removed from beneath the caisson, either by separating

FIG. 176.—Caisson Supported by Barges.
(Journal, Assoc. Eng. Soc., Sept., 1912, p. 48.)

the two ends of the pontoon or by using one end as a gate through which the caisson may be floated out.

The pontoon used in the construction of the large caissons for the Metropolis bridge over the Ohio River is shown in Fig. 177. The largest of these caissons was $60' \, 6'' \times 110' \, 6''$.

An interesting feature of this pontoon [13] was the use of a gate for submerging instead of the usual split design. One end was so constructed that it would float out when the pontoon had obtained a submergence of about 6 feet. Attached water-boxes containing about 75 tons when filled were used to sink the pontoon clear of the caisson, when it was easily floated out.

A caisson for a bridge at Manila, P. I., was built upon a barge arranged to permit of its being tipped so as to allow the caisson to slide off sideways.[14]

[13] Engineering Record, March 22, 1917.

[14] Engineering News-Record, May 17, 1917.

The caisson was 100 feet long, 35 feet wide, and 36 feet high. The walls of the lower portion of the caisson for a height of 14 feet were battered, and constructed of reinforced concrete 1 foot thick, while the remaining height was unbattered and built of double timber sheathing and one thickness of tarred paper. Three feet above the lower edge of the caisson was a 4-inch caulked plank floor supported by inverted timber trusses, which in turn rested on timber sills bolted to the upper edge of the concrete walls. The floor and trusses were designed to withstand water pressure during flotation. The scow upon which the caisson was built was divided longitudinally by a bulkhead, so that water could be admitted to one side and cause the scow to list. Between the scow deck and the caisson were pairs of skids that were well greased just before launching.

Twenty minutes after opening the valves, when the list was about 15°, the caisson slid smoothly into the water, reaching a maximum angle of inclination, due to the momentum of the slide, of about 45°, as was anticipated, just as soon as the caisson began to slide, the scow tipped quickly and was pushed from

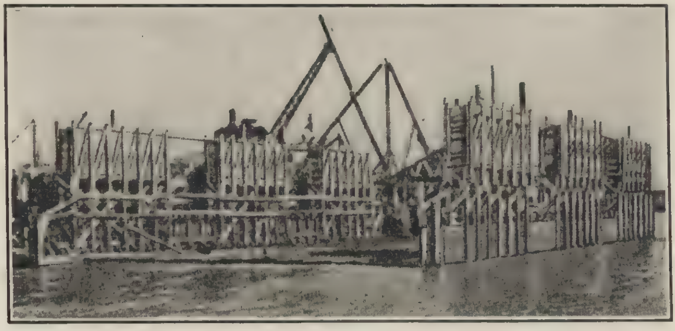


Fig. 177.—Pontoon for Launching a Caisson.
(Eng. News, March 22, 1917, p. 464.)

underneath with considerable force. Due to the low center of gravity, the caisson righted itself with long easy rolls and finally rested on an even keel at an immersed depth of $8\frac{1}{2}$ feet.

The floor and trusses were removed by divers, to be reused for a second caisson.

Caissons for Building Foundations.—For foundations of buildings, the caissons may usually be constructed in open excavation at about the level of ground water. For small caissons in New York City, steel cylinder caissons are usually constructed in a bridge shop and hauled to the site of the work by trucks. They are then placed in position in the excavations and a section of crib added on top. The sinking is usually started without the air pressure by sinking as far as possible in open excavation. Figure 178 shows a small pneumatic caisson for a building.

Timber is commonly used for caissons for rectangular form, which are handled in much the same manner, the caissons being constructed and hauled to the site. They are set in place in an area excavated

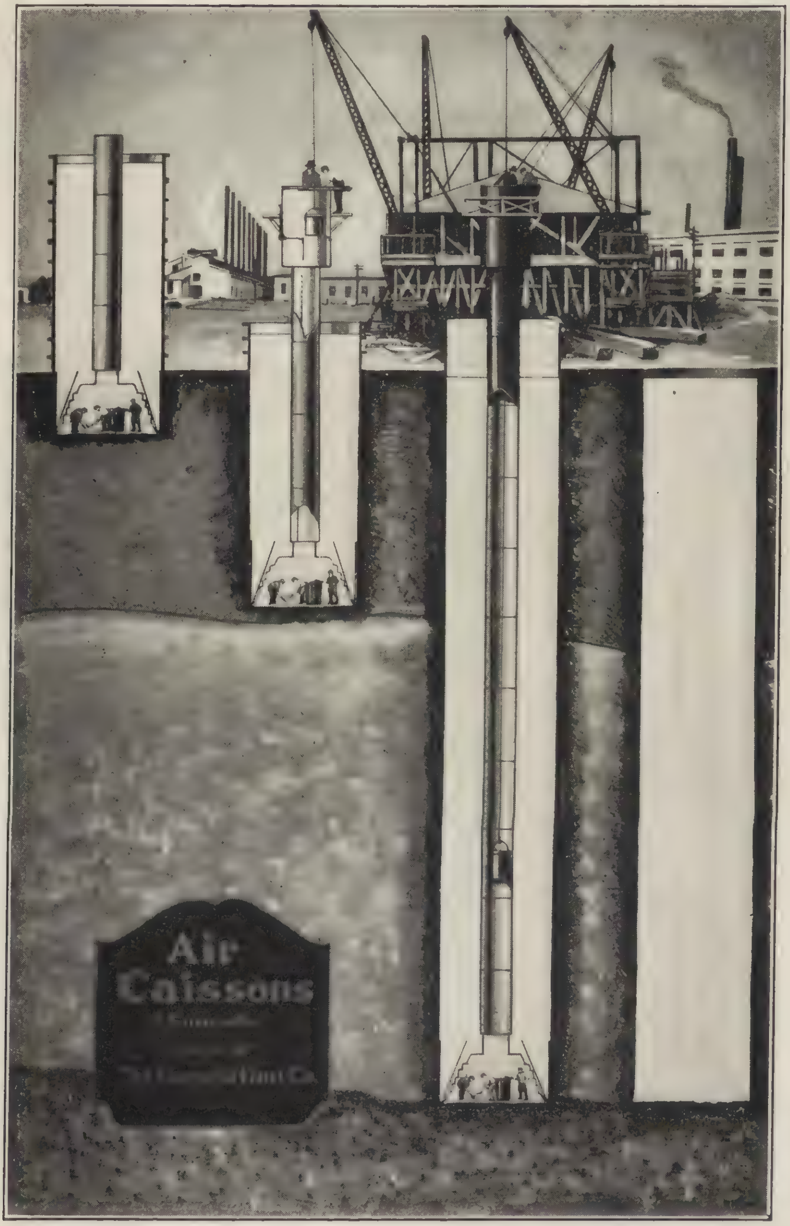


FIG. 178.—Small Pneumatic Caisson for a Building.
(Courtesy of The Foundation Company.)

to the level of ground water, and the sections of the crib added as needed.

Reinforced concrete is being used to a considerable extent for caissons of such size as to allow a sufficient thickness of wall without too greatly narrowing the working chamber. These are usually constructed in place, forms being built in the open excavation and the concrete poured for the whole caisson, or for a first sinking of considerable depth. If the caisson is not more than about 40 feet in height, the entire caisson is usually constructed before sinking begins. When the height is greater, it is usually handled in sections of 30 to 40 feet, each section being sunk to the ground level before the forms are placed for the succeeding section.

Placing Floating Caissons.—When a caisson needs to be floated to place, it is necessary to estimate carefully the weight of the caisson and its water displacement before launching in order to determine the depth of flotation. In many instances, it may be necessary to place a temporary floor under the working chamber to prevent the caisson sinking too deeply in the water. The crib or cofferdam must also be carried up sufficiently to give proper displacement so that there may be no danger of capsizing and the caisson may float in an upright position.

For bringing the floating caisson into position for sinking, clusters of piles are usually driven to which the caisson may be moored, and held in approximate position. Concrete is then placed in the crib until the caisson touches bottom. It is then brought accurately into position by the use of tackle, lifting the caisson a little if necessary by applying air pressure.

Where there is a swift current, particularly in a sedimentary stream, the obstruction caused by the caisson would be apt to induce scour of the bottom with possible undermining of the caisson. In such instances, willow matresses are often sunk covering a considerable area of the bottom about the site before the sinking of the caisson. The caisson is then sunk through the mattress. Sometimes a layer of sand bags is used for this purpose.

Where there is little current, it may be feasible to dredge the bottom, if of soft material, before placing the caisson and thus remove considerable material that would otherwise need to be excavated through the working chamber at greater cost.

When the bottom upon which the caisson must be placed is very uneven, much higher on one side than the other, there may be a tendency to throw the caisson out of position, and if the unevenness cannot be corrected by dredging before sinking, allowance is some-

FIG. 179.—Launching a Caisson from the Pontoon in which it was Built. Municipal Bridge.
(Courtesy of Jacoby and Davis, Foundations of Bridges and Buildings.)

times made for the probable shoving over of the caisson by moving the centers a few inches from the desired alignment.

Figure 179 shows the pontoon used for the caissons of the channel piers of the Municipal bridge at St. Louis, being removed from beneath one of the caissons.

The pontoon [15] from which the two caissons were launched was about 40×100 feet in plan and had sides 8 feet high. The sides and bottom were built of 4-inch plank, braced and caulked. The pontoon was divided into halves lengthwise; the two parts being bolted together from the inside.

After the caisson was built up to a height of about 17 feet above the cutting edge, and the caulking of the air chamber and friction planking was completed, the halves of the pontoon were unbolted, plugs removed from holes in the bottom and water allowed to enter. The pontoon slowly filled and sank, the buoyancy of the timber holding the two halves in position against the cutting edge of the caisson. One-half of the pontoon was anchored to clusters of piles, and a tug pulled the other half from under the caisson, which settled into the water clear of the pontoon. This is shown in Fig. 179. The caisson, now floating free, was built up to the full height, and towed by three tugs to the site, where it was anchored in approximate position to clusters of piles. The pontoon was towed to shore and the halves bolted together in readiness for the next caisson.

The river bed around the upstream end of the caisson and for some distance down each side, was first covered with sacks of sand, to prevent the bottom from scouring out from under the caisson. The sacks were deposited through a section of supply pipe 24 inches in diameter and long enough to reach the bottom. Concreting was now started, and the caisson gradually sank as the filling progressed until the river-bed was reached. Air was then turned on, the caisson raised slightly, moved into proper position by means of the cables, and the air released, allowing it to settle to the river bed. In placing the caisson, care was taken to bring its axes parallel to the true axes, although they might not exactly coincide. As the tendency of the caissons was to creep upstream and toward the east, they were set slightly to the south and west of true position.

In the new Castleton bridge of the N. Y. Central Railroad over the Hudson River, an ingenious plan was adopted to start a caisson in about 15 feet of water. The site was surrounded by a cofferdam of steel sheet piling, which was then filled with sand pumped from the river bed, to an elevation above water level. This formed an artificial island, upon which the caisson was constructed. Sinking was commenced by dredging through open wells in the caisson until an elevation of 20 feet below water level was reached. The air-locks were then put in place and the caisson was sunk under air about 31 feet to rock.

This method saved all trouble of launching the caisson, towing it to the site, and of getting it sunk to bottom in the correct location.

239. Sinking the Caisson.—After the caisson has been grounded in position, if in a stream, or when it has been carried as far as may

[15] Engineering News, March 16, 1911.

be feasible by open excavation, if on the land, the sinking may begin. Sinking is accomplished by excavating the material under the caisson and applying weight. The pockets in the crib are first filled sufficiently to hold the caisson to the bottom; air pressure is then applied until the water is driven out of the working chamber, after which men may enter and proceed with the excavation. During the sinking, filling is added in the pockets as may be necessary to keep the forces acting on the caisson balanced. The caisson is sustained

Fig. 180.—Sinking Caisson, Guaranty Trust Co. Building.
(Courtesy of The Foundation Co.)

by the air pressure in the working chamber, the resistance under the cutting edge and the friction upon the sides of the caisson. The weight of the caisson and filling must balance these, increasing with the depth.

In sinking small caissons for buildings and for cylinder piers

additional weights must usually be used, as the weight of the caisson and filling can not be made sufficient to overcome the frictional resistance. In sinking caissons for buildings in New York, large cast-iron weights are employed, weighing 3000 to 5000 pounds each. For the foundations of the Guaranty Trust Company building (see Fig. 180), over 2000 tons of such weights were used, being transferred from one caisson to another as needed. Usually six or eight caissons were being sunk at one time.

Where the caissons are of sufficient size to make the use of reinforced concrete caissons feasible, the necessary weighting is considerably reduced, on account of the greater weight of the caisson and crib.

Water jets are frequently used on the outside of caissons to reduce the frictional resistance in sinking. Sometimes also the sides of the caissons are greased for the same purpose.

Frictional Resistance upon the sides of caissons is very difficult to estimate. It depends upon many variable factors and there are few reliable data upon which to base an estimate; the results of recorded observations differing so widely for what seem to be similar conditions. Table LXX is taken from a discussion by Mr. H. L. Wiley in Transactions, American Society of Civil Engineers, Vol. 62, p. 133.

The skin friction on the lower section of the caisson increases directly as the depth sunk, but, unless the material is very unstable or practically in a liquid state, the friction at any given depth on successive sections of the caisson is not as great as that exerted on the cutting edge and lower section of the caisson while at that point; or, in other words, the passage of the lower part of the caisson smooths, lubricates, or in some other manner tends to decrease materially the friction on the following sections.

A wall section of 5 or 6 feet gives weight enough to overcome any friction that may develop ordinarily, unless the material penetrated be exceptionally difficult. Table LXX illustrates to some extent the wide variation in the amount of friction in such work.

The rate at which caissons may be sunk varies greatly, the rapidity of sinking the small caissons for building foundations being of course much greater than the large ones for bridge piers or abutments. In the New York Municipal building, caissons sunk about 100 feet to rock required an average of about one month; while one pier sunk about 40 feet was put down in about 24 hours after sinking began. In the New York Pier of the Manhattan bridge, the most rapid sinking was 7 feet in a week of six working days of 24 hours each, with a working force of twenty-five men.

The average rate of sinking for the caissons of the Municipal

bridge at St. Louis was about 1.26 feet per day. For the McKinley bridge at St. Louis, 2 feet per day, and for the Memphis bridge, 1.5 feet in sand and .31 foot in clay, per day of 24 hours. Probably the general average for large caissons would not be more than $\frac{3}{4}$ foot per day.

The excavation is usually carried a little ahead of the cutting

TABLE LXX.—VALUES OF SKIN FRICTION ON CAISSONS

Type of Caisson.	Method of Sinking.	Materials Penetrated.	Skin Friction, Lbs. per Sq. Ft.	Depth Below Low Water, Feet.	Area of Base, Sq. Ft.
Cast iron...........	Open excavation	Gravel, clay	240	60	125
Cast iron...........	Open excavation	Sand, clay	250	75	225
Cast iron...........	Open excavation	Sand	250	60	125
Wrought iron.......	Open excavation	Sand, clay	285	140	1000
Cast iron...........	Open excavation	Sand, clay, gravel	300	100	125
Cast iron...........	Open excavation	Sand	325	60	125
Cast iron...........	Open excavation	Silt	350	60	125
Steel construction...	Open excavation	Silt, sand, clay	375	55	190
Cast iron...........	Open excavation	Silt, mud, clay	390	75	100
Timber construction.	Open excavation	Sand	450	30	1300
Steel construction...	Open excavation	Silt, clay	450	60	700
Steel construction...	Open excavation	Silt, clay, sand	450	60	1200
Steel construction...	Open excavation	Mud, sand	450	65	1300
Steel construction...	Open excavation	Clay	450	75	1500
Iron construction....	Open excavation	Sand, gravel, clay	480	65	200
Cast iron...........	Open excavation	Clay	500	60	125
Steel construction...	Open excavation	Clay	700	65	1300
Masonry...........	Pneumatic	Sand, mud	205	40	75
Timber.............	Pneumatic.....	Clay	250	35	800
Steel construction...	Pneumatic	Clay, sand	275	60	150
Timber construction.	Pneumatic	Silt, sand, mud	310	75	2550
Timber construction.	Pneumatic	Sand, clay, gravel	350	100	1200
Timber construction.	Pneumatic	Sand, clay, boulders	400	48	1925
Timber construction.	Pneumatic	Clay, sand, gravel	400	95	4500
Timber construction.	Pneumatic	Sand, gravel, clay	425	55	1300
Steel construction...	Pneumatic	Sand, boulders	450	68	2700
Timber.............	Pneumatic	Silt, clay, gravel	500	75	1800
Iron cylinder.......	Pneumatic	Sand, shale	525	60	1200
Timber construction.	Pneumatic	Sand	540	75	1700
Timber.............	Pneumatic	Sand, clay	600	75	1400
Timber construction.	Pneumatic	Sand, gravel, clay	650	80	2000
Timber construction.	Pneumatic	Sand	650	90	1200
Timber construction.	Pneumatic	Sand, boulders	650	101	2100
Timber construction.	Pneumatic	Silt, sand, clay	900	54	1700

edge and somewhat deeper at the center than at the sides. When excavation is completed out to the cutting edges, a slight lowering of the air pressure causes the caisson to sink. Adding weight to the caisson at the same time may help to regulate the sinking. Reducing the air pressure by one pound per square inch has the same effect as adding a weight of 144 pounds per square foot over the area of the working chamber.

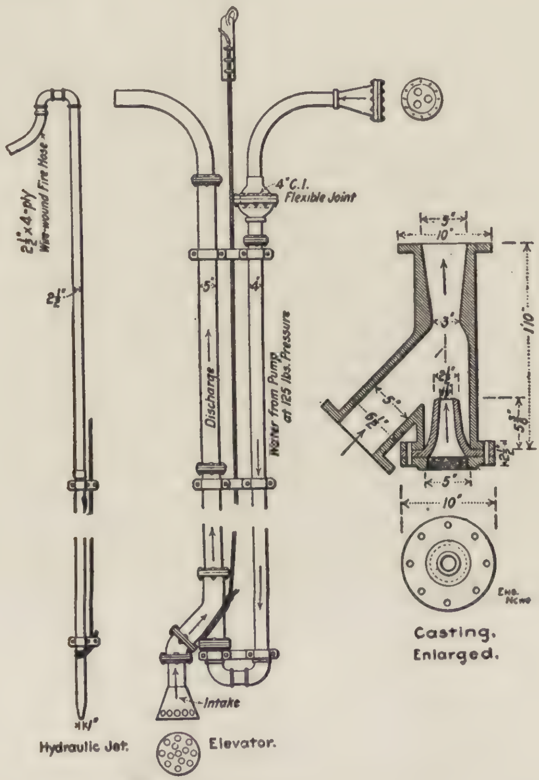


Fig. 181.—Details of Hydraulic Ejector.
(Courtesy of Jacoby and Davis, Foundations of Bridges and Buildings.)

The caisson is guided during the sinking by regulating the excavation; taking the material from under the cutting edge on the high side and leaving greater resistance on the low side when making a sinking. The weight of spoil on the outside may also be used to force the caisson into position, by depositing it on one side of the caisson. With all the precautions that may be taken, the caisson is likely to be somewhat out of position and perhaps not exactly plumb

when finally settled in place. It is therefore necessary to make allowance in the design of the caisson, and make it large enough to be certain that the base of the pier may be placed upon it.

Removing the Spoil.—The method of removing excavated material from the working chamber depends upon the character of the materials to be handled and the depth of sinking. Hard materials such as boulders and clay are removed in buckets through small shafts with special air-locks at the top. The buckets are usually operated by hoisting engines outside the shaft, but sometimes compressed air cylinders in the shaft are used for the purpose. Boulders of large size are broken up by blasting and loaded into the buckets, but if only a few are met, they may sometimes be carried down by excavating around them and be used in the foundation by bedding them in the concrete in sealing the bottom of the caisson.

The Mud and Sand Pump, or hydraulic ejector, is frequently used for removing sand and silt and sometimes wet clay. Figure 181 shows an ejector as used in constructing the Fraser River bridge at New Westminster, B. C. The water enters at high pressure at the bottom, producing a partial vacuum in the pump chamber and drawing the wet material from a sump in which the end of the suction is placed.

This device [16] could handle material of any size up to $2\frac{1}{2}$ inches, the diameter of the throat, and was found to be quite as effective at great depths as at smaller depths. The jet shown in the drawing, in connection with the ejector, was used as an agitator; when placed near the suction it greatly increased the efficiency of the work.

The Blowout Process is often used in removings and and silt. This consists in using the compressed air to drive the material through a 4-inch pipe leading from a sump in the working chamber into the open air. A small pit is formed in the bottom of the working chamber and the end of the hose leading to the blowout pipe is placed in this sump. The sand and silt are then shoveled about the end of this hose and the air pressure blows the materials out through the blowout pipe. Care must be used in regulating the amount of material placed about the end of the pipe, in order that a proper amount of air is admitted to the pipe to prevent clogging. When not in use the air is shut off by a valve in the working chamber. When the depth is not too great (below about 75 feet), this has been found a very satisfactory method of removing spoil. The difficulty of maintaining uniform pressure in the working chamber is its greatest disadvantage. The air compressors must be capable of supplying

[16] Engineering News, June 15, 1905.

air at such a rate as to maintain uniform pressure when the blowout is working. The rapid wear upon the elbow in the pipe, where it turns to the horizontal before discharging, has caused considerable difficulty. It is necessary to have this elbow made of metal which is specially resistant to abrasion and to renew it frequently.

In the caissons for the Brooklyn bridge, a vertical shaft extended from the top into a sump below the bottom of the working chamber. This sump was kept full of water and the shaft was open at the top and filled with water sufficiently to balance the air pressure in the working chamber. The excavated material was thrown into the pit under the end of the shaft and was removed by dredging with a clamshell bucket through the shaft.

Concreting the Working Chamber.—When the cutting edge has reached the stratum on which it is to rest, the caisson is sealed by cleaning off all loose material and depositing concrete over the bottom before removing the air pressure. If the cutting edge rests upon rock which is not level and uniform, concrete is filled under the cutting edges to support them uniformly. Sometimes concrete in bags is used for this purpose. When the surface of the rock is sloping, it may be necessary to cut it into steps to avoid any danger of slipping. In settling the caissons for the Municipal bridge at St. Louis, the rock was sloping and precautions were taken to insure uniform support.

As the rock was approached,[17] frequent soundings were made in the air chamber. When but 2 or 3 feet of sinking remained, the slope of the rock was accurately determined by soundings taken at close intervals all around the caisson, as close to the cutting edge as the slope of the roof would allow. The caisson was then tilted very slightly and allowed to settle slowly until the low cutting edge was landed on the side where the rock was highest. By doing this, any further slight settlement in coming to a bearing tended to level the caisson.

No attempt was made to level off the rock so as to bring the caisson to a bearing throughout; but where depressions occurred, sacks of concrete were deposited on the rock and tamped under the cutting edge. A wall was thus formed inside of which concrete was deposited in the usual manner.

In sealing, double gangs were used, working during the day only. Concrete was locked in through each material shaft, the lock being located at the extreme top of the shaft and being equipped with a hopper into which the concrete was dumped. The upper door of the lock was then raised, the lock equalized and the lower door dropped allowing the charge of concrete to fall down the shaft to the air chamber. There it was shoveled back and tamped under the shoulders and roof of the chamber.

Rather dry concrete is usually packed around the edges of the caisson and under the cross braces or bulkheads. This must be

[17] Engineering News, March 16, 1911.

carefully placed and well tamped in order to assure uniform support to the caisson. The concrete filling for the working chamber is then poured until the chamber is nearly full, leaving a space of a foot or 18 inches at the top to be filled after the main filling has set.

When this concrete has set, so that the shrinkage due to setting has taken place, and the chamber is sealed against the entrance of water the air pressure may be released and the rest of the concrete filling placed in the open. It is advisable to fill the upper part of the working chamber with concrete dry enough to pack well, which may be rammed into place to insure the complete filling of the chamber and the support of the roof, without danger that the concrete may contract in setting and leave a small crack under the roof of the chamber.

It is the practice of some engineers to make the final filling at the top of the working chamber with very wet concrete or grout which when poured into the material shaft will be under considerable pressure in the working chamber and forced up into the other shafts. The working chamber may be thoroughly filled in this manner provided arrangements are made for the escape of the air. The packing of dry concrete is rather a difficult and expensive matter, and the other method is considerably cheaper. The shrinkage of wet concrete in setting is, however, greater than that of dry concrete and if the ramming of the dry concrete is thoroughly done there is a certainty that the space is well filled.

240. Physiological Effects of Compressed Air.—The depth below water surface to which the pneumatic method may be employed is dependent upon the ability of men to work in compressed air. Experience has shown that under careful management, men in good physical condition may safely be subjected to an air pressure of about 45 or 50 pounds above atmospheric pressure, and work has been successfully carried out in several instances at maximum depths of 110 to 115 feet below water surface. Very careful attention to the physical condition of the men and to the methods used in entering and leaving the compressed air are necessary to prevent injurious results.

When the men enter the air-locks and the pressure is gradually increased, a sensation of giddiness, with pain in the ears, breathlessness, and inability to move about or to speak, is felt. The maintenance of equilibrium between the air pressure inside and outside the body is difficult. The pressure upon the ear drums is particularly unpleasant. This may be partially overcome by holding the nose and blowing, thus increasing the pressure in the tympanic cavity and equalizing the external and internal pressure

upon the ear. The action of swallowing may also tend to relieve the discomfort.

When equilibrium between the air pressures outside and inside the body has been reached, a feeling of exhilaration results while breathing the more dense air. The workmen appear to have more energy than when working in the open. They perspire freely in the high temperature under the air pressure. Labor in the compressed air is more exhausting than in outside air and is carried on in shorter shifts. It is important that the caisson be well ventilated and the air changed frequently to prevent general loss of vitality in the men, thus making them more likely to contract diseases other than those directly due to the compressed air.

On leaving the caisson, as the pressure is reduced, a feeling of intense cold is experienced. This is due to the expansion of the air in the lock and also to the liberation of gases from the body. The lowering of pressures and temperature of the air in the lock produces a disagreeable fog in the atmosphere of the lock. The men working in the high temperature of the compressed air are thinly clad, and should have warm clothing to put on in passing through the air-lock when being decompressed. There is also usually a sensation of itching under the skin, which disappears as soon as the pressures are fully equalized and the escape of gases from the body ceases. The intensity of these sensations depends largely upon the rate of decompression, which should be very slow to avoid ill effects upon the men. The size of the air-lock is also important, as sometimes, the men may be required to remain in a cramped position, without room to move about during a considerable time required in decompressing.

The sensations which have been described are those normally experienced by men entering, working in, and leaving the compressed air. Men commonly experience them, under right conditions without any serious effects upon health. It is, however, necessary that care be used in the selection of men for such work, and that the procedure in conducting the work be strictly regulated to prevent injury to the men.

Caisson Disease is a malady sometimes following work in compressed air. It is apt to develop soon after decompression; usually a few minutes after leaving the air-lock, seldom more than two or three hours later.

The symptoms of caisson disease [18] have been quite definitely established. First among these are neuralgic pains of an intermittent or paroxysmal charac-

[18] Engineering News, Vol. XLVI, page 157, Sept. 5, 1901.

ter, and of varying severity. In the worst instances these pains, or cramps, as they are commonly called—although they are rarely accompanied by muscular spasms—are so intense as to completely unnerve strong men. This symptom is very seldom absent, and from it comes the popular name of "bends" given to the disease. Another characteristic symptom which is always exhibited is a profuse cold perspiration. Another symptom which is of frequent occurrence, but which is not always exhibited, is pain at the pit of the stomach, usually, but not always, attended by vomiting. In about 50 per cent of the cases observed, paralysis has been a characteristic symptom. The degree of paralysis varies from slightly impaired sensation or numbness in the extremities to complete loss of sensation and motion in the affected parts, which are most frequently the legs and the lower part of the body. Finally the sufferer usually exhibits a number of transient symptoms, which have their origin in the brain; these are headache, dizziness, double vision, incoherence of speech, and sometimes unconsciousness. The duration of these symptoms varies from a few hours to several weeks in case of paralysis. In fatal cases congestion of the brain or spinal cord always exists. A very noticeable fact is that the attack of the disease never takes place while the subject is under air pressure, but always occurs while he is emerging from the compressed air chamber or after he has emerged.

Medical authorities seem now to be fairly well agreed that the caisson disease is caused by bubbles of gas (mostly nitrogen) in the blood and tissues. The gas, which has been absorbed by the blood under high pressure of the compressed air, is set free during the decompression and expands, producing bubbles in the blood vessels. If the lowering of the pressure is very slow, the gases may be thrown out of the blood at the lungs without developing large bubbles. With rapid lowering of the pressure, however, the bubbles expand in the blood vessels, obstructing the circulation and sometimes causing the vessels to burst.

The methods recommended for preventing caisson disease are:

1. Careful selection of the workmen to insure that only men in good physical condition are subjected to the compressed air;
2. Short periods of work in the compressed air;
3. Very slow decompression in leaving the compressed air;
4. Good ventilation in the working chamber and air-locks with frequent changes of air.

Selection of Workmen.—Competent medical supervision should always be provided for any project in which men must work under any considerable air pressure. Men with strong hearts, good circulation, and normal blood pressure should be employed. Fat men are more liable to the disease than others, as the fatty tissues absorb more of the gas than the blood and release it more slowly. Men addicted to alcoholic liquors should not be employed.

The broad proposition [19] that only men whose mental and bodily functions are in healthful operation should be allowed to work under air pressure is well established. In compressed air work, as in all other cases where abnormal environment or excessive exertion task physical endurance beyond the ordinary, the man who is physically fit is the one best able to endure the strain without injury. For the same reason, a carefully observed regimen of healthful diet, dress and sleep are important aids in preserving health during employment in the pneumatic caisson. A further precaution which experience has amply proved to be necessary is careful regard to the manner in which changes of pressure upon entering and emerging from high pressures are undergone.

The New York law [20] requires that medical officers be employed, and that an examination be made before admitting men to the compressed air. If absent from the work 10 days, they must be reexamined upon their return to work. No one addicted to intoxicants may be employed. Green men must work only one shift at first, and be reexamined before again entering the air. All men are to be reexamined after three months.

The Time of Compression in entering the compressed air is not considered of special importance and should be regulated so as to cause as little discomfort to the men as possible. The French Government regulations require at least 4 minutes to raise the pressure from 14 to 28 pounds above atmospheric pressure, and at least 5 minutes for each additional 14 pounds.

The Time of Working in caissons and the length of shifts varies with the pressure, becoming quite short for high pressures. The French law [21] fixes the following limits:

Below 28 lb.	pressure	.............8 hours;
28 to 35 lb.	"	.............7 hours;
35 to 42 lb.	"	.............6 hours;
42 to 49 lb.	"	.............5 hours;
49 to 56 lb.	"	.............4 hours.

The New York law is as follows:

Gage Pressure.	0–21	22–30	31–35	36–40	41–45	45–50
Time per day in caisson.......	8 hrs.	6 hrs.	4 hrs.	3 hrs.	2 hrs.	$1\frac{1}{2}$ hrs.
Number of shifts.	2 (minimum)	2	2	2 (min.)	2 (min.)	2
Length of shift..	....	3 hrs.	2 hrs.	$1\frac{1}{2}$ hrs. (max.)	1 hr. (max.)	$\frac{3}{4}$ hr.
Minimum time between shifts.	30 consecutive minutes	1 hr.	2 hrs.	3 hrs.	4 hrs.	5 hrs.

[19] Engineering News, Sept. 5, 1901.
[20] Engineering News, Aug. 14, 1913.
[21] Engineering News, Sept. 18, 1913.

This law has been criticised for dividing the work period into short shifts, thus requiring two decompressions daily, instead of using a single period and avoiding unnecessary decompressions. There seems to be considerable difference of opinion upon this point.

The Rate of Decompression has much to do with the development or prevention of caisson disease. Authorities differ as to the time that should be allowed, but nearly all recommend a longer period than is usually given. Some advocate a uniform rate of decompression, while others would divide the time required into stages with different rates of decompression; rapid reduction of pressure at first, followed by very slow final decompression.

It is claimed by those who advocate stage decompression that the gas in the blood does effervesce until a considerable reduction of pressure has been made and that a rapid reduction to this point is desirable. Dr. Haldane's experiments led to the belief that bubbles of gas are not released until there is a difference of pressure of more than 19 pounds between the pressure in the blood and the external pressure. He therefore concluded that the pressure might be reduced by that amount quickly in about 3 minutes. From this point the pressure should be reduced very slowly; from 2 to 9 minutes per pound of pressure. This is criticised by others as needlessly slow. Mr. Japp from his experience in the East River tunnel work [22] recommends reducing the pressure in the first stage, of 3 to 9 minutes, by 27 pounds and then allow 1½ to 2 minutes per pound for final reduction

The French Government rules require that the time of decompression shall not be less than:

20 minutes for each 14 lb., down to 42 lb.;
15 minutes for each 14 lb., between 42 and 28 lb.;
10 minutes for each 14 lb., below 28 lb.

The requirements of the New York law are as follows:

Gage pressures in lb., per sq. in.	10	15	20	25	30	36	40	50
Minutes in decompression	1	2	5	10	12	15	20	25

No pressure allowed above 50 pounds.

The pressures in the air-lock must be absolutely under the control of reliable persons, who will reduce the pressures at the specified rates. Workmen may sometimes rebel at the time required in passing through the locks, and may become reckless in obeying the rules prescribed by the medical authorities.

[22] Transactions, Am. Soc. C. E., Vol. 65, p. 17.

The passage through the lock [23] during the decompression is often uncomfortable and this explains the haste with which the men are anxious to lock out. The proper ventilation of the lock will improve the quality of the air as well as prevent condensation to some extent.

This condensation and the resulting cold are chiefly objected to by the workmen, and this seems plausible enough since instances have occurred where the temperature dropped from 36° C. to 5° C. This condensation, however, is due to a rapid decompression; when this decompression is effected uniformly and at a speed of one-tenth atmosphere per minute, there is no appreciable condensation and hardly any drop in temperature, the lowering of temperature being opposed by the radiation of 300 to 400 calories given off by the men, figured at an average of 75 calories per man per hour.

Ventilation.—Means of ventilating the working chamber should be provided, although usually the continual pumping required to keep up the pressure and the escape of air from the chamber will be sufficient to give proper ventilation. When concreting the working chamber, however, this escape would be cut off, and when considerable periods are required in the air-locks, ventilation is very important. The French law requires the height of the working chamber to be such that men can stand upright in it; that electric lights be used throughout; that the quantity of air be not less than 1400 cubic feet per man per hour; and that the CO_2 do not exceed 1 part in 1000.

It is well known that in a confined atmosphere, man sooner or later suffers from the accumulation of poisonous gases. The criterion of this pollution of the atmosphere is the amount of carbonic acid (CO_2) found present. When the percentage of CO_2 in the air rises above 0.1 per cent, evil effects are common. It should be clearly understood that these evil effects are not due to the carbonic acid itself, but to some other toxic property which the CO_2 content seems to run parallel with, and is, therefore, a measure of it. Now under pressure it is evident that such a gas will be still more dangerous. As a matter of fact, E. H. Snell reports that an "increase of CO_2 from 0.04 per cent to 0.1 per cent at 30 pounds pressure is the forerunner of much illness." He found that by free ventilation of the caisson, so as to remove this CO_2, the illness dropped from seven cases a day to one case in two days.[24]

Sometimes gases are met with in passing through material containing foul matter, or poisonous gases may be liberated from some strata, which may collect in the shaft and become dangerous unless thorough ventilation is provided for the shafts and air-locks as well as for the working chamber. In one instance men entering the shaft from outside were killed, while others in the working chamber were unharmed by poisonous gas.

[23] Engineering News, Sept. 18, 1913.

[24] Jour. Assoc. Eng. Soc., Nov., 1907, p. 301.

Treatment of Caisson Disease is nearly always that of returning the patient quickly to the compressed air and very slow decompression. For this purpose, medical locks are commonly provided, containing cots in which victims of the bends may be placed for treatment. The New York law requires that such locks shall be provided for all work in which a compression of more than 17 pounds is necessary. If this treatment is given before the rupture of the blood vessels has taken place, it is nearly always effective in relieving

FIG. 182.—Sinking Open Wells for Foundation Piers of Kinney Building, Newark, N. J.
(Courtesy of The Foundation Company.)

the patient, and no serious results follow. Since the introduction of these medical locks, the number of fatal cases has been greatly reduced.

NOTE.—In May, 1926, it is reported that the United States Government is experimenting successfully with a mixture of oxygen and helium under pressure for the treatment of patients suffering from "bends." This mixture seems to go in and out of solution in the blood more quickly than plain air under pressure.

ART. 61. OPEN WELL FOUNDATIONS

241. Wells with Sheet Piling.—Foundations for buildings where rock or other hard material is available at moderate depths, are often constructed in open excavations sheeted with piling.

This is much the same as the cofferdam process for piers, but these foundations are usually cylindrical wells of small diameter, the whole well being filled with concrete after the excavation is completed, and forming a solid pier for the support of the columns of the building.

Figure 182 shows the foundations of the Kinney building at Newark, N. J.

The foundation piers [25] were started in rectangular pits sheeted with 2-inch plank, and dug to the level of the tops of the concrete piers. The sheeting for the open shafts are of interlocking steel piles, assembled in these pits against wooden frames. They were clamped together by outside wire cables holding them close against the inside ranger frames spaced 5 feet apart at top to $2\frac{1}{2}$ feet at the bottom.

Excavation was made with pick and shovel, and lifted out in buckets by derricks. The water was kept down by steam syphons working in stages with a lift of 20 feet.

In the Railway Exchange Building at St. Louis,[26] steel sheet piling was driven 24 to 34 feet to rock in pits dug 8 feet to the water line and supported by wooden cofferdams, after which the interior was excavated by pumping. The well was then filled with concrete, the sheet piling being left as part of the foundation. (See Fig. 183.)

242. Open Wells with Sheeting.—This method, often called the Chicago method, consists in excavating the shaft in sections, a few feet at a time and sheeting each section before excavating the next. When the soil conditions are favorable, it is a very economical method of procedure. In Chicago, where this method has been used for many heavy buildings, the soil for a few feet below the surface is loam and made ground. Underneath this top layer, a bed of clay, 70 to 80 feet deep, extends to rock or to a layer of very hard clay. The upper part of the clay is quite hard while lower down it is wet and much softer.

Concrete wells as now built [27] are put down 4 or 5 feet and then lined with wooden lagging 2 to 3 inches thick, tongued and grooved, and either 4 feet or 5 feet 4 inches long. Each section of this lagging is held in place by two steel bars, generally 4 inches wide by $\frac{3}{4}$ inch thick made semicircular, in two pieces turned up at the ends, so that they can be bolted together.

When the first section is lined, a new section is dug and lined up, and so on down to the hard clay, 60 to 70 feet or to the rock, as the case may be.

When the wells are only carried down to the hard clay, they are belled out at the bottom to about twice the diameter of the shaft.

[25] Engineering Record, Oct. 19, 1912.
[26] Engineering Record, Sept. 7, 1912.
[27] Engineering Record, July 29, 1905.

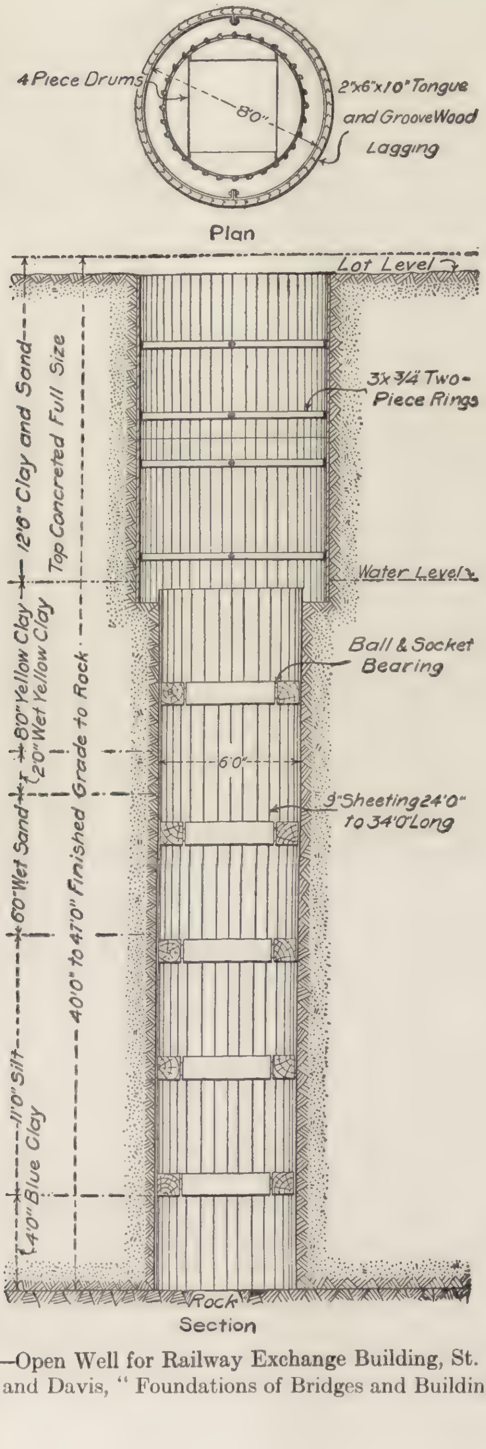


FIG. 183.—Open Well for Railway Exchange Building, St. Louis. (Jacoby and Davis, " Foundations of Bridges and Buildings.")

When the holes are excavated to the bottom, the concreting is begun and the rings are taken out as the work of filling progresses.

243. Freezing and Grouting Processes.—*The Poetsch freezing process* is a method of excavating wet soil by freezing the water in the surrounding soil, thus forming a solid dam around the site to be excavated. The method has been applied to foundations in very few instances and under special circumstances, although it is reported to have been used for some deep mine shafts in Germany. It is too slow and expensive, requiring a refrigerating plant, for use in ordinary foundation work.

In the process as invented by Dr. Poetsch, water-tight tubes,

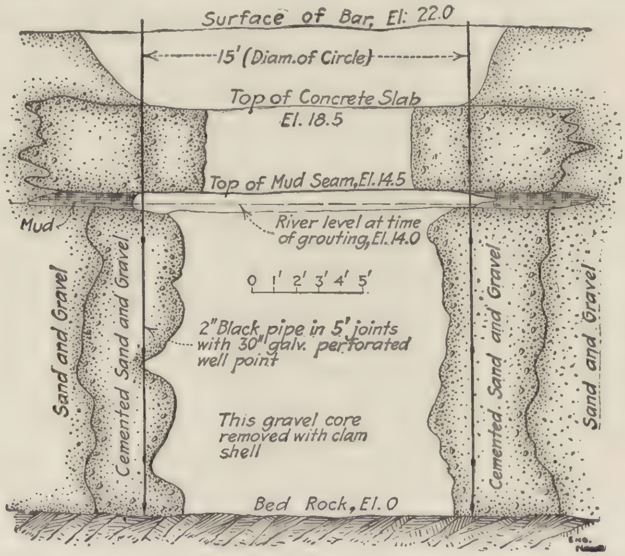


FIG. 184.—Typical Cross-section of Concrete Cylinder Formed by the Circle of Grouping Pipes.
(Jacoby and Davis, "Foundations of Bridges and Buildings.")

4 to 6 inches in diameter and closed at the lower end are driven about 3 feet apart around the area to be excavated. Small pipes, 1 to $1\frac{1}{2}$ inches in diameter, open at the bottom, are placed in these tubes. These pipes are then connected up with the pumps, and the cold brine from the refrigerating plant is driven through the small pipes and returned through the larger tubes to the plant. This slowly freezes the ground and prevents the surrounding material from entering the excavation.

The Grouting Process consists in solidifying the soil by driving cement grout through it. It may be used either to solidify a bed

of gravel into concrete upon which to place a foundation, or to form a dam about the site to be excavated as with the freezing process. The method consists in driving pipes into the ground to be cemented, and forcing the grout down under pressure. The difficulty in distributing the grout uniformly through the soil, and the lack of uniformity in the material to be grouted, makes the use of this method for securing satisfactory bases upon which to place structures rather uncertain. The method could only be used in that way where the soil was a gravel from which good concrete might be made. Figure 184 shows the results of an experiment made by the Louisville and Nashville Railroad to test the efficiency of the method.[28]

On a gravel bar, where the rock was 23 feet below the surface and water level 8 feet below, 2-inch pipes were driven to the rock on the circumference of a circle 15 feet in diameter, spaced every 3 feet. The pipes were gradually drawn up as grout was driven in, sufficient grout being furnished at each level to fill all the material within a radius of about 2 feet. After allowing time for the cement to set, the center was excavated, and the work examined. The bottom below the water level was rather poor concrete; the silt had been carried up by the grout and deposited at the level of water surface in a layer of mud containing no grout; the upper part above the water level was good concrete.

For description of the "Sinking Pneumatic and Open Caisson Foundations for Philadelphia-Camden Bridge" see Engineering News-Record, June 4, 1925, p. 920.

[28] Engineering News, May 8, 1913.

CHAPTER XV

BRIDGE PIERS AND ABUTMENTS

ART. 62. BRIDGE PIERS AND ABUTMENTS

244. Locations and Dimensions for Piers.—In fixing the locations for piers of a bridge, it may be necessary to take into consideration several factors. In a navigable stream, the piers must be arranged so as to obstruct the channel as little as possible and meet the regulations imposed by the Government. This may sometimes determine positions, length of span, and height of structure. The waterway requirements and possibility of the piers restricting the waterway to a serious extent must always be considered. The character of the foundation along the line of the bridge, and probable difficulty of placing foundations at various locations may sometimes influence the choice of positions for piers.

Financial considerations are always important. The total cost of the structure including piers and superstructure should be the minimum consistent with properly meeting the other requirements. The cost of superstructure increases approximately as the square of the length of span, while the cost of piers may be nearly proportional to their number. An arrangement may therefore be worked out in each instance which will give a minimum of cost for the entire structure.

However, it will usually be found that the Government requirements for purposes of navigation and adequate waterway will take precedence over economic considerations in determining the spacing of piers. In the preliminary plans for the proposed Hudson River bridge at 57th Street, New York, the requirement that the main channel between pier head lines should not be obstructed by piers necessitated a design with a main span of 3240 feet.

Aesthetic considerations may also have an influence on pier location; the appearance of the structure is always an important matter and may sometimes control the design. The arrangement of spans to secure symmetry in the whole structure, with proper placing of dominating features ought to be carefully considered.

In settled sections, bridges are built to conform to established

lines of communication, and their locations are thus very closely determined. In undeveloped country, however, some latitude is

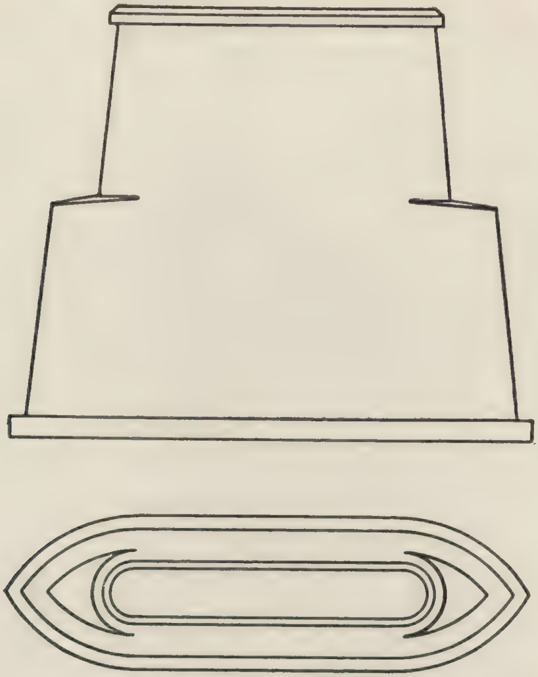


FIG. 185.—Typical Bridge Pier.

permitted. In such cases a crossing of moderate width should be chosen. If too wide, the structure will be unduly costly on account of length; if too narrow, a swift current may be encountered, that will add greatly to the difficulty and cost of the foundations.

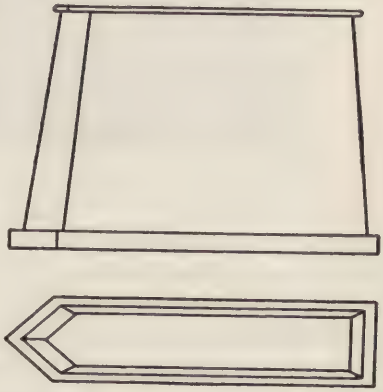


FIG. 186.—Pier with Pointed Starling.

The shape to be given to a pier is determined by the requirements of each particular case. It must be designed safely to transmit to the foundation, the loads brought upon it, and to resist any lateral pressure due to wind or current, and the form to be given a horizontal section should offer as little

resistance as possible to the flow of the stream in which it may be placed.

The most common form for piers in streams is that of a rectangle of length a little more than the width of the bridge, with triangular or curved ends. The pointed ends below high water are known as

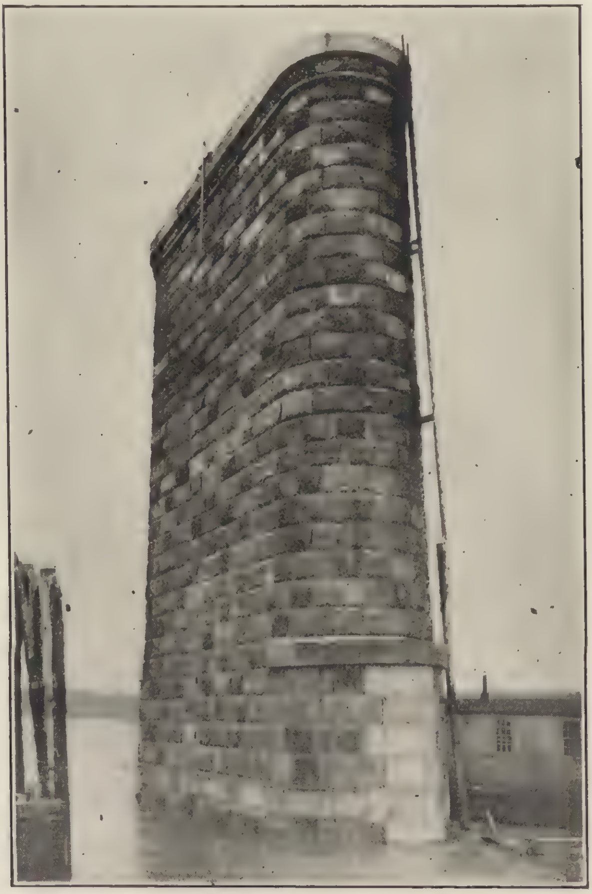


FIG. 187.—Pier No. 2 of Cantilever Bridge over Mississippi River, Thebes, Illinois. (Courtesy of Mr. Ralph Modjeski, Consulting Engineer.)

starlings, and are intended to reduce the disturbance to the stream flow and sometimes to act as ice breakers. Sometimes starlings are used only on the upstream end of the pier, but more commonly the horizontal section is made symmetrical. The downstream starling serves to prevent eddies below the pier, and to equalize the load over the foundation area. Starlings are necessary only below high water,

and the upper part of the pier is sometimes made rectangular, but more commonly the ends are semicircular (see Fig. 185) or the shape of the starling is continued to the top.

Figure 186 shows a simple form of concrete pier in which a triangular starling is used upon the upstream end only and is continued to the top of the pier. In Fig. 185 the horizontal section of the star-

FIG. 188.—Pier No. 3 of McKinley Bridge over Mississippi River, St. Louis, Missouri.
(Courtesy of Mr. Ralph Modjeski, Consulting Engineer.)

ling is composed of two intersecting circular arcs of radius equal to the width of the pier, while the upper part of the pier has semicircular ends.

George S. Morison, the well-known bridge engineer, in describing the attempt to design an elaborate pier for a large bridge, said that

the final and best design was the simplest, all non-essentials having been gradually dropped as the plan progressed. The pier as built was, from high water down to the river bottom, best adapted to pass the water with the least disturbance to the flow; it had parallel sides and the ends were formed of two circular arcs meeting in a point. Above high water the ends were semicircular. The batter was uniform and was one in twenty-four. A coping 2 feet wider than the body of the pier projected far enough to shed water, and this projection was divided between the coping and the belting course below it. Another coping with a smaller projection surmounted the pointed ends of the starlings. This design was so satisfactory and presented such a pleasing appearance that it was later used for 43 piers of eleven different bridges. Figure 187 exemplifies Mr. Morison's conclusions. It shows Pier No. 2 of the cantilever bridge over the Mississippi River at Thebes, Ill. This bridge was designed by Noble and Modjeski, April, 1905.

Figure 188 shows a slight modification of Mr. Morison's suggestion. The McKinley Bridge over the Mississippi River at St. Louis, Mo., was designed in May, 1909. The photograph of Pier No. 3 illustrates the use of a conical top for the starling.

The dimensions required for the top of a pier are usually fixed mainly by the area of the bearings needed for the superstructure. A coping not less than 1 foot in thickness is placed on the top of the pier, projecting 3 to 6 inches beyond the top of the masonry beneath. The dimensions of the top of the pier should be such that the base plate of the superstructure shall not come within 4 to 6 inches of the edges of the masonry under the coping. The width of the top of the pier under the coping is required to be at least 4 feet, and at least 1 foot more than is needed for the base plate.

A batter of at least $\frac{1}{2}$ inch to 1 foot, or sometimes 1 inch to 1 foot, is given to the surfaces of the pier. Footing courses may be employed at the base of the pier to distribute the loads over a larger area of the foundation, being commonly stepped off, projecting about a foot horizontally and with a depth about twice the width. When of reinforced concrete the projecting steps may be designed as cantilever slabs.

Where the location is not absolutely fixed by local conditions, several possible sites may be compared as to foundations by a series of borings or core drillings taken in the line of the proposed bridge. This will determine the general suitability of each site. After the location has been selected and the pier spacing worked out, a number of holes should be put down at the site of each pier. These should

never be less than four in number, one at each corner of the pier, and if there is any great variation among these as to the depth of rock, a number of intermediate holes should be drilled.

The weight of a bridge is communicated to the pier or abutment in the following manner: The load at the end of the truss is carried on a strong, ribbed casting, known as a shoe, or pedestal. The base of this shoe rests upon a steel bearing plate, with an area of contact equal to the load divided by the safe crushing strength of the plate. Supporting this is the bridge seat, which is a block of granite or other stone of high crushing strength. The area of the plate is computed from the safe unit load on the stone. The latter rests on the pier masonry, the area of bearing between them being proportioned to distribute the load to the masonry without exceeding its safe bearing value.

Cylinder piers are frequently used when the sectional area of a single solid pier is not necessary to stability. These consist of a pair of cylinders arranged so that each may carry the ends of the trusses upon one side of the bridge, and are connected by bracing near the top to give rigidity transversely to the length of the bridge. They are either thin steel shells filled with concrete, or monolithic concrete shafts, reinforced near the outer surfaces.

245. Stability of Piers.—A masonry pier is a vertical column carrying both horizontal and transverse loads. The vertical loads carried by any horizontal section of the pier consist of the weight of the superstructure with its live load and the weight of the pier above the section considered. The effect of impact is not usually considered. although a small allowance for impact is sometimes added for the upper part of railroad bridge piers.

The wind and current pressures are horizontal forces which tend to produce bending moments in any horizontal section of the pier, and in the foundation, in a direction normal to the bridge. The wind load upon the superstructure and upon a railway train upon the bridge may be taken the same as in designing the superstructure. Wind upon the end of the pier is commonly taken at about 20 pounds per square foot of vertical section for semicircular ends, but may be reduced to 15 pounds for pointed ends, and should be increased to 30 pounds for rectangular piers.

The pressure of a current of water upon the end of a pier cannot be accurately determined; in pounds per square foot of vertical section, it is frequently taken at about $.75v^2$ (where v is the surface velocity of the stream in feet per second) for curved or pointed starlings and about twice this amount for rectangular piers. The center of

pressure is assumed to be at one-third the depth from the surface to the bottom of the stream.

The pressure exerted by ice depends upon the thickness of the ice and the shape of the upstream end of the pier, and is greatest when the ice is breaking up and a large body of floating ice is being cut by the pier. Where ice 10 or 12 inches thick may form, a pressure of 45,000 to 50,000 pounds per foot of width of pier is often assumed, considered as concentrated at the level of high water. For other thicknesses, the pressure is somewhat proportional to the thickness. Where heavy ice is likely to form, the use of ice breakers, or starlings with edges inclined to the vertical (as shown in Fig. 189) may materially decrease the pressure.

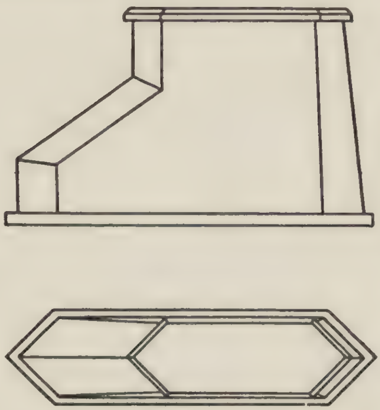


Fig. 189.—Pier with Ice-breaker.

The tractive force in the piers of a railway bridge is a horizontal force acting parallel with the length of the bridge at the level of the rail, and therefore produces moments in the horizontal sections of the pier and foundation at right angles to those due to wind and current. The tractive force is commonly taken at 2/10 of the moving load on one track.

It is essential to stability that the maximum compressive stress upon any horizontal section due to the vertical loads combined with that due to the moments of the horizontal forces shall not exceed the safe compressive strength of the masonry. The maximum unit pressure upon the foundation must not exceed a safe value. No tension should exist in the masonry at any section under any possible loading, unless it be reinforced concrete designed for tension, and compression must always exist over the whole area of the foundation.

The horizontal forces must not be sufficient to produce sliding upon any joint in the masonry or foundation, or to shear any section of concrete.

Ordinary solid piers dimensioned to give sufficient bearing area at the top and slightly battered will usually be amply strong. The distribution of loads over the foundation should, however, be carefully looked after.

Large masonry piers are frequently built hollow. The masonry under the base plates of the superstructure is considered to act as

columns which transmit the vertical loads to the foundation, and the central part of the pier is regarded as bracing to stiffen the columns and carry the lateral loads. A part of the masonry at the center of the pier may be left out without appreciably reducing its strength, thus reducing the weight upon the foundation and saving a considerable volume of masonry or concrete. Such an arrangement is shown in Fig. 190.

Hollow piers of reinforced concrete have been occasionally used. These have been designed in a number of ways, columns being used under the base plates of the superstructure, connected in some way by reinforced bracing. The exterior shape of these piers below high water is made the same as solid piers in order to produce minimum disturbance of stream flow.

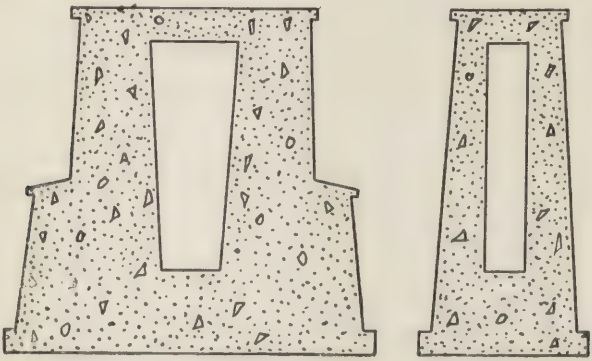


Fig. 190.—Hollow Pier.

The pier below high water is hollow, with reinforced side walls connecting the towers at the ends. Above high water, the towers are separate and connected by a reinforced arch at the top. Openings are provided through the walls to admit water to the interior spaces.

246. Construction of Piers.—Solid bridge piers are constructed of concrete or of concrete with facing of cut stone. The use of rubble masonry as backing in such work has given way to concrete on account of its lower cost and greater ease of handling.

Stone masonry facing has the advantage of presenting a pleasing appearance, and offering good resistance to the abrasion of the stream and of floating debris. In constructing a pier by its use forms are unnecessary, which frequently results in lessened cost of construction, although the cost of the masonry itself is greater than that of concrete. First-class ashlar masonry is required in such work, and the stone must be well bonded into the concrete backing. In im-

portant work carrying heavy loadings, the facing stones are tied to the concrete by the use of steel rods attached to the stretchers at frequent intervals and extending well into the concrete.

When piers are wholly of concrete, it is desirable to place light reinforcement near the surface in the face of the pier to prevent surface cracks, which usually develop in any large exposed surface of concrete. This would require horizontal bars not more than 1 foot apart, and vertical bars every 2 or 3 feet, embedded about 2 inches in the concrete. The top of the coping should be similarly reinforced.

Cylinder piers are most commonly formed by constructing a cylindrical shell of steel and filling it with concrete. Reinforced concrete cylinders are also coming into use, and have the advantage of not requiring painting to prevent rust. A pair of cylinders is generally used for a pier and they are connected by bracing near the top or at two points for high piers. This bracing may be of reinforced concrete, or sometimes a steel truss inclosed in concrete.

The masonry of a pier may be supported upon a caisson, or upon hard material or piles in a cofferdam. When the pier rests upon a caisson, a cofferdam is built upon the top of the caisson and the masonry built inside the cofferdam after filling the caisson with concrete. When the pier rests upon piles, the tops of the piles extend upward into and are inclosed by the concrete in the base of the pier. In such work, it is desirable to place reinforcement in the bottom of the footing of the pier between the piles.

247. Types of Bridge Abutments.—A bridge abutment is a combination of a pier with a retaining wall; it carries the weight of one end of the bridge with its moving load and retains the bank of earth sustaining the roadway leading to the bridge, the requirements for stability being the same as those for a retaining wall. The weight of the bridge with its live load is brought upon the abutment near the top, and the thrust of the earth filling with that of the load upon the roadway is brought upon the back of the abutment, as shown in Fig. 191. These, with the weight of the pier itself, must give a proper distribution of pressures upon the foundation and safe stresses at any point in the masonry.

On any other than a rock or very solid foundation, the resultant line of pressure should preferably lie near the center of the base than at the edge of the middle third, as is considered satisfactory for most masonry structures. This is in order to secure a uniform pressure on the soil, with no tendency to settlement towards one side. To accomplish this, the toe of the footing may be extended, the batter of the front face increased, and the load placed well to the rear.

The *type* of abutment is ordinarily designated by the means adopted to retain the end of the embankment supporting the roadway. This embankment usually has side slopes of about 1.5 horizontal

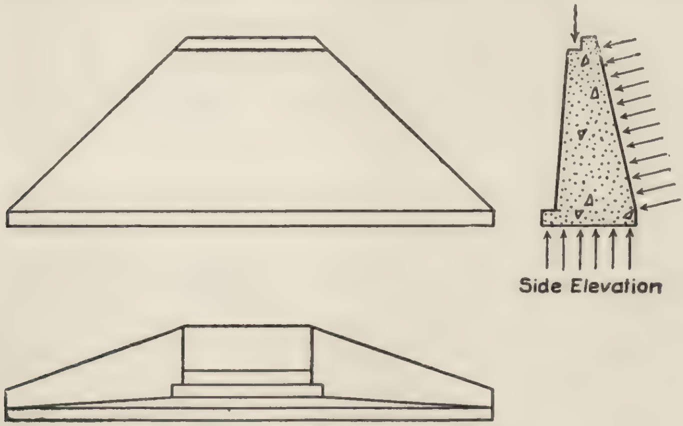


Fig. 191.—Straight Abutment.

to 1 vertical, which must be sustained by walls joined to the abutment. Abutments are therefore divided according to the method used (for

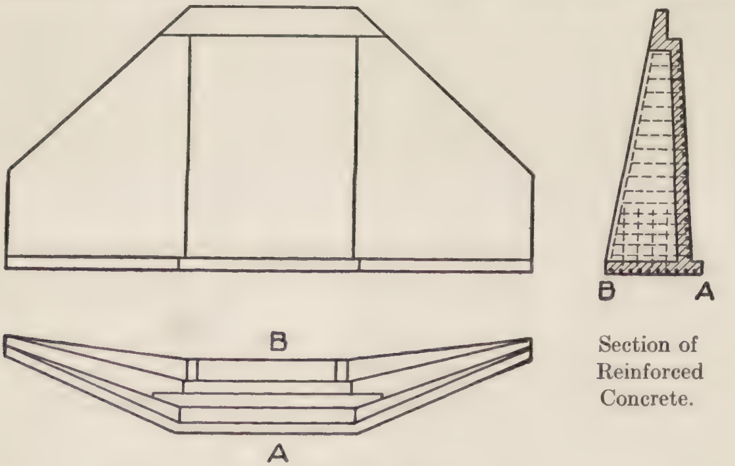


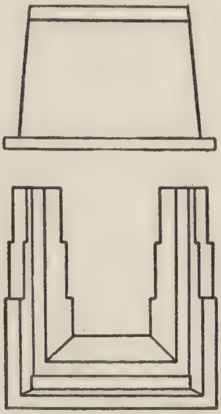
Fig. 192.—Wing Abutment.

supporting the side slopes) into straight abutments, wing abutments, U-abutments, and T-abutments.

Straight abutments are those in which the walls retaining the side slopes are continuations of the abutments in the same lines, as shown in Fig. 191.

Wing abutments are those in which the side slopes are retained by wing walls, making an angle, usually about 30°, with the face of abutment (see Fig. 192). This type of abutment is selected where a stream flows past the face of the abutment, as it disturbs the flow of the stream to the least extent and protects the abutment against the stream getting behind it. The wing walls may be shorter and require somewhat less masonry than the walls of straight abutments, when the bottom of the sloping earth is held back to the line of the face of the abutment.

U-*Abutments* are those in which the walls are turned at right angles to the abutment as shown in Fig. 193. The earth slope is then upon the face of the wall. This type may be economical when the abutment is on the edge of a bluff so that the depth of the wall may be reduced by running into the face of the bluff, and where the material is such that the footing may be stepped up toward the rear of the abutment as shown.



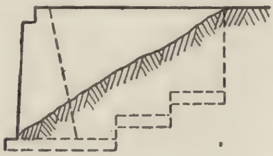


Fig. 193.—U-abutment.

The U-abutment is used very effectively in the architectural treatment of monumental masonry arches, in which a heavy balustrade commences at the end of the abutment and is carried straight through to the opposite side.

The T-*abutment* is provided with only one wall, which lies directly under the roadway, and therefore makes no effort to confine the earth of the embankment, which surrounds the abutment, and slopes away from its face towards the bed of the stream. Like the U-abutment, this type may be economical when built into a steep bank.

The *buried abutment* comprises a simple pier to support the end of the bridge, and as in the above type, the earth fill is not supported, but spills around the front face. In the design, some weight may be saved by deducting from the earth pressure on the back of the wall, the compensating effect of the fill in front. Sometimes this abutment is combined with a pier at the foot of the slope in front, with a short span between the two. This reduces the length of main span otherwise required. Such combination abutments have been much used in highway crossings of the New York Barge Canal.

The use of reinforced concrete has made possible a number of

variations in the standard types of abutments. In most of these there is a considerable economy of masonry, since the earth filling supplies a large portion of the weight required for stability.

The ordinary wing abutment may be built of reinforced concrete, as shown in Fig. 192. The design is similar to that of a retaining wall with counterforts, except that these latter also act as columns in supporting the bridge seat and the loads carried on it. This type is much cheaper than the mass abutment where the height is considerable.

There is also a *cellular* or *box abutment* of reinforced concrete that is very economical of material, much of its weight being secured by the earth filling.

A novel design is the combination of the U-abutment with a relieving platform, or deck, such as is used in bulkhead walls, near the top. This deck carries the fill of the roadway, and relieves the front wall of all pressure, the earth sloping from the rear of the platform to the foot of the wall. The front and side walls may therefore be pierced by arches, leaving columns between to carry the deck. The result is a light, open structure, affording a great saving of material.

"Foundations of Bridges and Buildings," Second Edition, by Jacoby and Davis, New York, 1925, gives a complete description of the various methods of constructing foundations, with detailed descriptions of many important constructions.

A very complete work on this subject, "Engineering and Building Foundations," in three volumes, by Charles Evan Fowler, is in preption. Vol. I—"Ordinary Foundations," was published in 1920.

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